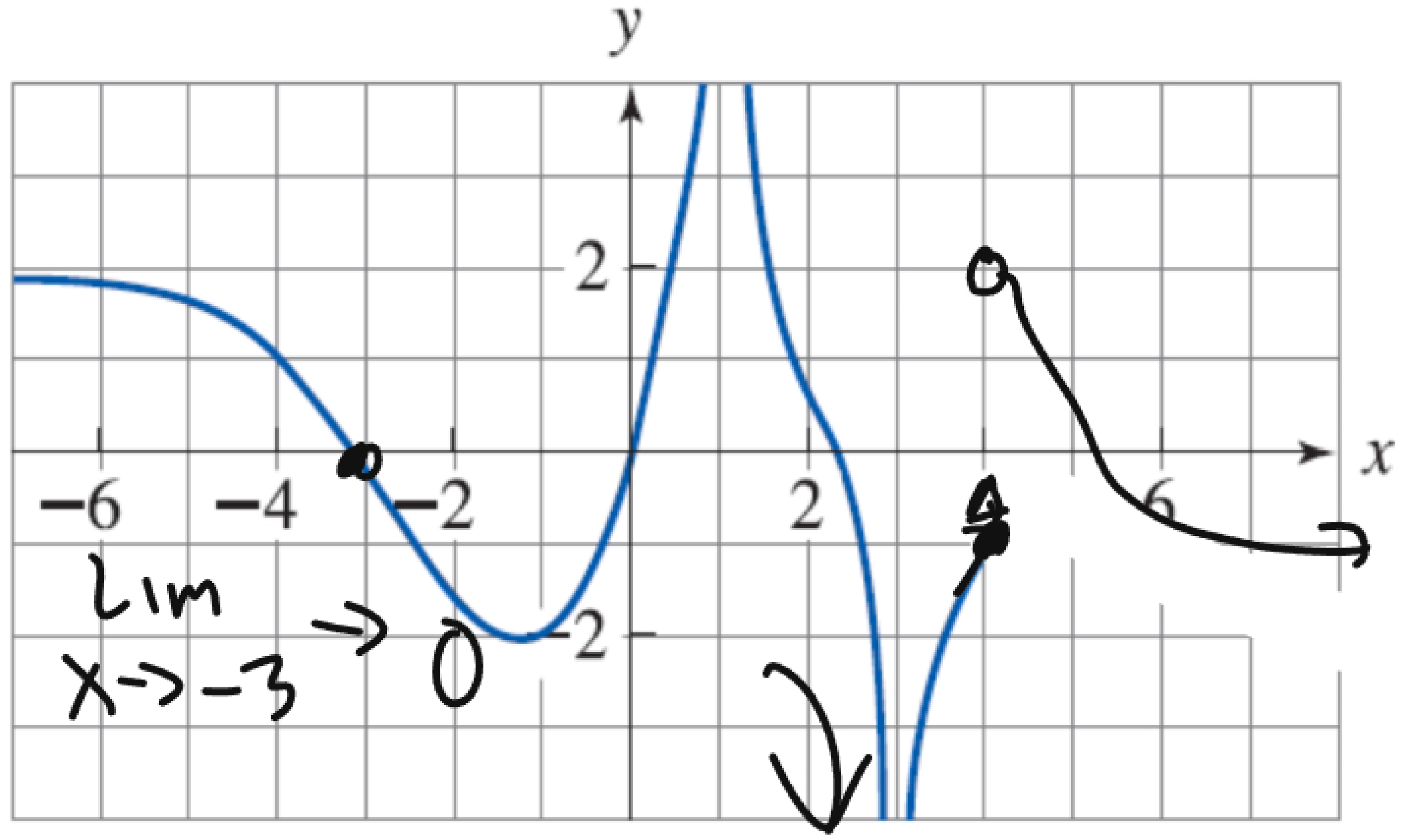
1) For the function f whose graph is given, state the following:



$\lim_{x \to \infty} f(x) \to -2$
Horizontial Asymptote

$\lim_{x \to -\infty} f(x) \to 2$
H.A.

a) $\lim_{x \to \infty} f(x)$
b) $\lim_{x \to -\infty} f(x)$
c) $\lim_{x \to 1} f(x) \to \infty$ VA
d) $\lim_{x \to 3} f(x) \to -\infty$ VA
e) What are the asymptotes of the function?

$y = -2$ Asy
$y = 2$
$x = 1$ $x = 3$

$\lim_{x \to 1} \to \lim_{x \to 1^+} \infty$

$\lim_{x \to 1^-} \infty$

2) $g(x) = \frac{5}{8}x^2 - 8x + 17$

$f(x) = x^n$

$f'(x) = nx^{n-1}$

$g'(x) = \frac{5}{8}(2)x^{2-1} - 8(1)x^{1-1}$

$g'(x) = \frac{5}{4}x - 8$

3) $f(x) = (x + 7\sqrt{x})e^{x}$

$f'g + fg'$

$f = (x + 7\sqrt{x}) = x + 7\boxed{x^{1/2}} \xrightarrow{d/dx} \frac{1}{2\sqrt{x}}$

$f' = 1 + \frac{7}{2\sqrt{x}}$

$g = e^{x}$

$g' = e^{x}$

$f'(x) = \left(1 + \frac{7}{2\sqrt{x}}\right)e^{x} + (x + 7\sqrt{x})\, e^{x}$

$= e^{x}\left(1 + \frac{7}{2\sqrt{x}} + x + 7\sqrt{x}\right)$

$$x-3 \qquad \frac{f'g - fg'}{g^2}$$

$$f = \sqrt[3]{x} = x^{1/3} \to f' = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$$

$$g = x-3 \to g' = 1$$

$$m'(x) = \frac{\left(\frac{1}{3}x^{-2/3}\right)(x-3) - \left(x^{1/3}\right)(1)}{(x-3)^2}$$

$$= \frac{\boxed{x^{1/3}\left[\left(\frac{1}{3}x^{-1}\right)(x-3) - 1\right]}}{}$$

$$\boxed{= \frac{\sqrt[3]{x}\left(\frac{1}{3} - \frac{1}{x} - 1\right)}{(x-3)^2}}$$

$$x^2(x-3) - x^2$$

$$x^2((x-3) - 1)$$

$$5)\quad y=\frac{2x}{9-\tan(x)}$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}(\sin x)=\cos x \qquad \frac{d}{dx}(\csc x)=-\csc x\cot x$$

$$\frac{d}{dx}(\cos x)=-\sin x \qquad \frac{d}{dx}(\sec x)=\sec x\tan x$$

$$\frac{d}{dx}(\tan x)=\sec^2 x \qquad \frac{d}{dx}(\cot x)=-\csc^2 x$$

$f=2x \Rightarrow f'=2$

$g=9-\tan x \Rightarrow g'=-\sec^2 x$

$$y'=\frac{2(9-\tan x)-(2x)(-\sec^2 x)}{(9-\tan x)^2}$$

$$y'=\frac{18-2\tan x+2x\sec^2 x}{(9-\tan x)^2}$$

$$g = \left(\frac{x^3-2}{x^3+2}\right)$$

$$n = x^3 - 2 \rightarrow n' = 3x^2$$
$$d = x^3 + 2 \rightarrow d' = 3x^2$$

$$g' = \frac{12x^2}{(x^3+2)^2}$$

$$3x^2(x^3+2) - (3x^2)(x^3-2)$$
$$3x^5 + 6x^2 - 3x^5 + 6x^2 = 12x^2$$

$$h'(x) = \boxed{\left(\frac{12x^2}{(x^3+2)^2}\right)}\boxed{(8)\left(\frac{x^3-2}{x^3+2}\right)^7}$$

$$h'(x) = \boxed{\frac{96x^2}{(x^3+2)^2}\left(\frac{x^3-2}{x^3+2}\right)^7} = \frac{96x^2(x^3-2)^7}{(x^3+2)^9}$$

$$F'(x) = \boxed{f'(g(x))} \cdot \boxed{g'(x)}$$

Derivatives of Inverse Trigonometric Functions

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$$

$$\boxed{\frac{d}{dx}(\ln x) = \frac{1}{x}}$$

$$\frac{d}{dx}(b^x) = \ln(b)\, b^x$$

$$6xy' + 24yy' = -2x - 6y$$

$$y'(6x+24y) = -2x-6y$$

$$y' = \frac{-2x-6y}{6x+24y} \qquad (2, 1)$$

$$y' = \frac{-2(2)-6(1)}{6(2)+24(1)} = \frac{-10}{36} = \boxed{\frac{-5}{18}}$$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - 1 = \frac{-5}{18}(x-2)}$$

$$y = mx + b$$

$$1 = -\frac{5}{18}(2) + b$$

$$1 = \frac{-5}{9} + b$$

$$\frac{14}{9} = b$$

$$\boxed{y = \frac{-5}{18}x + \frac{14}{9}}$$

8) Find the limit:

$$\lim_{x \to 1} \frac{\sin(x-1)}{x^3+2x-3} \to \frac{\sin(0)}{(1)^3+2(1)-3} = \frac{0}{0}$$

$$\sin(x-1) \to \cos(x-1)$$

$$x^3+2x-3 \to 3x^2+2$$

$$\lim_{x \to 1} \frac{\cos(x-1)}{3x^2+2} \to \frac{\cos(0)}{5} = \boxed{\frac{1}{5}}$$

9) Given the function $f(x) = \frac{x-4}{x^2}$ answer the following questions:

a. What is the domain of the function?
b. What is the x-intercepts? $y = 0$
c. What is the y-intercepts? $x = 0$
d. Find the interval on which *f* is increasing?
e. Find the interval on which *f* is decreasing?

first derivative

f. Find the interval on which *f* is concave up?
g. Find the interval on which *f* is concave down?

second derivative

h. Find the local maximum for *f*?
i. Find the local minimum for *f*?

first derivative

$$f(x) = \frac{x-4}{x^2}$$

1) cant divide by zero

2) even roots ≥ 0

3) logs > 0

$x^2 \neq 0 \rightarrow x \neq 0$

$(-\infty, 0) \cup (0, \infty)$

$$f(x) = \frac{x-4}{x^2}$$

$$f(0) = \frac{0-4}{0} = \frac{-4}{0} \text{ DNE}$$

No y-Int

$$0 = \frac{x-4}{x^2} \rightarrow 0 = x-4 \quad x=4$$

X-Intercept

$$f(x) = \frac{x-4}{x^2}$$

$$\lim_{x \to \infty} \frac{x-4}{x^2} \overset{H}{=} \frac{1}{2x} \to 0$$

$$\lim_{x \to -\infty} \frac{x-4}{x^2} \overset{H}{=} \frac{1}{2x} \to 0$$

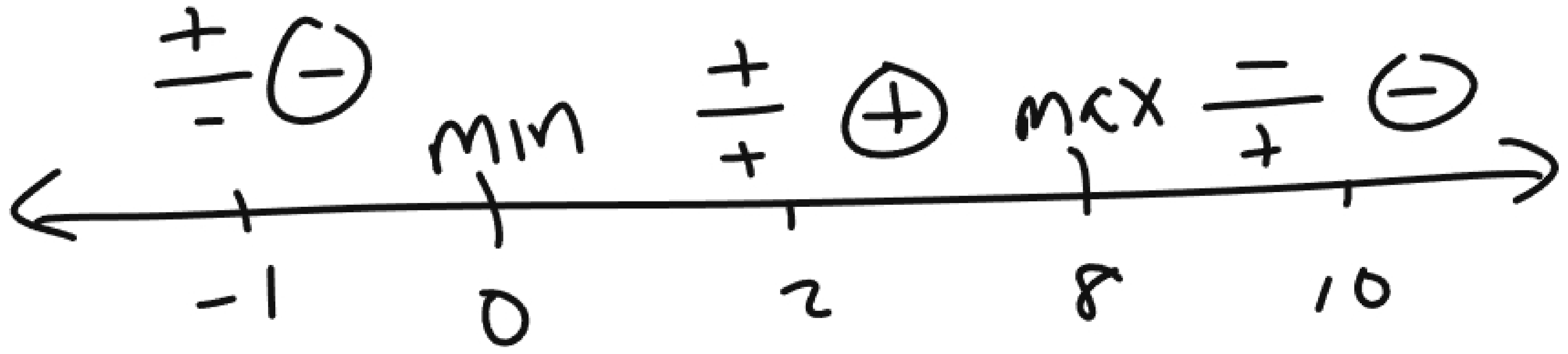
$$f(x)=\frac{x-4}{x^2} \qquad f'(x)=\frac{(1)(x^2)-(2x)(x-4)}{x^4}$$

$$= x^2-2x^2+8x = -x^2+8x$$

$$f'(x)=\frac{x(8-x)}{x^4}=\frac{8-x}{x^3}$$

$$0=\frac{8-x}{x^3} \to 0=8-x \to x=8$$

$x=0$ DNE



D $\to (-\infty, 0) \cup (8, \infty)$

I $\to (0, 8)$

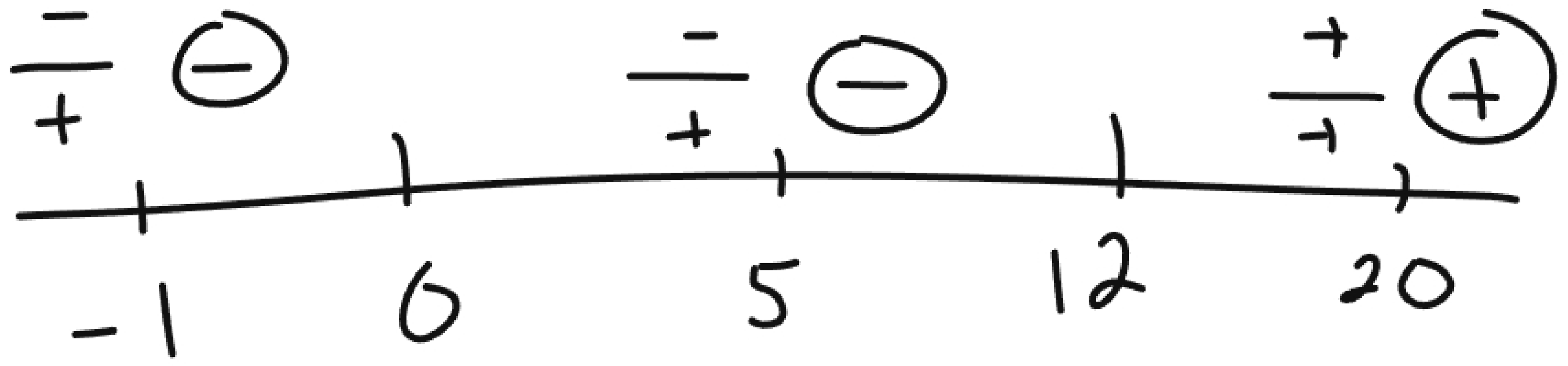
no min $\qquad f(x)=\frac{x-4}{x^2} \qquad$ max

$$= \frac{2x^3 - 24x^2}{x^6} = \frac{x^2(2x-24)}{x^6}$$

$$f''(x) = \frac{2x-24}{x^4}$$

$$0 = \frac{2x-24}{x^2} \rightarrow 2x - 24 = 0 \rightarrow x = 12$$

DNE $x = 0$



Concave down $(-\infty, 0) \cup (0, 12)$

up $(12, \infty)$