Find the derivative of the following using the rule given:

1) $m(x) = x^2 + 4x + 4$ (power)

2) $n(x) = (x+2)^2$ (product)

3) $p(x) = \sin^2 x$ (product)

Find the derivative of the following using the rule given:

1) $m(x) = x^2 + 4x + 4$ $\quad m'(x) = 2x + 4$

2) $n(x) = (x+2)^2$ $\quad n'(x) = 1(x+2) + (1)(x+2)$

$= 2x + 4$

3) $p(x) = \sin^2 x$ $\quad p'(x) = \cos x \sin x + \cos x \sin x$

$= 2 \cos x \sin x$

The Chain Rule If g is differentiable at x and f is differentiable at $g(x)$, then the composite function $F = f \circ g$ defined by $F(x) = f(g(x))$ is differentiable at x and F' is given by the product

$$\boxed{1} \qquad F'(x) = f'(g(x)) \cdot g'(x)$$

$$h(x) = (x+2)^2$$

$$h'(x) = (1)(2(x+2)) = 2x + 4$$

$m(x) = \sqrt{x+3}$

$f(x) = \sqrt{x}$

$g(x) = x+3$

$(f \circ g)(x)$

Differentiate $y = \boxed{(x^3 - 1)}^{100}$.

$g(x) = x^3 - 1$

$f(x) = x^{100}$

$100x^{99}$

$y' = (3x^2)\left[100\left(\boxed{x^3 - 1}\right)^{99}\right]$

$y' = 300x^2(x^3 - 1)^{99}$

Find $F'(x)$ if $F(x) = \sqrt{\boxed{x^2 + 1}}$.

$$\boxed{m} = x^2 + 1$$

$$F(m) = \sqrt{\boxed{m}}$$

$$= m^{1/2}$$

$$m' = 2x$$

$$F'(m) = \frac{1}{2} m^{-1/2} = \frac{1}{2\sqrt{m}}$$

$$F'(x) = (2x)\left[\frac{1}{2\sqrt{x^2+1}}\right] = \frac{x}{\sqrt{x^2+1}}$$

$$r(x) = \sqrt[3]{(3x^2+4x-2)^5}$$

$$r(x) = (\boxed{3x^2+4x-2})^{5/3}$$

$g(x) =$
$3x^2+4x-2$

$$r'(x) = (6x+4)\left(\frac{5}{3}(3x^2+4x-2)^{2/3}\right)$$

$g'(x) =$
$6x+4$

$$r'(x) = \frac{5}{3}(6x+4)\sqrt[3]{(3x^2+4x-2)^2}$$

$y^{5/3} \rightarrow \frac{5}{3}y^{2/3}$

Find $f'(x)$ if $f(x) = \dfrac{1}{\sqrt[3]{x^2 + x + 1}}$.

$$f(x) = (x^2 + x + 1)^{-1/3}$$

$$g(x) = x^2 + x + 1$$

$$g'(x) = 2x + 1$$

$$f'(x) = (2x+1)\left(\frac{-1}{3}(x^2+x+1)^{-4/3}\right)$$

$$y^{-1/3} \rightarrow \frac{-1}{3} y^{-4/3}$$

$$f'(x) = \frac{-(2x+1)}{3\sqrt[3]{(x^2+x+1)^4}}$$

$n(x) = \sin^2 x$

$= (\sin x)^2$

(chain rule)

$g(x) = \sin x$

$g'(x) = \cos x$

$n'(x) = (\cos x)(2 \sin x)$

$= 2 \sin x \cos x$

$y^2 \to 2y$

Differentiate $y = (2x + 1)^5(x^3 - x + 1)^4$.

$$m(x) = \boxed{(\boxed{2x+1})^5} \qquad n(x) = \boxed{(\boxed{x^3 - x + 1})^4}$$

$$m'(x) = 2\left(5(2x+1)^4\right)$$

$$= 10(2x+1)^4$$

$$n'(x) = (3x^2 - 1)\left[4(x^3 - x + 1)^3\right]$$

Differentiate $y = (2x + 1)^5(x^3 - x + 1)^4$.

$m(x) = (2x+1)^5 \qquad m'(x) = 10(2x+1)^4$

$n(x) = (x^3 - x + 1)^4 \qquad n'(x) = 4(3x^2 - 1)(x^3 - x + 1)^3$

$$y' = \left[10(2x+1)^4\right]\left[(x^3 - x + 1)^4\right] + \left[4(3x^2-1)(x^3-x+1)^3\right]\left[(2x+1)^5\right]$$

$$g(t) = \left(\boxed{\frac{t-2}{2t+1}}\right)^9$$

$$\frac{f'g - g'f}{g^2}$$

$$m(t) = \frac{\boxed{t-2}}{\boxed{2t+1}} \begin{matrix} \longrightarrow (1) \\ \longrightarrow (2) \end{matrix}$$

$$2t+1-2t+4$$

$$m'(t) = \frac{(1)(2t+1)-(2)(t-2)}{(2t+1)^2} = \frac{5}{(2t+1)^2}$$

$$g(t) = \boxed{\left(\boxed{\frac{t-2}{2t+1}}\right)^9}$$

$$m'(t) = \frac{5}{(2t+1)^2}$$

$$g'(t) = \left(\frac{5}{(2t+1)^2}\right)\left(9\left(\frac{t-2}{2t+1}\right)^8\right)$$

$$g'(t) = \frac{45(t-2)^8}{(2t+1)^{10}}$$

1) $\ln(e^x) = x \iff e^{\ln(x)} = x$

2) $\left[e^{\ln(x)} = x\right]^p \to e^{p\ln(x)} = x^p$

$$\boxed{b^x = e^{x\ln(b)}}$$

find $\frac{d}{dx}$

$$\frac{d}{dx}\left(b^x\right) = \frac{d}{dx}\left(e^{x\ln(b)}\right) \rightarrow \ln(b)$$

$$\frac{d}{dx}\left(b^x\right) = (\ln b)\boxed{\left(e^{x\ln(b)}\right)} \leftarrow b^x$$

$$\frac{d}{dx}\left(b^x\right) = (\ln b)\left(b^x\right)$$

$$\frac{d}{dx}\left(e^x\right) = (\ln e)\left(e^x\right) = e^x$$

$$\frac{d}{dx}\left(2^x\right) = (\ln 2)\left(2^x\right)$$

$f(x) = 5^x$ $\quad f'(x) = \ln(5)(5^x)$

$g(x) = 7^x$ $\quad g'(x) = \ln(7)(7^x)$

$h(x) = 5^{x^2}$ $\quad h'(x) = (2x)\ln(5)(5^{x^2})$

$$f(x) = \boxed{\sin(\boxed{\cos(\tan x)})}$$

$$g(x) = \cos(\boxed{\tan x})$$

$$g'(x) = \sec^2(x)\,(-\sin(\tan x))$$

$$f'(x) = \left(\sec^2(x)\,(-\sin(\tan x))\right)\left(\cos(\cos(\tan x))\right)$$