2.3. Calculating Limits Using the Limit Laws

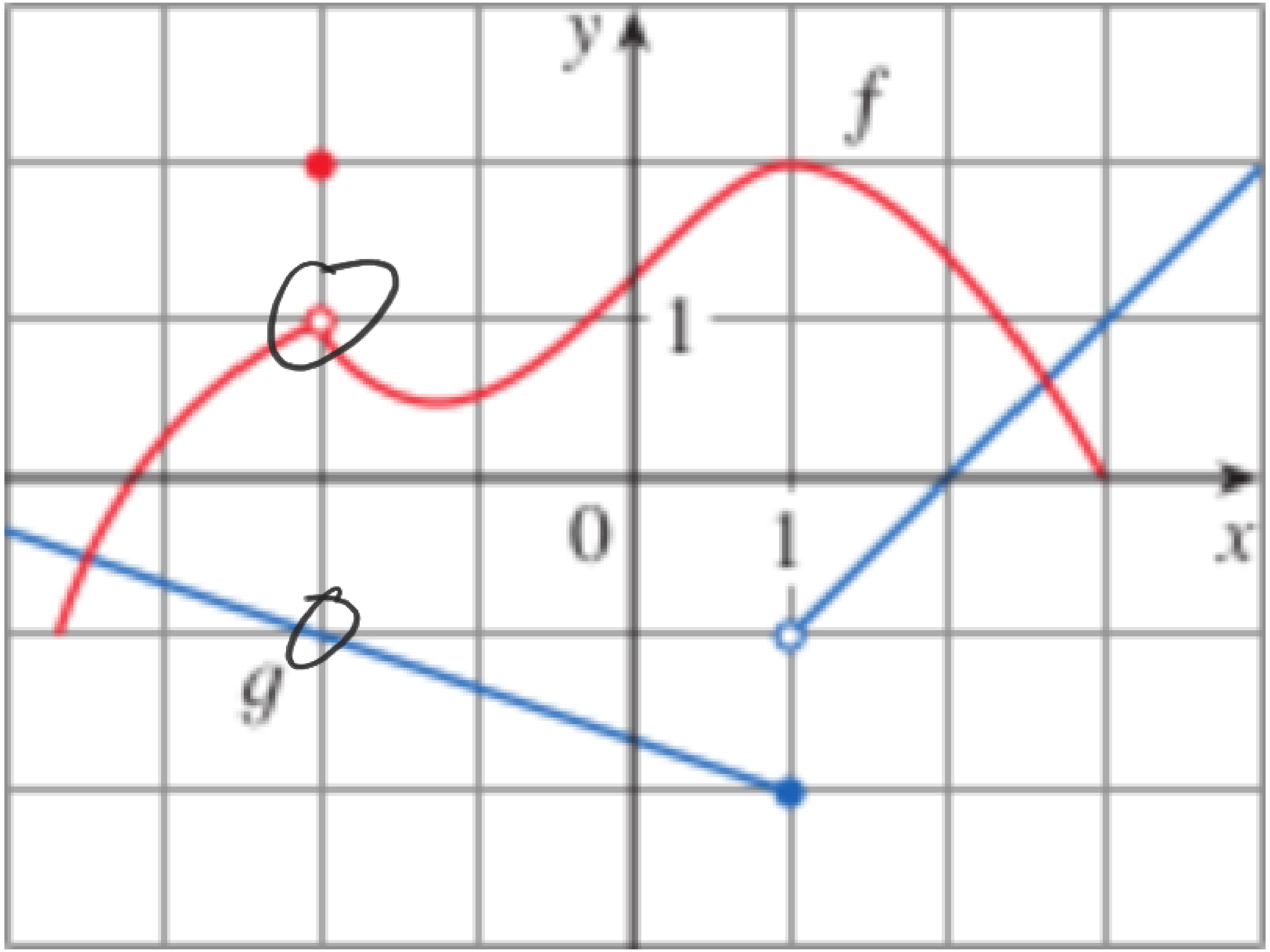
1. $$\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

2. $$\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$$

3. $$\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$$

4. $$\lim_{x \to a} [f(x)g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$

5. $$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \text{ if } \lim_{x \to a} g(x) \neq 0$$



(a) $\lim\limits_{x \to -2} [f(x) + 5g(x)]$

$1 + 5(-1)$

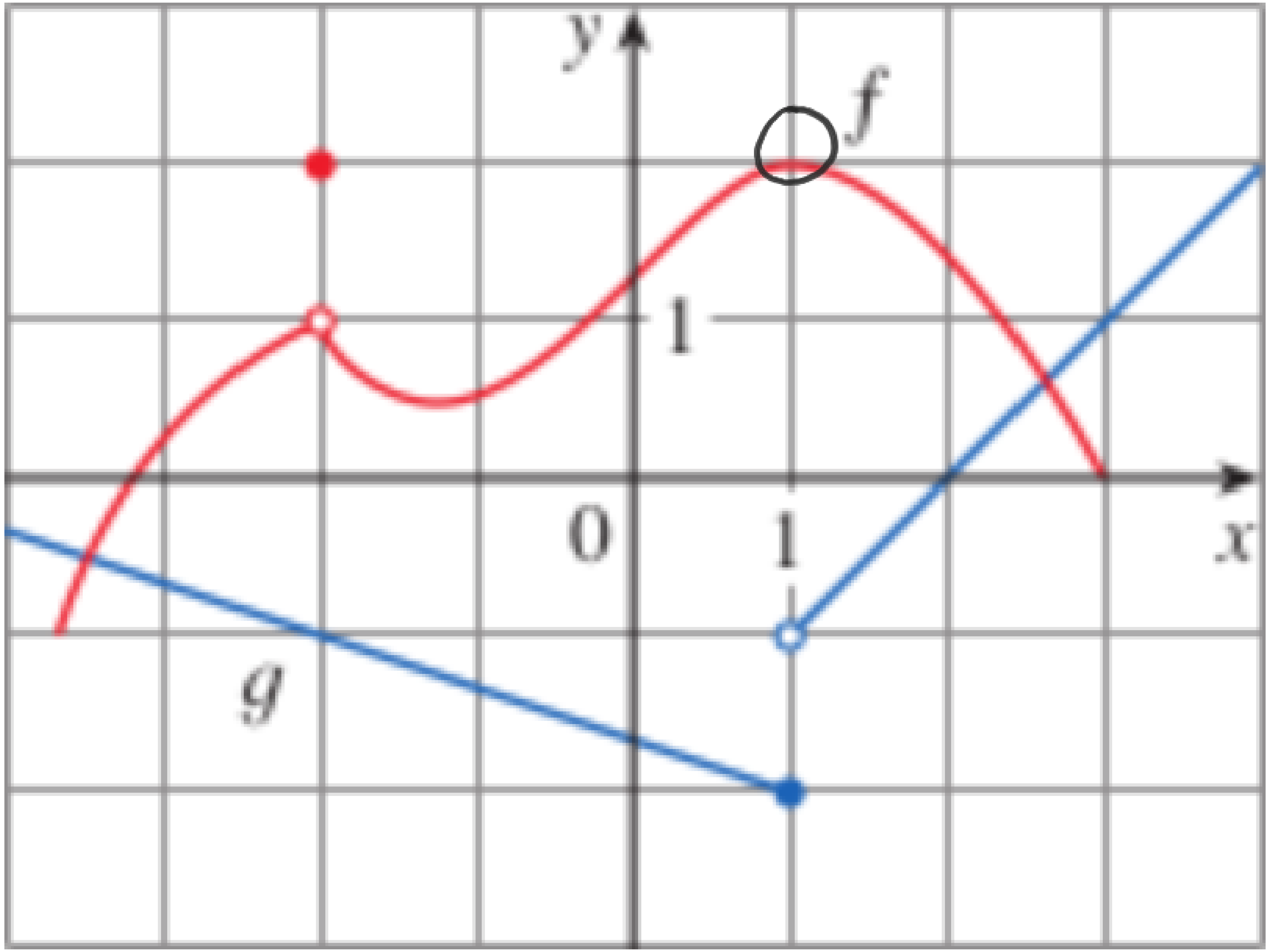
(b) $\lim\limits_{x \to 1} [f(x)g(x)]$

(c) $\lim\limits_{x \to 2} \dfrac{f(x)}{g(x)}$

a) $\lim\limits_{x \to -2} f(y) = 1$

$\lim\limits_{x \to -2} g(x) = -1$

$a = -4$



(a) $\lim_{x \to -2} [f(x) + 5g(x)]$

(b) $\lim_{x \to 1} [f(x)g(x)]$ DNE

(c) $\lim_{x \to 2} \frac{f(x)}{g(x)}$

$\lim_{x \to 1} f(x) = 2$

$\lim_{x \to 1} g(x) = \text{DNE}$

6. $\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$ where n is a positive integer

7. $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$ where n is a positive integer

8. $\lim_{x \to a} c = c$

9. $\lim_{x \to a} x = a$

10. $\lim_{x \to a} x^n = a^n$ where n is a positive integer

11. $\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$ where n is a positive integer

$$\lim_{x \to 5} (2x^2 - 3x + 4) = 39$$

$$2(5)^2 - 3(5) + 4$$

$$50 - 15 + 4$$

$$\boxed{39}$$

$$\lim_{x \to -2} \frac{x^3 + 2x^2 - 1}{5 - 3x} = -\frac{1}{11}$$

$$\lim_{x \to -2} \frac{x^3 + 2x^2 - 1}{5 - 3x} = \frac{\lim_{x \to -2} (x^3 + 2x^2 - 1)}{\lim_{x \to -2} (5 - 3x)} \qquad \text{(by Law 5)}$$

$$= \frac{\lim_{x \to -2} x^3 + 2 \lim_{x \to -2} x^2 - \lim_{x \to -2} 1}{\lim_{x \to -2} 5 - 3 \lim_{x \to -2} x} \qquad \text{(by 1, 2, and 3)}$$

$$= \frac{(-2)^3 + 2(-2)^2 - 1}{5 - 3(-2)} \qquad \text{(by 10, 9, and 8)}$$

$$= -\frac{1}{11}$$

Find $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1}$.

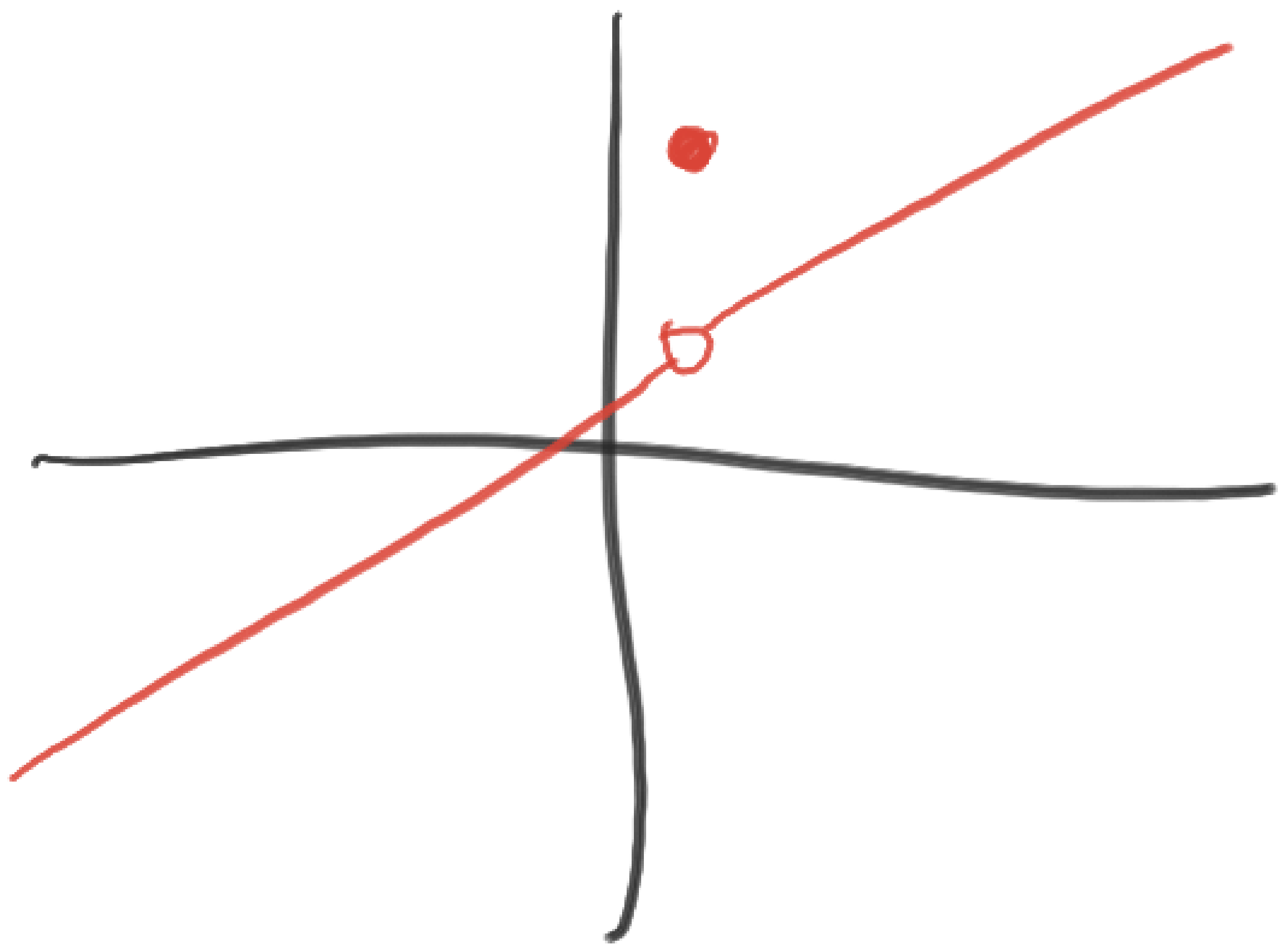
$$\frac{(1)^2 - 1}{1 - 1} = \frac{0}{0}$$

$$\frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} = x + 1$$

$$\lim_{x \to 1} x + 1 = \boxed{2}$$

$$g(x) = \begin{cases} x+1 & \text{if } x \neq 1 \\ \pi & \text{if } x = 1 \end{cases}$$

$$\lim_{x \to 1} g(x) = 2$$



$$\lim_{x \to 2} h(x) = 1$$

$$h(x) = \begin{cases} x^2 - 3 & x \neq 2 \\ 5 & x = 2 \end{cases}$$

$$\lim_{\boxed{x \to 0}} h(x) = -3$$

Evaluate $\lim_{h\to 0} \frac{(3+h)^2 - 9}{h} = 6$

$(3+h)^2 = (3+h)(3+h) = 9 + 6h + h^2$

$9 + 6h + h^2 - 9 = \frac{6\cancel{h} + h^{2}}{\cancel{h}} = 6 + h$

$\frac{\cancel{h}(6+h)}{\cancel{h}}$

$\lim_{h\to 0} 6 + h = 6$

Find $\displaystyle \lim_{t \to 0} \frac{\sqrt{t^2+9}-3}{t^2} \cdot \frac{\left(\sqrt{t^2+9}+3\right)}{\left(\sqrt{t^2+9}+3\right)}$

$$\left(\sqrt{t^2+9}\right)^2 \cancel{+3\sqrt{t^2+9}} \cancel{-3\sqrt{t^2+9}} - (3)^2$$

$$t^2+9-9 = t^2$$

$$\text{Find } \lim_{t\to 0} \boxed{\frac{\sqrt{t^2+9}-3}{t^2} \cdot \frac{(\sqrt{t^2+9}+3)}{(\sqrt{t^2+9}+3)}}$$

$$\lim_{t\to 0} \frac{\cancel{t^2}}{\cancel{t^2}(\sqrt{t^2+9}+3)} = \frac{1}{\sqrt{t^2+9}+3}$$

$$= \boxed{\frac{1}{6}}$$

1 Theorem

$$\lim_{x \to a} f(x) = L$$

if and only if

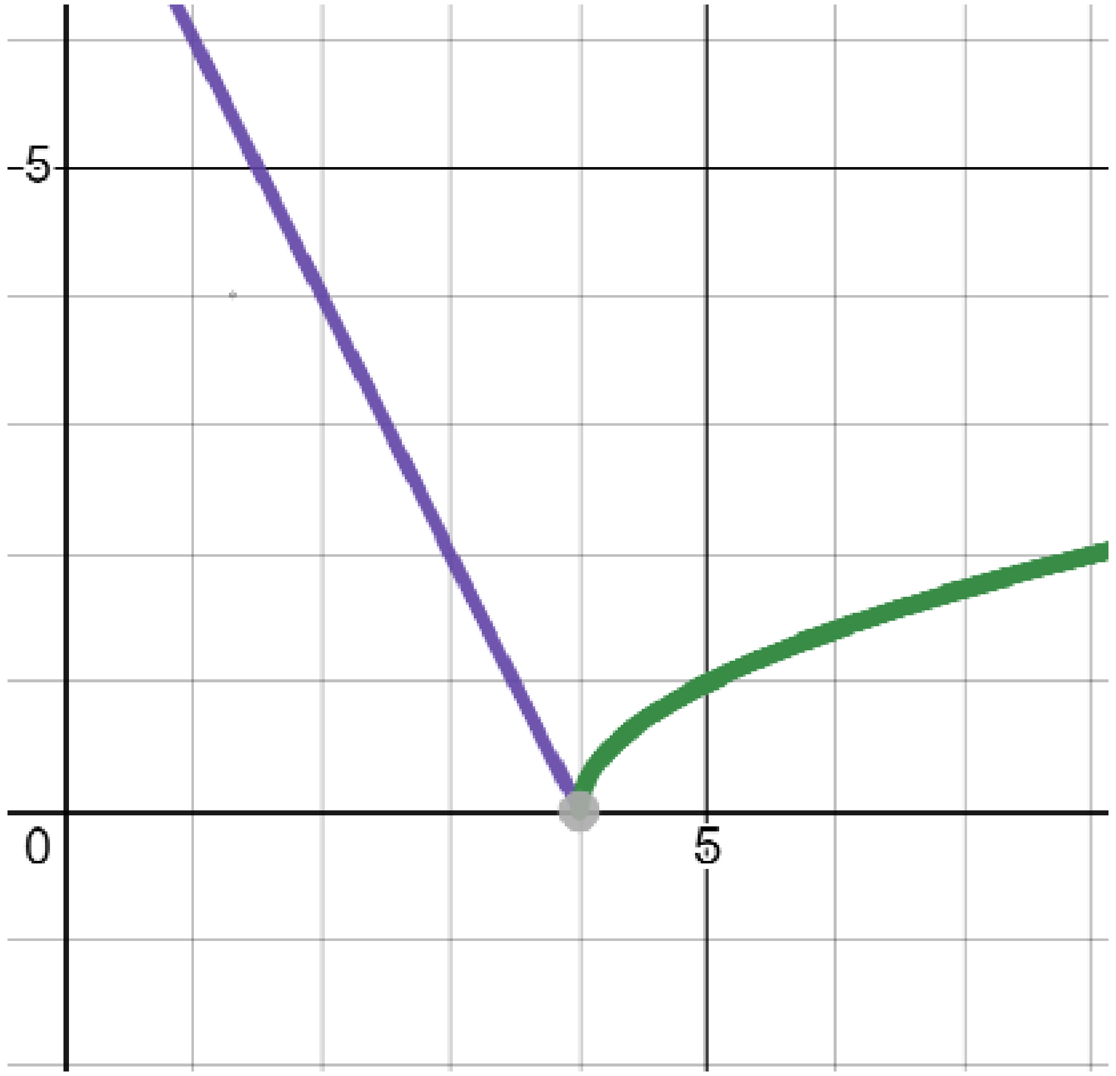
$$\lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$$

$$f(x) = \begin{cases} \sqrt{x-4} & \text{if } x > 4 \quad R \\ 8-2x & \text{if } x < 4 \quad L \end{cases}$$

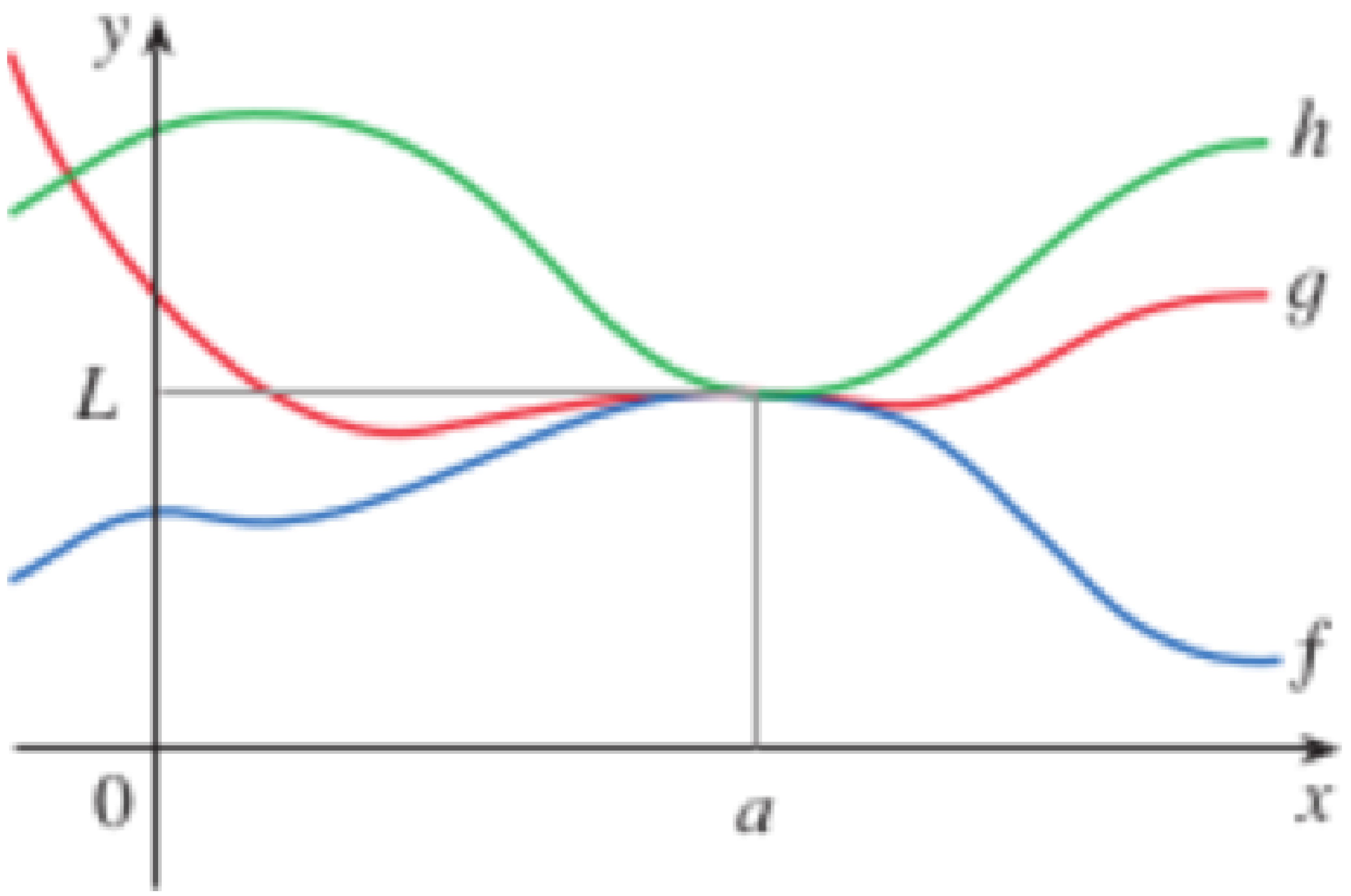
$$\lim_{x \to 4} f(x) = 0$$

$$\lim_{x \to 4^+} f(x) = \sqrt{x-4} = 0$$

$$\lim_{x \to 4^-} f(x) = 8-2x = 0$$



The Squeeze Theorem



Show that $\lim_{x \to 0} x^2 \sin \frac{1}{x} = 0$.

$$\left(\lim_{x \to 0} x^2\right)\left(\lim_{x \to 0} \sin \frac{1}{x}\right)$$

Show that $\lim_{x \to 0} x^2 \boxed{\sin \frac{1}{x}} = 0$.

$$-1 \le \sin \frac{1}{x} \le 1$$

$$-x^2 \le \boxed{x^2 \sin \frac{1}{x}} \le x^2$$

$$\lim_{x \to 0} -x^2 \le \lim_{x \to 0} x^2 \sin \frac{1}{x} \le \lim_{x \to 0} x^2$$

$$0 \le \lim_{x \to 0} x^2 \sin 1/x \le 0$$

$$\lim_{x\to 2} \frac{2-x}{\sqrt{x+2}-2}$$

$$\lim_{t\to 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t}\right)$$

$$\lim_{x\to -4} (|x+4| - 2x) = 8$$

$$\lim_{x\to 2} \frac{2-x}{(\sqrt{x+2}-2)} \frac{(\sqrt{x+2}+2)}{(\sqrt{x+2}+2)} = x-2$$

$$\lim_{x\to 2} \frac{(2-x)(\sqrt{x+2}+2)}{\cancel{x-2}} \qquad -1(\cancel{x-2})$$

$$\lim_{x\to 2} -1(\sqrt{x+2}+2) = -4$$

$$\lim_{t\to 0}\left(\frac{1}{t\sqrt{1+t}}-\frac{1}{t}\right) \to \frac{1-\sqrt{1+x}}{x\sqrt{1+x}}$$

$$\lim_{x\to 0} \frac{1-\sqrt{1+x}}{x\sqrt{1+x}} \quad \frac{(1+\sqrt{1+x})}{(1+\sqrt{1+x})}$$

$$\lim_{x\to 0} \frac{1-(1+x)}{(x\sqrt{1+x})(1+\sqrt{1+x})} \leftarrow -x$$

$$= \frac{-1}{(\sqrt{1+x})(1+\sqrt{1+x})} = \frac{-1}{2}$$