(a) If $f(x) = x^3 - x$, find a formula for $f'(x)$.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{(\cancel{x^3} + 3x^2 h + 3xh^2 + h^3 \cancel{- x} - h) - (\cancel{x^3} \cancel{- x})}{h}$$

$$\lim_{h \to 0} 3x^2 + 3xh + h^2 - 1 = \boxed{3x^2 - 1}$$

Derivative of a Constant Function

$$\frac{d}{dx}(c) = 0$$

The Power Rule (General Version) If n is any real number, then

$x^3 \rightarrow 3x^2$

$$\boxed{\frac{d}{dx}(x^n) = nx^{n-1}}$$

$f(x) = x^7 \qquad f'(x) = 7x^6$

$h(x) = x^{100} \qquad h'(x) = 100x^{99}$

$g(x) = x^1 \qquad g'(x) = 1x^0 = 1$

$m(x) = x^{-3} \qquad m'(x) = -3x^{-4}$

$$f(x) = \sqrt{x} = x^{1/2}$$

$$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

The Constant Multiple Rule If c is a constant and f is a differentiable function, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

The Sum and Difference Rules If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

$$f(x) = 5x^6$$

$$= 5\left(\frac{d}{dx}x^6\right)$$

$$f'(x) = 5\left(6x^5\right)$$

$$= 30x^5$$

$$g(x) = \frac{3}{4}x^{5/4}$$

$$= \frac{3}{4}\left(\frac{d}{dx}x^{5/4}\right)$$

$$g'(x) = \frac{3}{4}\left(\frac{5}{4}x^{1/4}\right)$$

$$= \frac{15}{16}x^{1/4}$$

$$f(x) = 5x^2 + x - 3$$

$$f'(x) = 5\left(\frac{d}{dx}x^2\right) + \frac{d}{dx}(x) - \frac{d}{dx}(3)$$

$$f'(x) = 10x + 1$$

$$f''(x) = 10 \qquad f'''(x) = 0$$

$$f(x) = x^5 - 7x^3 + x^2 - 5$$

$$f'(x) = 5x^4 - 21x^2 + 2x$$

$$g(m) = \frac{1}{m} + \frac{1}{m^2}$$

$$= m^{-1} + m^{-2}$$

$$g'(m) = -1m^{-2} + (-2)m^{-3}$$

$$= \frac{-1}{m^2} - \frac{2}{m^3}$$

$$h(w) = \sqrt{2}w - \sqrt{2} \qquad f(w) = w^1$$

$$h'(w) = \sqrt{2}(1) - 0 \qquad f(w) = 1w^0$$

$$= \boxed{\sqrt{2}} \qquad = 1(1)$$

$$= 1$$

$$h(x) = \pi x^3 + 5x^2$$

$$h'(x) = 3\pi x^2 + 10x$$

$$g(x) = \sqrt{2x} = (2x)^{1/2} = \sqrt{2}\, x^{1/2}$$

$$g'(x) = \sqrt{2}\left(\frac{1}{2} x^{-1/2}\right) = \boxed{\frac{\sqrt{2}}{2\sqrt{x}}} = \frac{\sqrt{2x}}{2x}$$

$$f(x) = \frac{1 + 5x^3}{2x^2} = \frac{1}{2x^2} + \frac{5x^3}{2x^2}$$

$$= \frac{1}{2}x^{-2} + \frac{5}{2}x$$

$$f'(x) = \frac{1}{2}(-2x^{-3}) + \frac{5}{2}(1)$$

$$= \boxed{\frac{-1}{x^3} + \frac{5}{2}}$$

$$m(x) = (3x^2 - 2)(5x + 1)$$

$$m(x) = 15x^3 + 3x^2 - 10x - 2$$

$$m'(x) = 45x^2 + 6x - 10$$

59. Find the points on the curve $y = x^3 + 3x^2 - 9x + 10$ where the tangent is horizontal.

$y' = 3x^2 + 6x - 9$

$0 = 3x^2 + 6x - 9$

$0 = 3(x^2 + 2x - 3)$

$0 = 3(x+3)(x-1)$

$x = -3$

$x = 1$

39. $y = x + \dfrac{2}{x}$, $(2, 3)$ $m = \frac{1}{2}$

$y = x + 2x^{-1}$ $y' = 1 + 2(-1)(x^{-2})$

$\boxed{y' = 1 - \dfrac{2}{x^2}}$

$y'(2) = 1 - \dfrac{2}{4} = \dfrac{1}{2}$

$y = mx + b$

$3 = \frac{1}{2}(2) + b$

$2 = b$

$y = \frac{1}{2}x + 2$

$$f(x) = b^x$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{\boxed{b^{x+h}} - b^x}{h} \longrightarrow \lim_{h \to 0} \frac{\boxed{b^x b^h} - b^x}{h}$$

$$\lim_{h \to 0} b^x \left(\frac{b^h - 1}{h} \right) \rightarrow b^x \boxed{\lim_{h \to 0} \frac{b^h - 1}{h}}$$

$$b^x \boxed{\left(\lim_{h\to 0} \frac{b^h - 1}{h}\right)} =$$

$$b^x \boxed{f'(0)} \longrightarrow b^x$$

When $b = e$

$$f(x) = e^x \longrightarrow f'(x) = e^x$$

$$f(x) = e^{x} - x$$

$$f'(x) = e^{x} - 1$$

$$g(x) = 3e^{x} - x^{2}$$

$$g'(x) = 3e^{x} - 2x$$

34. $y = \boxed{e^{x+1}} + 1$

$$e^{x+1} = (e^x)(e^1) = (e)(e^x)$$

$$y' = e e^x = \boxed{e^{x+1}}$$

$$\cancel{e^{x^2+2x} = (e^{x^2})(e^{2x})}$$

53. The equation of motion of a particle is $s = t^3 - 3t$, where s is in meters and t is in seconds. Find

(a) the velocity and acceleration as functions of t,

(b) the acceleration after 2 s, and

(c) the acceleration when the velocity is 0.

$$3t^2 - 3 = 0$$
$$t^2 = 1$$
$$t = \pm 1$$

Position $= S = t^3 - 3t$

velocity $= V = S' = 3t^2 - 3$

acceleration $= a = V' = S'' = 6t$