Find $\frac{dy}{dx}$ of $b^y = x$ (Look back to notes from 3.1)

$$e^x = e^x$$

$$b^x = b^x(\ln b)$$

$$\boxed{\frac{dy}{dx} = y'}$$

$$\frac{d}{dx}\left(b^y\right) = \frac{d}{dx}(x)$$

$$b^y(\ln b)\frac{dy}{dx} = 1 \rightarrow \frac{dy}{dx} = \frac{1}{b^y \ln b} = \frac{1}{x \ln b}$$

$$y = \log_b x \Longleftrightarrow b^y = x$$

$$\boxed{y' = \frac{1}{x \ln b}}$$

$$\boxed{y = \ln x \Longleftrightarrow y' = \frac{1}{x}}$$

Differentiate $y = \ln(x^3 + 1)$.

$x^3+1 \to 3x^2$

$\ln(x) = \frac{1}{x}$

$y' = (3x^2)\left(\frac{1}{x^3+1}\right)$

$m = x^3 + 1$

$m' = 3x^2$

$\boxed{y' = \frac{3x^2}{x^3+1}}$

$y' = m'\left(\frac{1}{m}\right)$

Find $\frac{d}{dx} \ln(\sin x)$.

$\sin x \rightarrow \cos x$

$$\frac{d}{dx} = (\cos x)\left(\frac{1}{\sin x}\right) = \frac{\cos x}{\sin x} = \cot x$$

BitPaper Share Knowledge

Differentiate $f(x) = \log_{10}(2 + \sin x)$

$2 + \sin x \to \cos x$

$$f'(x) = (\cos x)\left(\frac{1}{(2+\sin x)\boxed{\ln 10}}\right)$$

$$g(x) = \log_{2}(2+\sin x) \to \frac{\cos x}{(2+\sin x)\ln(2)}$$

$$y = 2^x \rightarrow y' = 2^x \ln(2)$$

$$y = \log_8(x^2 + 3x)$$

$$y' = (2x+3)\left(\frac{1}{(x^2+3x)(\ln 8)}\right)$$

$$\boxed{y' = \frac{2x+3}{(x^2+3x)(\ln 8)}}$$

Break up so there are no exponents

$$\ln \frac{(3x+4)^3}{x\sqrt{x+5}} \qquad \ln x^3$$

$$\ln(3x+4)^3 - \ln\left(x\sqrt{x+5}\right)$$

$$\ln(3x+4)^3 - \left[\ln(x) + \ln\sqrt{x+5}\right]$$

$$3\ln\left(\boxed{3x+4}\right) - \ln(x) - \frac{1}{2}\ln(x+5)$$

$$3\ln(3x+4) - \ln(x) - \frac{1}{2}\ln(x+5)$$

$$\frac{3(3)}{3x+4} - \frac{1}{x} - \frac{(1/2)(1)}{x+5}$$

$$\boxed{\frac{9}{3x+4} - \frac{1}{x} - \frac{1}{2(x+5)}}$$

Differentiate $y = \dfrac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}$.

$$\ln y = \boxed{\ln\left(\frac{x^{3/4}(x^2+1)^{1/2}}{(3x+2)^5}\right)}$$

$$\ln y = \ln\left(x^{3/4}\right) + \ln(x^2+1)^{1/2} - \ln(3x+2)^5$$

$$\ln y = \frac{3}{4}\ln(x) + \frac{1}{2}\ln(x^2+1) - 5\ln(3x+2)$$

$$\ln y = \frac{3}{4}\ln(x) + \frac{1}{2}\ln(x^2+1) - 5\ln(3x+2)$$

$$\frac{1}{y}y' = \frac{3}{4}\left(\frac{1}{x}\right) + \frac{1}{2}\left(\frac{2x}{x^2+1}\right) - 5\left(\frac{3}{3x+2}\right)$$

$$y' = y\left(\frac{3}{4x} + \frac{x}{x^2+1} - \frac{15}{3x+2}\right)$$

$$\boxed{y' = \left(\frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}\right)\left(\frac{3}{4x} + \frac{x}{x^2+1} - \frac{15}{3x+2}\right)}$$

Constant base, constant exponent 2^3	**1.** $\frac{d}{dx}(b^n) = 0$ (b and n are constants)
Variable base, constant exponent x^3	**2.** $\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1}f'(x)$
Constant base, variable exponent 3^x	**3.** $\frac{d}{dx}[b^{g(x)}] = b^{g(x)}(\ln b)g'(x)$
Variable base, variable exponent x^x	**4.** To find $(d/dx)[f(x)]^{g(x)}$, logarithmic differentiation

$$y = \sin^{-1} x \quad \leftrightarrow \quad \sin y = x$$

$$\sin y = x$$

$$\sin^2 y = x^2$$

$$\boxed{1 - \sin^2 y} = 1 - x^2$$

$$\sin^2 y + \cos^2 y = 1$$

$$\sqrt{\cos^2 y} = \sqrt{1 - x^2}$$

$$\frac{dy}{dx} \cos y = 1$$

$$\frac{dy}{dx} = \frac{1}{\cos y}$$

$$\frac{dy}{dx} = \boxed{\frac{1}{\sqrt{1-x^2}}}$$

$$y = \sin^{-1}(x)$$

Derivatives of Inverse Trigonometric Functions

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2 - 1}}$$

$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2 - 1}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1 + x^2} \qquad \frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1 + x^2}$$

Differentiate $g(x) = \sec^{-1}(x^2)$.

$$\sec^{-1}(x) = \frac{1}{x\sqrt{x^2-1}}$$

$$g'(x) = (2x)\left(\frac{1}{x^2\sqrt{(x^2)^2-1}}\right) = \boxed{\frac{2}{x\sqrt{x^4-1}}}$$

$y = x^2 \ln x,$

$f'(x)g(x) + g'(x)f(x)$

$y' = 2x \ln(x) + \left(\frac{1}{x}\right)(x^2)$

$y' = 2x\ln x + x$

$f(x) = \sin x + \ln x,$

$f'(x) = \cos x + \frac{1}{x}$

46. $y = \dfrac{e^{-x}\cos^2 x}{x^2 + x + 1}$

$$\ln e^{-x} + \ln|\cos x|^2 - \ln(x^2 + x + 1) = -x + 2\ln|\cos x| - \ln(x^2 + x + 1)$$

$$\frac{1}{y}\,y' = -1 + 2\cdot\frac{1}{\cos x}(-\sin x) - \frac{1}{x^2 + x + 1}(2x + 1)$$

$$y' = y\left(-1 - 2\tan x - \frac{2x+1}{x^2 + x + 1}\right)$$

$$y' = -\frac{e^{-x}\cos^2 x}{x^2 + x + 1}\left(1 + 2\tan x + \frac{2x+1}{x^2 + x + 1}\right)$$