2.2 The Limit of a Function

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Suppose $f(x)$ is defined when x is near the number a. (This means that f is defined on some open interval that contains a, except possibly at a itself.) Then we write

$$\lim_{x \to a} f(x) = L$$

$$\lim_{\boxed{x \to a}} f(x) = L$$

and say

> "the limit of $f(x)$, as x approaches a, equals L"

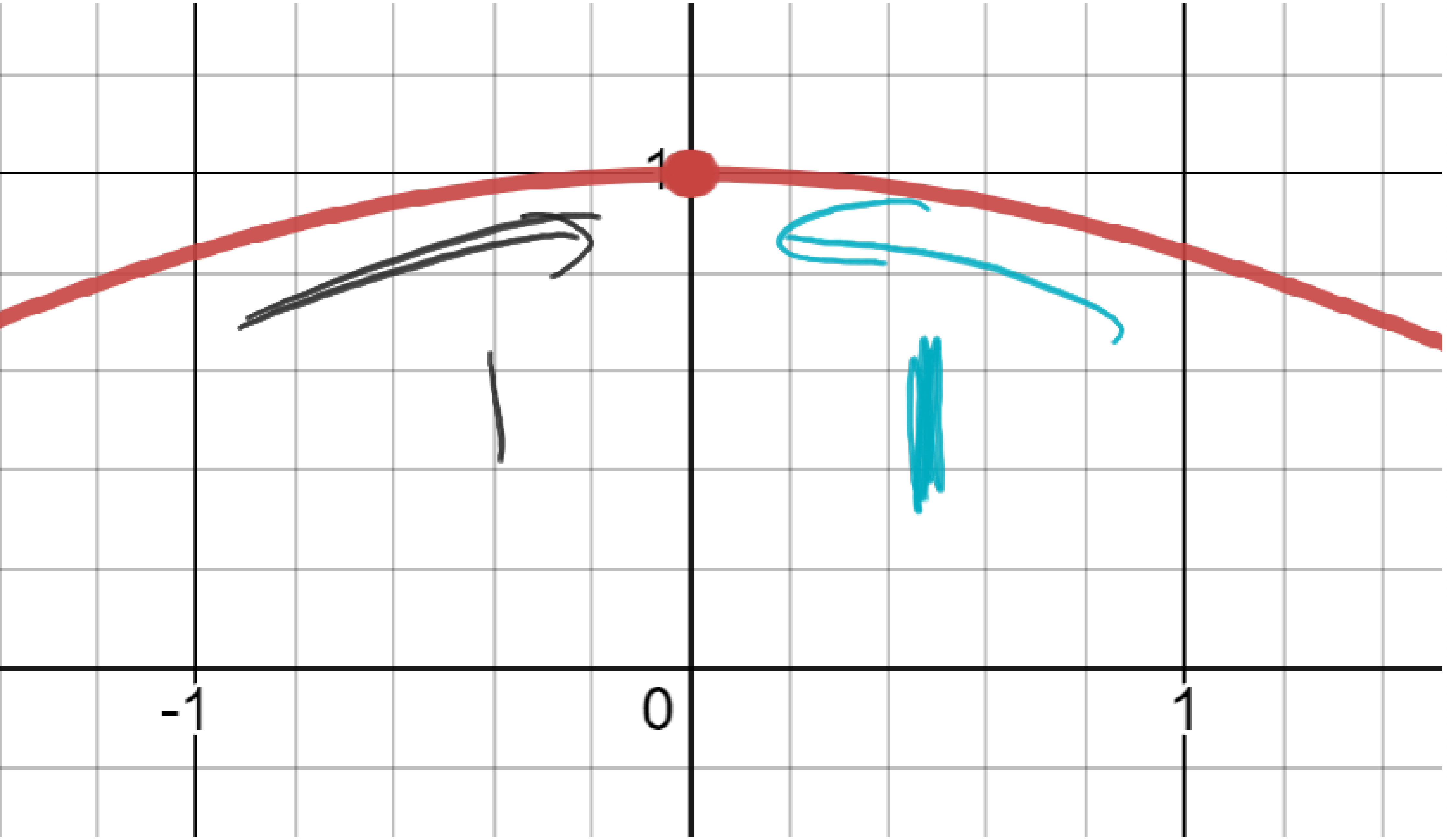
if we can make the values of $f(x)$ arbitrarily close to L (as close to L as we like) by restricting x to be sufficiently close to a (on either side of a) but not equal to a.

e of $\lim_{t \to 0} \frac{\sqrt{t^2+9}-3}{t^2} = \frac{1}{6}$

$0.1\overline{66}$

x	$\frac{\sqrt{x^2+9}-3}{x^2}$
0.1	0.1666204
0.01	0.1666662
0.001	0.16666666
0.0001	0.16666668
0.00001	0.16666668

ue of $\lim_{x \to 0} \frac{\sin x}{x} = 1$



x_1	$\frac{\sin(x_1)}{x_1}$
-0.01	0.99998333
-0.001	0.99999983
-0.0001	1
0	undefined
0.0001	1
0.001	0.99999983
0.01	0.99998333

Find $\lim_{x \to 0} \left(x^3 + \frac{\cos 5x}{10,000} \right) = \frac{1}{10000}$

$= .0001$

$= 1 \times 10^{-4}$

x_1	$x_1^3 + \frac{\cos 5x_1}{10000}$
-0.01	9.887503×10^{-5}
-0.001	9.999775×10^{-5}
-0.0001	9.999999×10^{-5}
0	1×10^{-4}
0.0001	9.999999×10^{-5}

We write

$$\lim_{x \to a^-} f(x) = L$$

$$\lim_{x \to a^-} f(x) = L$$

Left

and say that the **left-hand limit of** $f(x)$ as x approaches a [or the limit of $f(x)$ as x approaches a *from the left*] is equal to L if we can make the values of $f(x)$ arbitrarily close to L by restricting x to be sufficiently close to a with x *less than* a.

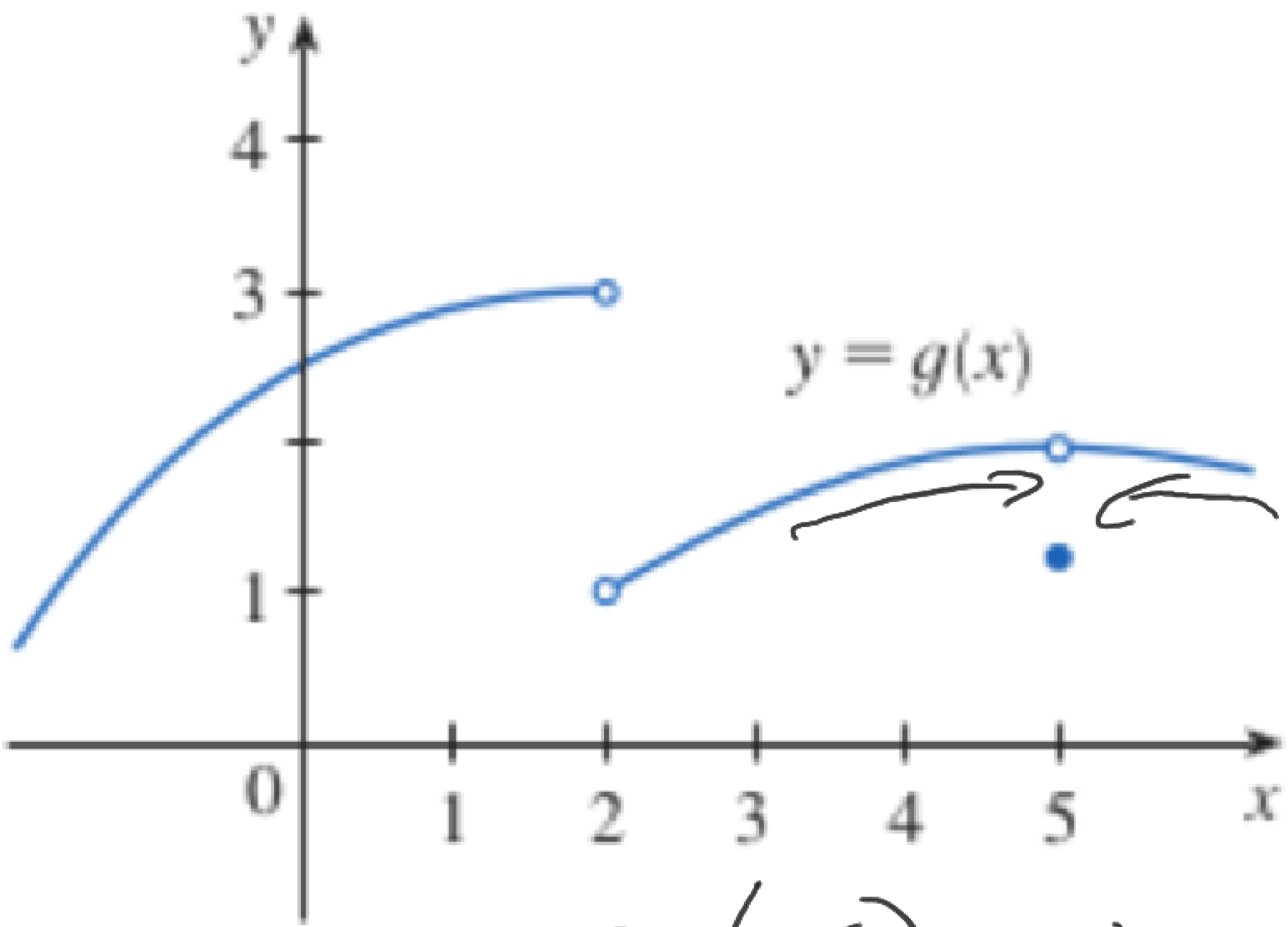
We write

$$\lim_{x \to a^+} f(x) = L$$

$$\lim_{x \to a^+} f(x) = L$$

Right

and say that the **right-hand limit of** $f(x)$ as x approaches a [or the limit of $f(x)$ as x approaches a *from the right*] is equal to L if we can make the values of $f(x)$ arbitrarily close to L by restricting x to be sufficiently close to a with x *greater than* a.



$g(5) = 1$

(a) $\lim_{x \to 2^-} g(x) = 3$

(b) $\lim_{x \to 2^+} g(x) = 1$

(c) $\lim_{x \to 2} g(x) = \text{DNE}$

(d) $\lim_{x \to 5^-} g(x) = 2$

(e) $\lim_{x \to 5^+} g(x) = 2$

(f) $\lim_{x \to 5} g(x) = 2$

e $\lim_{x \to 0} \sin \frac{\pi}{x}$

DNE

Find $\lim_{x \to 0} \dfrac{1}{x^2}$ if it exists.

$$\lim_{x \to 0} \frac{1}{x^2} \to \infty$$

Does the curve $y = \dfrac{2x}{x-3}$ have a vertical asymptote?

$$\lim_{x \to 3} \frac{2x}{x-3} = \text{DNE or } \infty$$

$$\lim_{x \to 3^-} = \quad \begin{array}{c} x \\ 2.9 \\ 2.99 \\ 2.999 \end{array} \qquad \lim_{x \to 3^+} = \quad \begin{array}{c} 3.1 \\ 3.01 \\ 3.001 \end{array}$$

Does the curve $y = \dfrac{2x}{x-3}$ have a

$$\lim_{x \to 3} \frac{2x}{x-3} = \text{DNE}$$

V.A. $\to x = 3$

x	$\dfrac{2x}{x-3}$
2.9	−58
2.99	−598
2.999	−5998
3	undefined
3.001	6002
3.01	602
3.1	62

$$\lim_{x \to 4} \frac{\ln x - \ln 4}{x - 4}$$

$$\frac{\ln(x) - \ln(4)}{x - 4}$$

x	$\frac{\ln(x) - \ln(4)}{x - 4}$
3.9	0.25317808
3.99	0.25031302
3.999	0.25003126
4	undefined
4.001	0.24996876
4.01	0.24968802
4.1	0.24692613

$$\lim_{x\to(\pi/2)^+} \frac{1}{x} \sec x$$

$L = -\infty$

$\frac{\pi}{2}$	undefined
$\frac{\pi}{2} + 0.0001$	-6365.7925
$\frac{\pi}{2} + 0.0001$	-6365.7925
$\frac{\pi}{2} + 0.01$	-63.260311
$\frac{\pi}{2} + 0.01$	-63.260311
$\frac{\pi}{2} + 0.1$	-5.9951569
$\frac{\pi}{2} + 1$	-0.46226731

$$\lim_{x\to 0}\left(x^2 - \frac{2^x}{1000}\right) = -\frac{1}{1000}$$

$\lim_{x \to 7}$

6.9

6.99

6.99.

7

7.001

7.01

7.1

$$\lim_{x \to 0}$$

$-0{,}1$

$-0{,}01$

$-0{,}001$

0

$0{,}001$

$0{,}01$

$0{,}1$