$y = (4x^2 + 3)(2x + 5)$ find y'

$y = 8x^3 + 20x^2 + 6x + 15$

$f(x) = x^n$

$f'(x) = nx^{n-1}$

$y' = 24x^2 + 40x + 6$

Product Rule

$$\frac{d}{dx}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x)$$

$$y = (4x^2 + 3)(2x + 5)$$

$f(x) = 4x^2 + 3$

$f'(x) = 8x$

$g(x) = 2x + 5$

$g'(x) = 2$

$y' = (8x)(2x+5) + (4x^2+3)(2)$

$f'(x)\,g(x) + f(x)\,g'(x)$

$y' = 16x^2 + 40x + 8x^2 + 6 = \boxed{24x^2 + 40x + 6}$

$f(x) = \boxed{xe^x}$, find $f'(x)$. $(x)(e^x)$

$m = x^1$ $m' = 1$ $n = e^x$ $n' = e^x$

$$f'(x) = (1)(e^x) + (e^x)(x)$$

$$(m')(n) + (n')(m)$$

$$f'(x) = e^x + e^x(x) = \boxed{e^x(1+x)}$$

$$m'(x) = e^x(1+x)$$

$$m''(x) = e^x(1+x) + (1)(e^x)$$

$$= e^x + xe^x + e^x = 2e^x + xe^x$$

$$= e^x(2+x)$$

$$f(x) = e^x \qquad g(x) = 1+x$$

$$f'(x) = e^x \qquad g'(x) = 1$$

$$m'''(x) = e^x(3 + x)$$

$$m^{(4)}(x) = e^x(4 + x)$$

$$m^{(5)}(x) = e^x(5 + x)$$

$$m^{(n)}(x) = e^x(n + x)$$

$$h(w) = (w^2 + 3w)(w^{-1} - w^{-4})$$

$$f(w) = w^2 + 3w \qquad g(w) = w^{-1} - w^{-4}$$

$$f'(w) = 2w + 3 \qquad g'(w) = -1w^{-2} + 4w^{-5}$$

$$h'(w) = (2w+3)(w^{-1} - w^{-4}) + (-w^{-2} + 4w^{-5})(w^2 + 3w)$$

$$= 2 - 2w^{-3} + 3w^{-1} - 3w^{-4} + (-1) - 3w^{-1} + 4w^{-3} + 12w^{-4}$$

$$= 1 + 2w^{-3} + 9w^{-4} = \boxed{1 + \frac{2}{w^3} + \frac{9}{w^4}}$$

In prime notation the Quotient Rule is written as

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

$$y = \frac{x^2 + x - 2}{x^3 + 6}. \quad y' =$$

$f = x^2 + x - 2$ $\qquad$ $g = x^3 + 6$ $\qquad$ $\dfrac{f'g - fg'}{g^2}$

$f' = 2x + 1$ $\qquad$ $g' = 3x^2$

$$y' = \frac{(2x+1)(x^3+6) - (x^2+x-2)(3x^2)}{(x^3+6)^2}$$

$$y' = \frac{\boxed{(2x+1)(x^3+6) - (x^2+x-2)(3x^2)}}{(x^3+6)^2}$$

$$(2x^4+12x+x^3+6) - (3x^4+3x^3-6x^2)$$

$$-3x^4 \; -3x^3 \; +6x^2$$

$$\boxed{\frac{-x^4-2x^3+6x^2+12x+6}{(x^3+6)^2}}$$

$$\frac{3x^2 + 2\sqrt{x}}{x} = \frac{3x^2}{x} + \frac{2\sqrt{x}}{x^1} \rightarrow x^{1/2} \quad x^{1/2 - 1}$$

$$= 3x + 2x^{-1/2}$$

$$= 3 + 2\left(-\frac{1}{2}\right)x^{-3/2}$$

$$\boxed{= 3 - x^{-3/2}}$$

$$y = \frac{x^2 - 5x + 6}{x-2} = \frac{\cancel{(x-2)}(x-3)}{\cancel{x-2}}$$

$$y = X - 3 \longrightarrow \boxed{1}$$

$$m(x) = \left(\frac{3}{x}\right)\left(\frac{x+2}{x^3}\right) = \frac{3x+6}{x^4}$$

$$3x^{-3} + 6x^{-4}$$

$$m = \frac{5t}{t^3 - t - 1}$$

$f = 5t$ $\quad g = t^3 - t - 1$

$f' = 5$ $\quad g' = 3t^2 - 1$

$$m' = 5(t^3 - t - 1) - (5t)(3t^2 - 1)$$

$$(5t^3 - 5t - 5) - (15t^3 - 5t)$$

$$\boxed{m' \frac{-10t^3 - 5}{(t^3 - t - 1)^2}}$$

In this exercise we estimate the rate at which the total personal income is rising in Boulder, Colorado. In 2015, the population of this city was 107,350 and the population was increasing by roughly 1960 people per year. The average annual income was $60,220 per capita, and this average was increasing at about $2250 per year (a little above the national average of about $1810 yearly). Use the Product Rule and these figures to estimate the rate at which total personal income was rising in Boulder in 2015. Explain the

Pop = 107350

Pop' = 1960

Income = 60220

Income' = 2250

(Pop)(Income) = (Pop')(Income) + (Pop)(Income)'

$(1960)(60220) + (107350)(2250)$

$$1960(60220)+107350(2250) \quad - \quad 359568700$$

$359{,}568{,}700$

Pop =
107350

Pop' =
1960

Income =
60220

Income' =
2250

$$(\text{Pop})(\text{Income})' = \boxed{(\text{Pop}')(\text{Income}) + (\text{Pop})(\text{Income})'}$$

The *biomass* $B(t)$ of a fish population is the total mass of the members of the population at time t. It is the product of the number of individuals $N(t)$ in the population and the average mass $M(t)$ of a fish at time t. In the case of guppies, breeding occurs continually. Suppose that at time $t = 4$ weeks the population is 820 guppies and is growing at a rate of 50 guppies per week, while the average mass is 1.2 g and is increasing at a rate of 0.14 g/week. At what rate is the biomass increasing when $t = 4$?

$f = 820$ $\quad m = 1.2$

(fish)(mass) $\quad f' = 50$ $\quad m' = .14$

$$B'(4) = N(4)\,M'(4) + M(4)\,N'(4) = 820(0.14) + 1.2(50) = 174.8 \text{ g/week}.$$