2.5. Continuity

Continuity of a Function

Press Record!

1 Definition

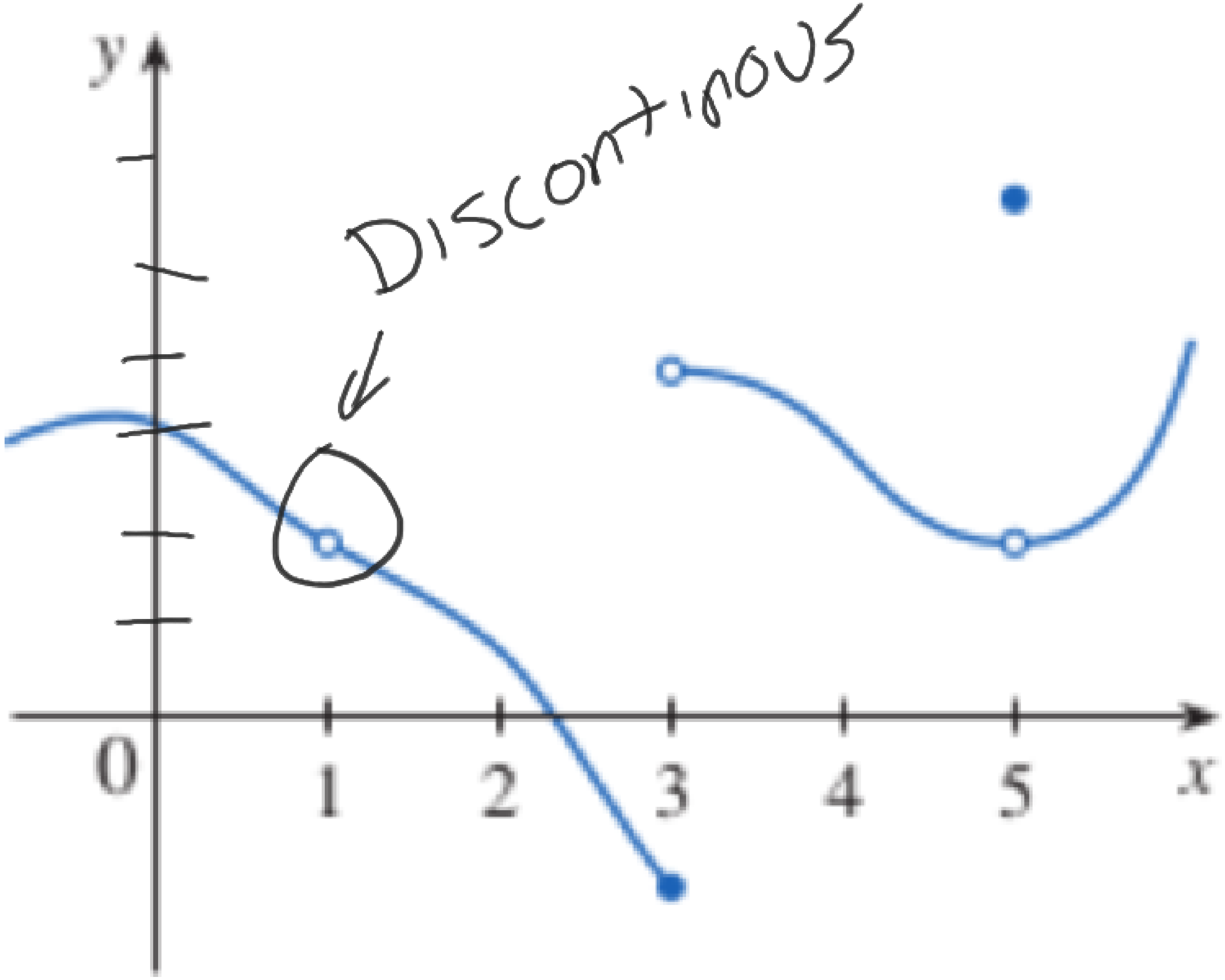
A function f is **continuous at a number** $\boldsymbol{a}$ if

$$\lim_{x \to a} f(x) = f(a)$$

Notice that Definition 1 implicitly requires three things if f is continuous at a :

1. $f(a)$ is defined (that is, a is in the domain of f)
2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

the graph of a function f . At which numbers is f discontinuous? Why?



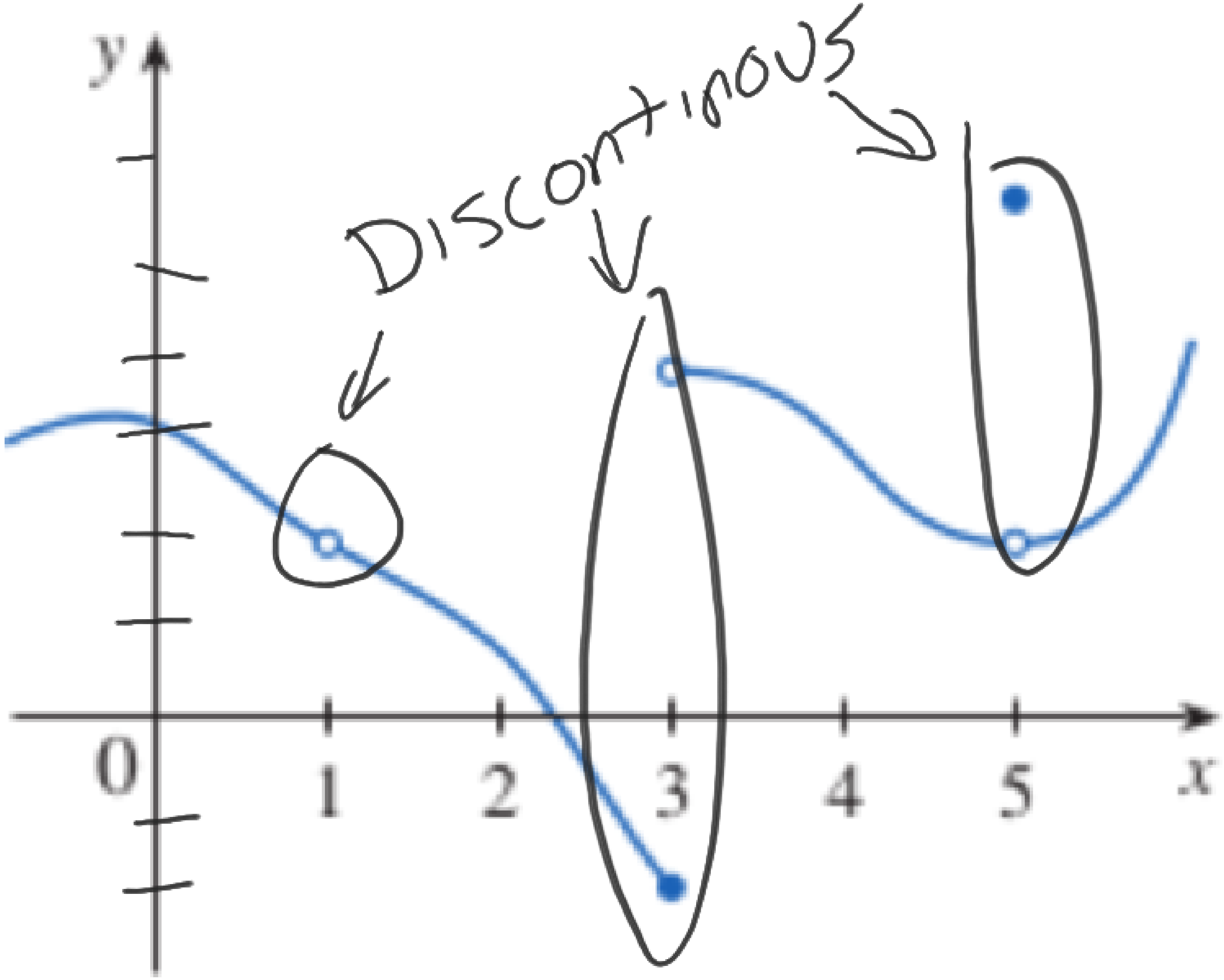
$a = 1, 3, 5$

$\lim_{x \to a} f(x) = f(a)$

$\lim_{x \to 1} f(x) = f(1)$

2 DNE

the graph of a function f . At which numbers is f discontinuous? Why?



$a = 1, 3, 5$

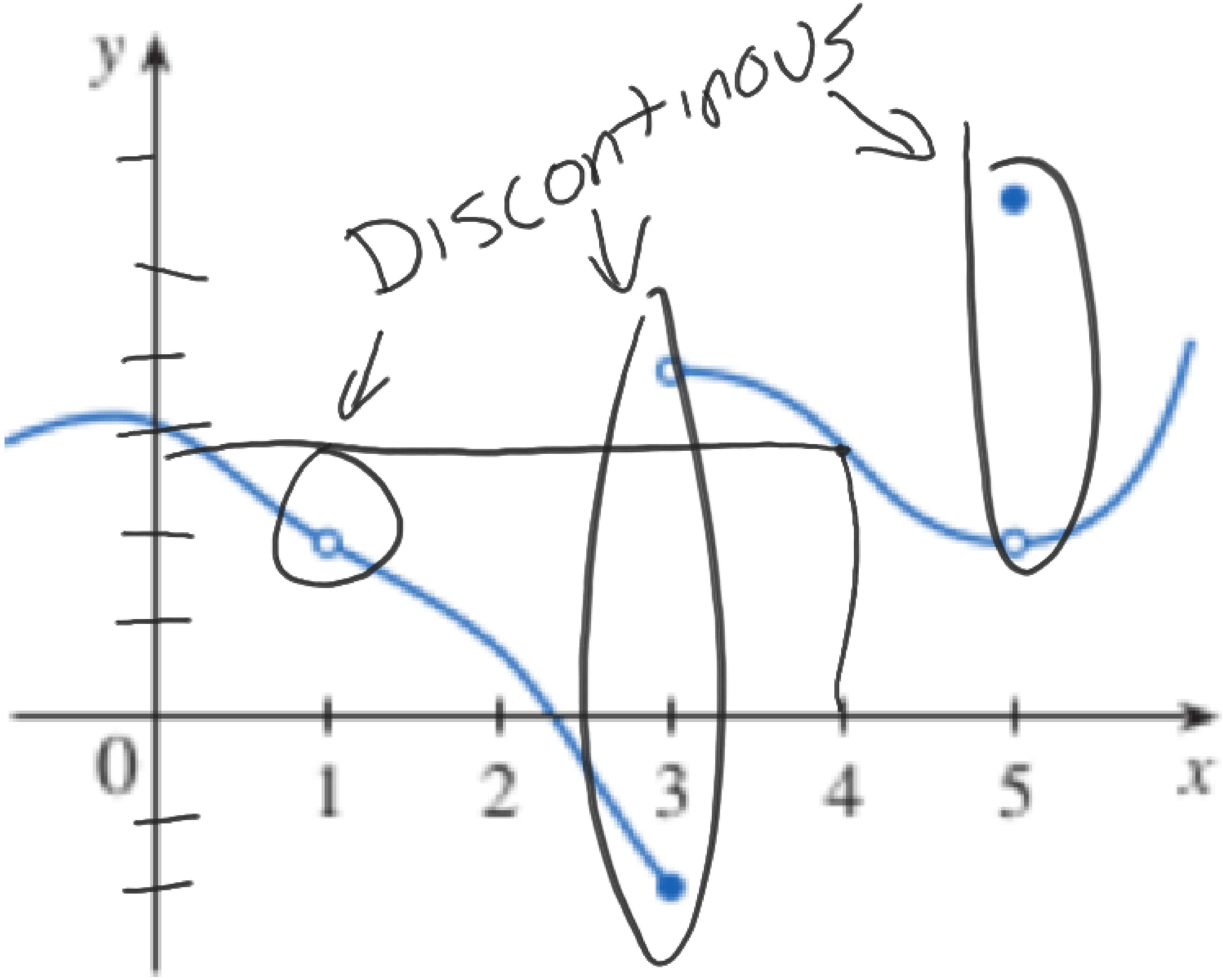
$\lim_{x \to 3} f(x) = f(3)$

DNE $\quad -2$

$\lim_{x \to 5} f(x) = f(5)$

$2 \neq 6$

the graph of a function f. At which numbers is f discontinuous? Why?



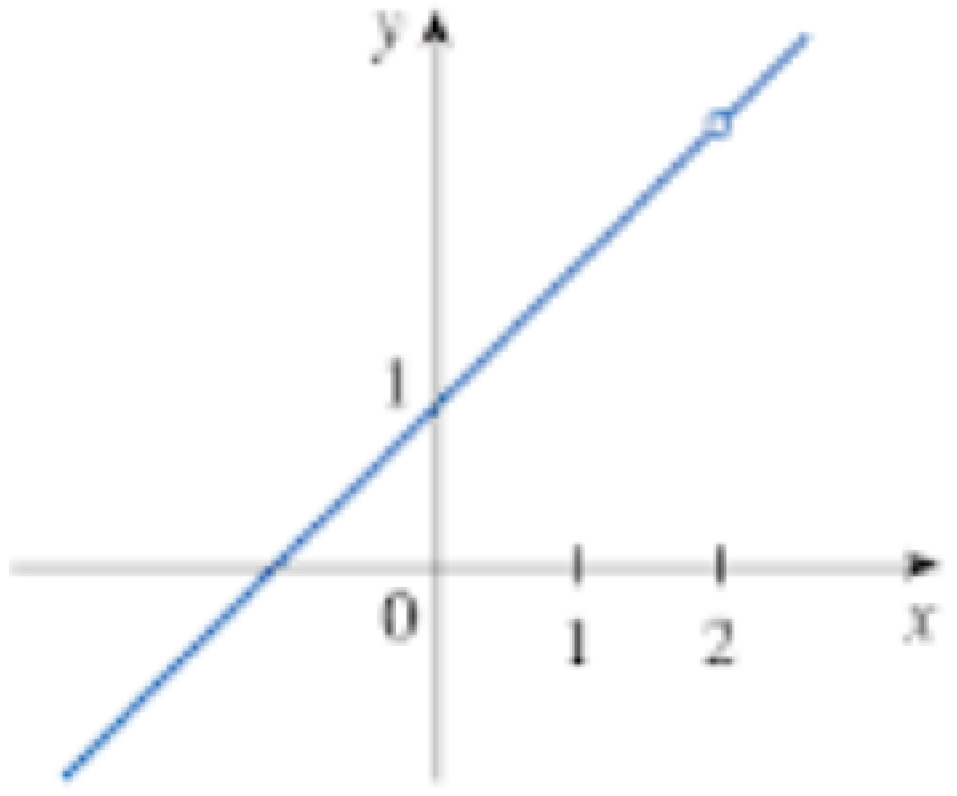
$a = 1, 3, 5$

$\lim_{x \to 4} f(x) = f(4)$

$3 = 3$

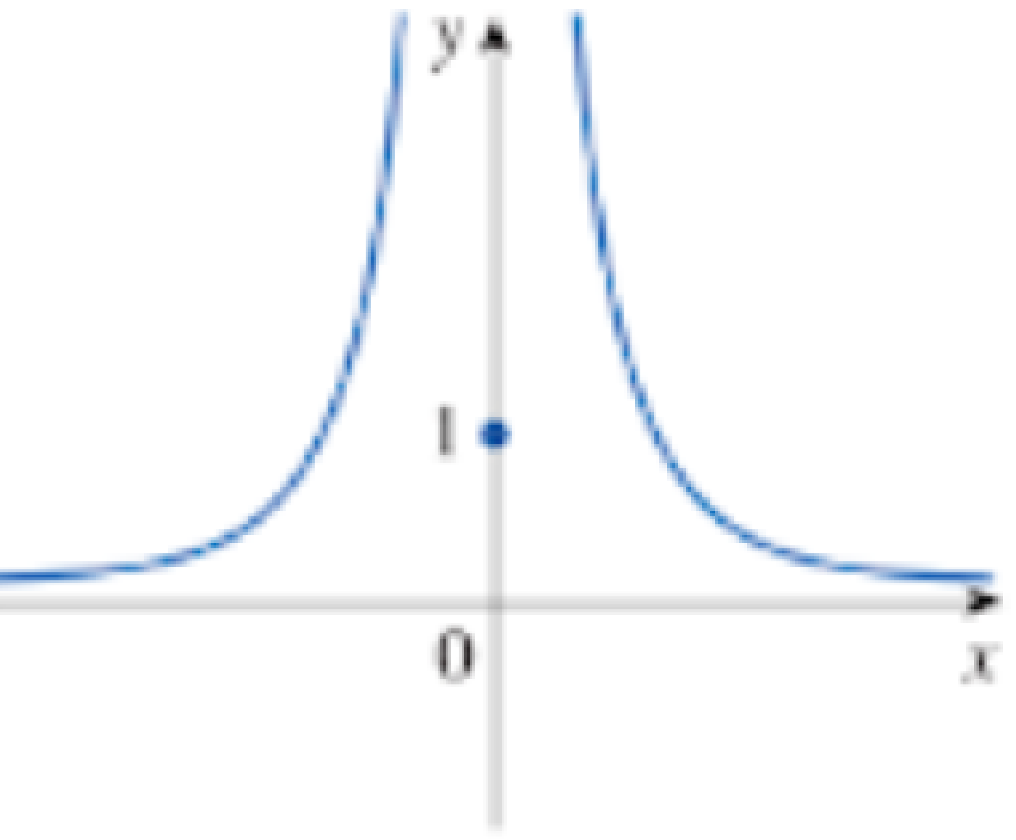
Continous

$x = 3$

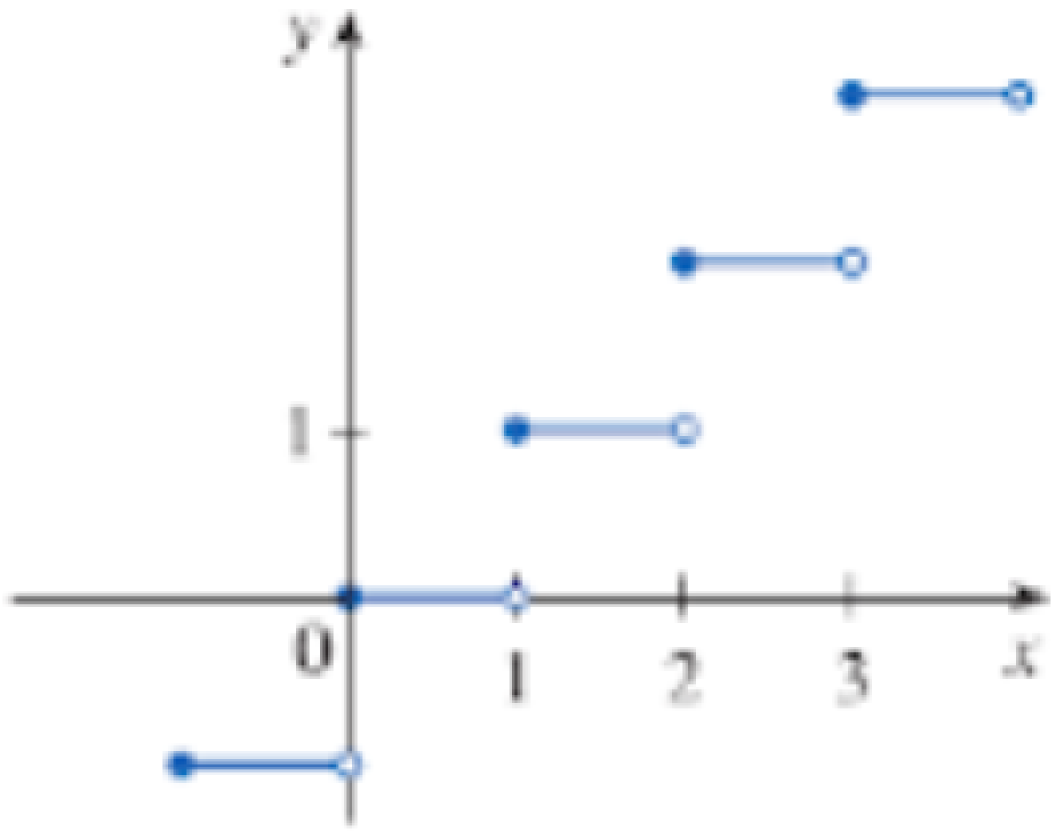


(a) A removable discontinuity

(b) A removable discontinuity



(c) An infinite discontinuity



(d) Jump discontinuities

Where are each of the following functions discontinuous?

$$f(x) = \frac{x^2 - x - 2}{x - 2} = \frac{(x+1)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$x = 2$ discontinuity

$$f(x) = \begin{cases} \dfrac{x^2 - x - 2}{x - 2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$$

$$\lim_{x \to 2} f(x) = 3$$

$$f(2) = 3$$

$$f(x) = \begin{cases} \dfrac{x^2 - x - 2}{x - 2} & \text{if } x \neq 2 \\ 3 & \text{if } x = 2 \end{cases}$$

$$f(x) = [\![x]\!]$$

$$f(1) = [\![1]\!] = 1$$

$$f(1.25) = [\![1.25]\!] = 1$$

$$f(1.75) = [\![1.75]\!] = 1$$

12. Explain why each function is continuous or discontinuous.

a. The temperature at a specific location as a function of time

b. The temperature at a specific time as a function of the distance due west from New York City

c. The altitude above sea level as a function of the distance due west from New York City

d. The cost of a taxi ride as a function of the distance traveled

e. The current in the circuit for the lights in a room as a function of time

A function f is **continuous from the right at a number a** if

$$\lim_{x \to a^+} f(x) = f(a)$$

and f is **continuous from the left at a** if

$$\lim_{x \to a^-} f(x) = f(a)$$

A function f is **continuous on an interval** if it is continuous at every number in the interval. (If f is defined only on one side of an endpoint of the interval, we understand *continuous* at the endpoint to mean *continuous from the right* or *continuous from the left*

Show that the function $f(x) = 1 - \sqrt{1 - x^2}$ is continuous on the interval $[-1, 1]$

$$\lim_{x \to a} f(x) = f(a) \qquad \boxed{-1 < a < 1}$$

$$\lim_{x \to a} 1 - \sqrt{1 - x^2} \rightarrow \lim_{x \to a} 1 - \lim_{x \to a} \sqrt{1 - x^2}$$

$$1 - \lim_{x \to a} \sqrt{1 - x^2} \rightarrow 1 - \sqrt{\lim_{x \to a} (1 - x^2)}$$

$$1 - \sqrt{1 - \lim_{x \to a} x^2} \rightarrow \boxed{1 - \sqrt{1 - a^2}}$$

Show that the function $f(x) = 1 - \sqrt{1 - x^2}$ is continuous on the interval $[-1, 1]$

$$\lim_{x \to -1^+} f(x) = f(-1)$$

$$1 = 1 \checkmark$$

$[-1$

$$\boxed{[-1, 1]}$$

$$\lim_{x \to 1^-} f(x) = f(1)$$

$$1 = 1$$

$1]$

The following types of functions are continuous at every number in their domains:

- polynomials
- rational functions
- root functions
- trigonometric functions
- inverse trigonometric functions
- exponential functions
- logarithmic functions

Find $\lim_{x \to -2} \frac{x^3 + 2x^2 - 1}{5 - 3x}$

$x \neq \frac{5}{3}$

$$\lim_{x \to -2} \frac{(-2)^3 + 2(-2)^2 - 1}{5 - 3(-2)} = \frac{-1}{11}$$

Where is the function $f(x) = \dfrac{\ln x + \tan^{-1} x}{x^2 - 1}$ continuous?

$\ln(x) \rightarrow \boxed{(0, \infty)}$

$\tan^{-1}(x) \rightarrow (-\infty, \infty) \rightarrow \mathbb{R}$ (all Real #'s)

$(x^2 - 1) \rightarrow (-\infty, \infty)$

$\boxed{= \cancel{(-\infty, -1)} \cup (-1, 1) \cup (1, \infty)}$

$\boxed{\text{Domain } (0, 1) \cup (1, \infty)}$

10 The Intermediate Value Theorem

Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.

Show that there is a solution of the equation

$$4x^3 - 6x^2 + 3x - 2 = 0$$

between 1 and 2 .

$f(1) = 4(1)^3 - 6(1)^2 + 3(1) - 2 = -1$

$f(2) = 4(2)^3 - 6(2)^2 + 3(2) - 2 = 12$

$[1, 2]$ $f(a) = -1$ $f(b) = 12$