find the derivative of

$$y = \sqrt{x^2+25} = (x^2+25)^{1/2}$$

$f(x) = x^{1/2}$

$g(x) = x^2 + 25$

$$y' = (2x)\left(\frac{1}{2\sqrt{(x^2+25}}\right)$$

$\frac{dy}{dx} =$

$$\boxed{y' = \frac{x}{\sqrt{x^2+25}}}$$

SECTION 3.5 Implicit Differentiation

$x^2 + y^2 = 25,$ $\frac{d}{dx}$ derivative

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(25)$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(25)$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(25)$$

$$2x + 2y\boxed{\left(\frac{dy}{dx}\right)} = 0$$

$$-2x \qquad\qquad -2x$$

$$y = \pm\sqrt{x^2+25}$$

$$\frac{2y\left(\frac{dy}{dx}\right)}{2y} = \frac{-2x}{2y}$$

$$\boxed{dy/dx = \frac{-x}{y}} = \boxed{\frac{-x}{\pm\sqrt{x^2+25}}}$$

EXAMPLE 1

If $x^2 + y^2 = 25$, find $\dfrac{dy}{dx}$. Then find an equation of the tangent to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$.

$$\frac{dy}{dx} = \frac{-x}{y} = \boxed{\frac{-3}{4}}$$

Slope = m

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{-3}{4}(x - 3)$$

$$y - 4 = \frac{-3}{4}x + \frac{9}{4}$$

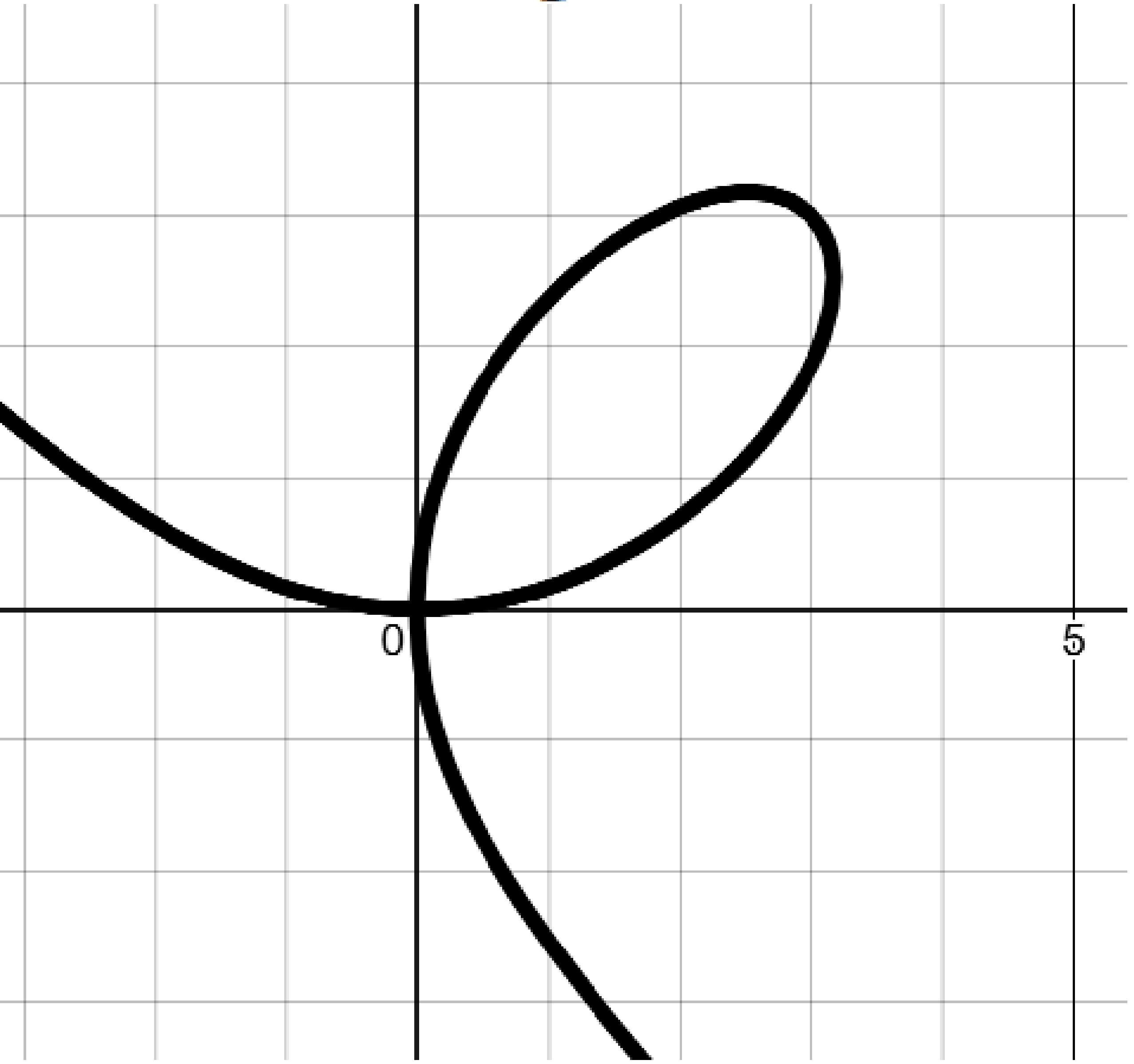
$$\boxed{y = \frac{-3}{4}x + \frac{25}{4}}$$

EXAMPLE 2

(a) Find y' if $x^3 + y^3 = 6xy$.

(b) Find the tangent to the folium of Descartes $x^3 + y^3 = 6xy$ at the point (3, 3).

(c) At what point in the first quadrant is the tangent line horizontal?



$$\frac{dy}{dx} = y'$$

$$x^3 + y^3 = \boxed{6xy}$$

$$3x^2 + 3y^2 y' = 6y + 6xy'$$

$$6x \to 6 \qquad y \to y'$$

$$3x^2+3y^2\boxed{y'}=6y+6x\boxed{y'}$$

$$3x^2-6y+3y^2y'=6xy'$$

$$3x^2-6y=6xy'-3y^2y'$$

$$\frac{3x^2-6y}{6x-3y^2}=\frac{y'(6x-3y^2)}{(6x-3y^2)}$$

$$y'=\frac{3x^2-6y}{6x-3y^2}=\boxed{\frac{x^2-2y}{2x-y^2}}$$

EXAMPLE 2

(a) Find y' if $x^3 + y^3 = 6xy$.

x, y

(b) Find the tangent to the folium of Descartes $x^3 + y^3 = 6xy$ at the point $(3, 3)$.

(c) At what point in the first quadrant is the tangent line horizontal?

a) $y' = \dfrac{x^2 - 2y}{2x - y^2}$

b) $m = \dfrac{(3)^2 - 2(3)}{2(3) - (3)^2} = \dfrac{3}{-3} = -1$

$y - 3 = -1(x - 3) \Rightarrow \boxed{y = -x + 6}$

EXAMPLE 2

(a) Find y' if $x^3 + y^3 = 6xy$.

(b) Find the tangent to the folium of Descartes $x^3 + y^3 = 6xy$ at the point (3, 3).

(c) At what point in the first quadrant is the tangent line horizontal?

a) $y' = \dfrac{x^2 - 2y}{2x - y^2}$

c) $0 = \dfrac{x^2 - 2y}{2x - y^2} \rightarrow 0 = x^2 - 2y$

$2y = x^2$

$y = \dfrac{x^2}{2}$

$$x^3 + y^3 = 6xy$$

$$x^3 + \left(\frac{x^2}{2}\right)^3 = 6x\left(\frac{x^2}{2}\right)$$

$$x^3 + \frac{x^6}{8} = 3x^3$$

$$\frac{x^6}{8} - 2x^3 = 0$$

$$x^3\left(\frac{1}{8}x^3 - 2\right) = 0$$

$$x^3 = 0 \qquad x = 0$$

$$1/8\, x^3 - 2 = 0$$

$$1/8\, x^3 = 2$$

$$x^3 = 16$$

$$x = 16^{1/3} = \boxed{2^{4/3}}$$

$$y = \frac{x^2}{2} = \frac{(2^{4/3})^2}{2}$$

$$y = 2^{8/3 - 1} = \boxed{2^{5/3}}$$

Find y'' if $x^4 + y^4 = 16$.

$$4x^3 + 4y^3y' = 0$$

$$4y^3y' = -4x^3$$

$$\boxed{y' = \frac{-x^3}{y^3}}$$

$$y' = \frac{-x^3}{y^3}$$

$N \to -x^3 \to -3x^2$

$D \to y^3 \to 3y^2 y'$

$$y'' = \frac{(-3x^2)(y^3) - (3y^2 y')(-x^3)}{y^6}$$

$$y'' = \frac{-3x^2 y^3 + 3x^3 y^2 y'}{y^6} \quad \left(-x^3/y^3\right)$$

$$y'' = \frac{-3x^2 y^3 - 3x^6 y^{-1}}{y^6}$$

$$\boxed{\sin(xy)} = \sin x + \sin y$$

$\sin(xy) \to \cos(xy)(y + y'x)$

$xy \to y + y'x$

$\sin(m) \to \cos m$

$$\sin(xy) = \boxed{\sin x + \sin y}$$

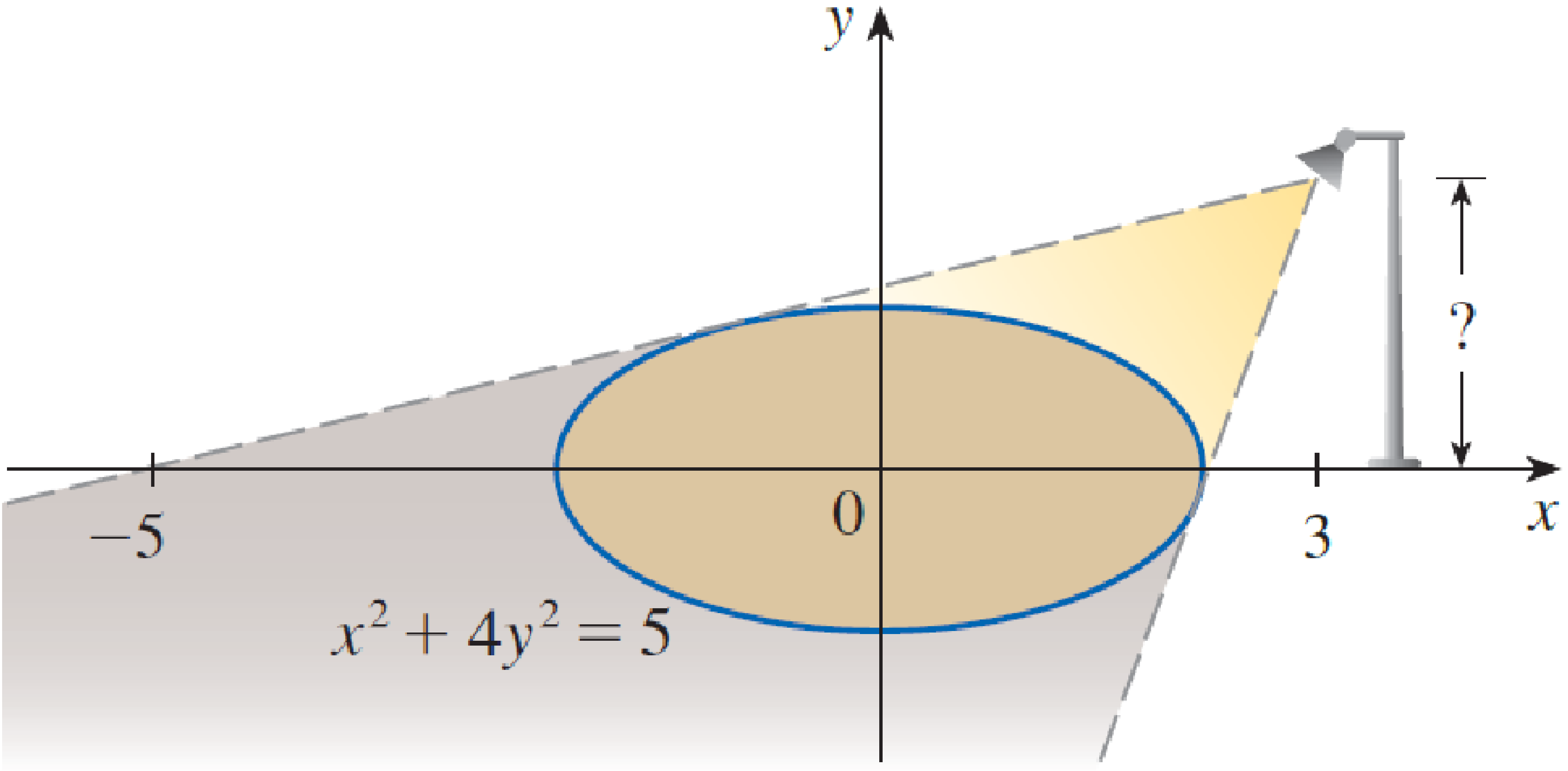
$$(y + xy')\cos(xy) = \cos x + y'\cos y$$

$$y\cos(xy) + xy'\cos(xy) = \cos x + y'\cos y$$

$$\underline{xy'\cos(xy)} - \underline{y'\cos y} = \cos x - y\cos(xy)$$

$$y' = \frac{\cos x - y\cos(xy)}{x\cos(xy) - \cos y}$$

67. The figure shows a lamp located three units to the right of the y-axis and a shadow created by the elliptical region $x^2 + 4y^2 \leqslant 5$. If the point $(-5, 0)$ is on the edge of the shadow, how far above the x-axis is the lamp located?



$$x^3 - \boxed{xy^2} + y^3 = 1$$

xy^2

$x \to 1 \quad y^2 \to 2yy'$

$$3x^2 - (y^2 + 2xyy') + 3y^2y' = 0 \qquad 1y^2 + (2yy')(x)$$

$$3x^2 - y^2 - 2xyy' + 3y^2y' = 0$$

$$y'(-2xy + 3y^2) = -3x^2 + y^2$$

$$y' = \frac{-3x^2 + y^2}{-2xy + 3y^2}$$