1 Definition Let c be a number in the domain D of a function f. Then $f(c)$ is the

- **absolute maximum** value of f on D if $f(c) \geqslant f(x)$ for all x in D.
- **absolute minimum** value of f on D if $f(c) \leqslant f(x)$ for all x in D.

2 Definition The number $f(c)$ is a

- **local maximum** value of f if $f(c) \geqslant f(x)$ when x is near c.
- **local minimum** value of f if $f(c) \leqslant f(x)$ when x is near c.

3 The Extreme Value Theorem If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

$$f(x) = 3x^4 - 16x^3 + 18x^2 \qquad \boxed{-1 \leqslant x \leqslant 4}$$

$f'(x) = 12x^3 - 48x^2 + 36x$

$0 = 12x^3 - 48x^2 + 36x$

$= 12x(x^2 - 4x + 3)$

$= 12x(x-3)(x-1)$

$x = 0 \quad x = 3 \quad x = 1$

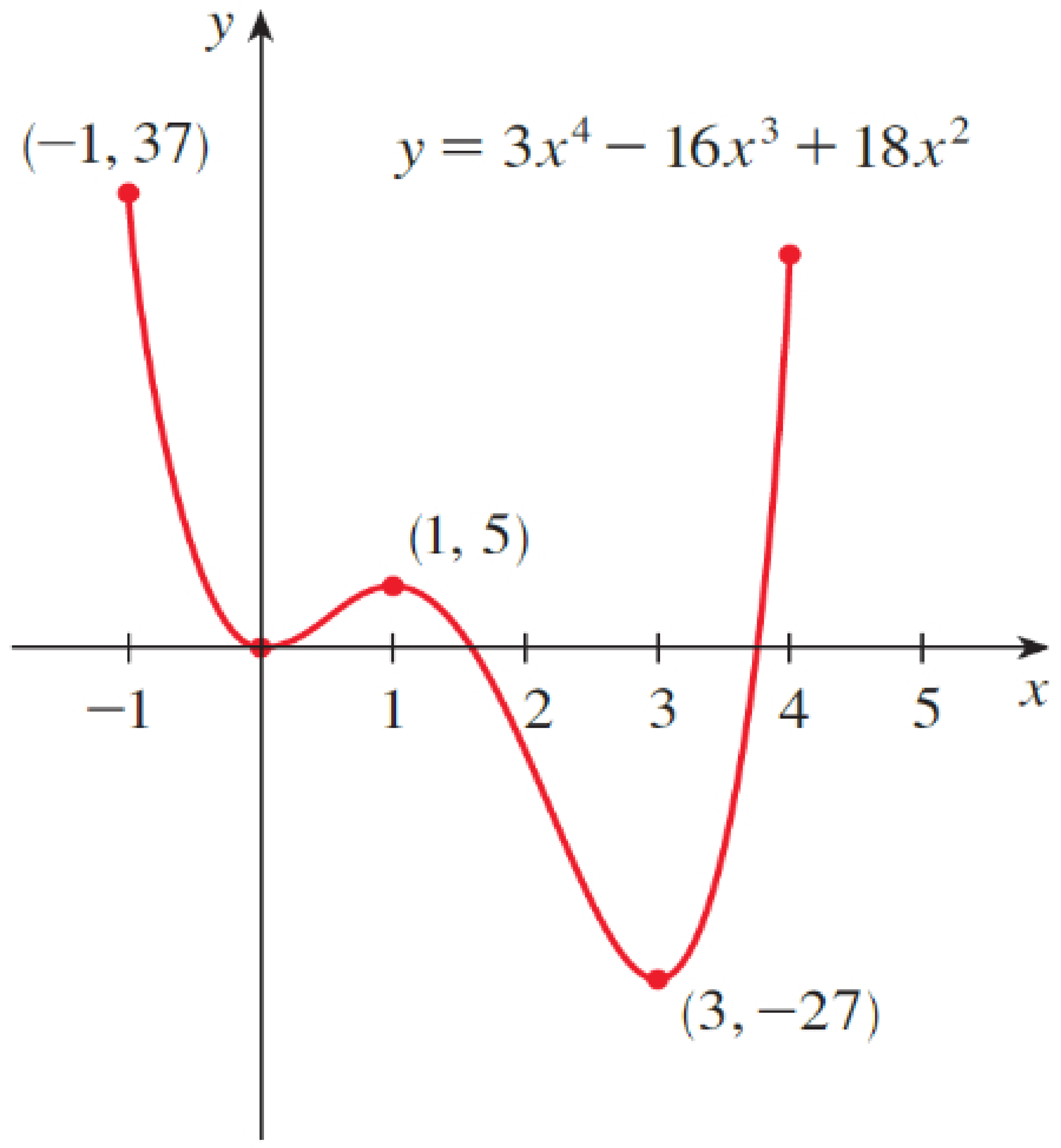
$\boxed{f(-1) = 37}$ A. Max

$f(4) = 32$ None

$f(0) = 0$ L. Min

$f(3) = -27$ A. Min

$f(1) = 5$ L. Max



4 Fermat's Theorem If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

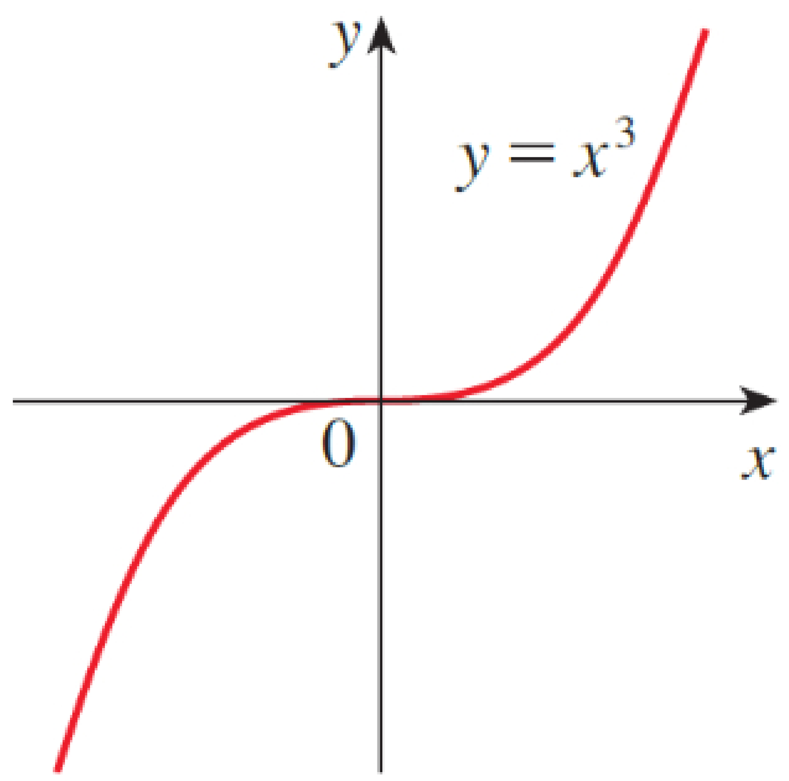
6 Definition A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

$$f(x) = x^3$$

$f'(x) = 3x^2$

Critical #'s $\rightarrow$ $x = 0$

$(0, 0)$



EXAMPLE 10 The Hubble Space Telescope was deployed on April 24, 1990, by the space shuttle *Discovery*. A model for the velocity of the shuttle during this mission, from liftoff at $t = 0$ until the solid rocket boosters were jettisoned at $t = 126$ seconds, is given by

$$v(t) = 0.001302t^3 - 0.09029t^2 + 23.61t - 3.083$$

(in feet per second). Using this model, estimate the absolute maximum and minimum values of the *acceleration* of the shuttle between liftoff and the jettisoning of the boosters.

$$a(t) = 0.003906t^2 - 0.18058t + 23.61$$

$$a'(t) = 0.007812t - 0.18058$$

$$a(0) = 23.61$$

$$a(23.12) = 21.52$$

$$a(126) = 62.87$$

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40. $g(x) = \sqrt[3]{4 - x^2} \rightarrow (4-x^2)^{1/3}$

$$g'(x) = (-2x)\left(\tfrac{1}{3}\right)(4-x^2)^{-2/3}$$

$$g'(x) = \frac{-2x}{3(4-x^2)^{2/3}}$$

$-2x = 0$

$$\boxed{\boxed{x = 0} \quad x = 2 \quad x = -2}$$

$4 - x^2 = 0$

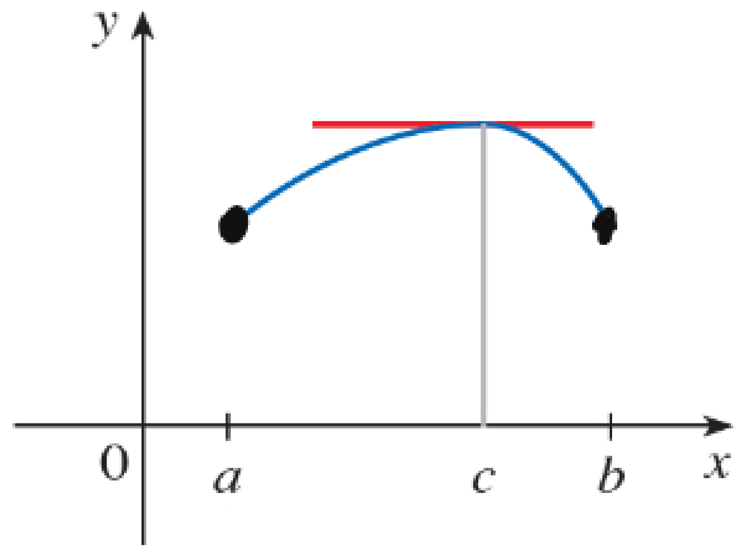
$-x^2 = -4$

$x^2 = 4 \rightarrow x = \pm 2$

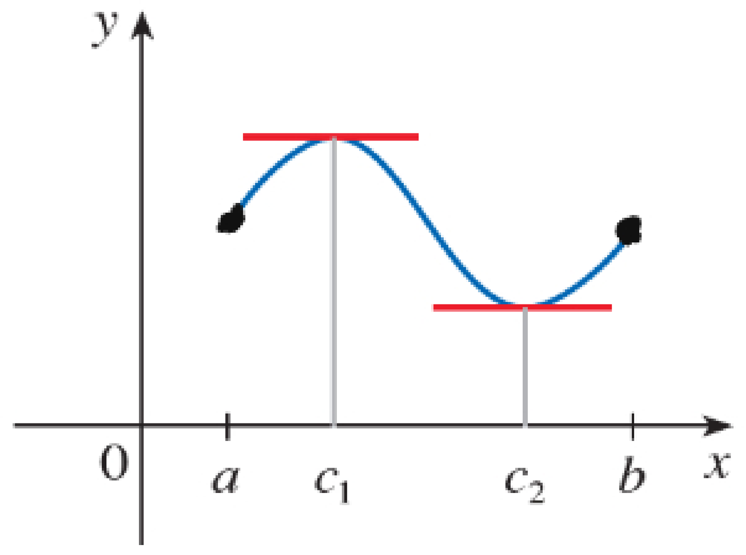
Rolle's Theorem Let f be a function that satisfies the following three hypotheses:

1. f is continuous on the closed interval $[a, b]$.
2. f is differentiable on the open interval (a, b).
3. $f(a) = f(b)$

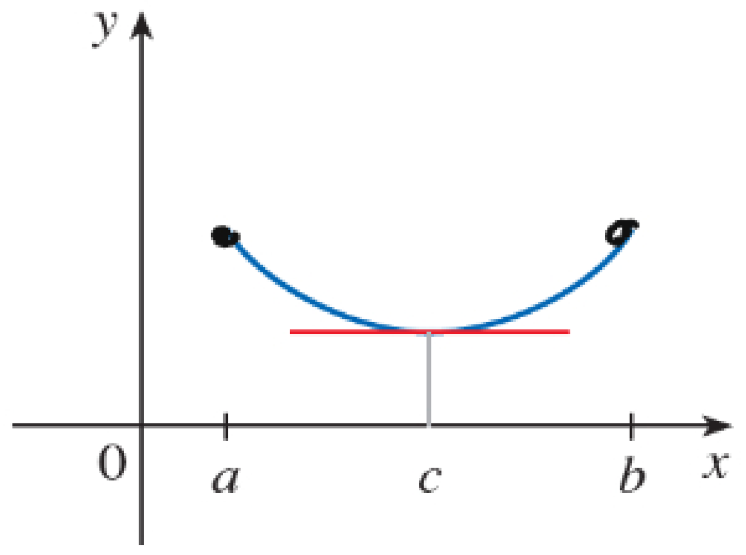
Then there is a number c in (a, b) such that $f'(c) = 0$.



(b)



(c)



(d)

The Mean Value Theorem Let f be a function that satisfies the following hypotheses:

1. f is continuous on the closed interval $[a, b]$.

2. f is differentiable on the open interval (a, b).

Then there is a number c in (a, b) such that

$$\boxed{1} \qquad f'(c) = \frac{f(b) - f(a)}{b - a}$$

or, equivalently,

$$\boxed{2} \qquad f(b) - f(a) = f'(c)(b - a)$$

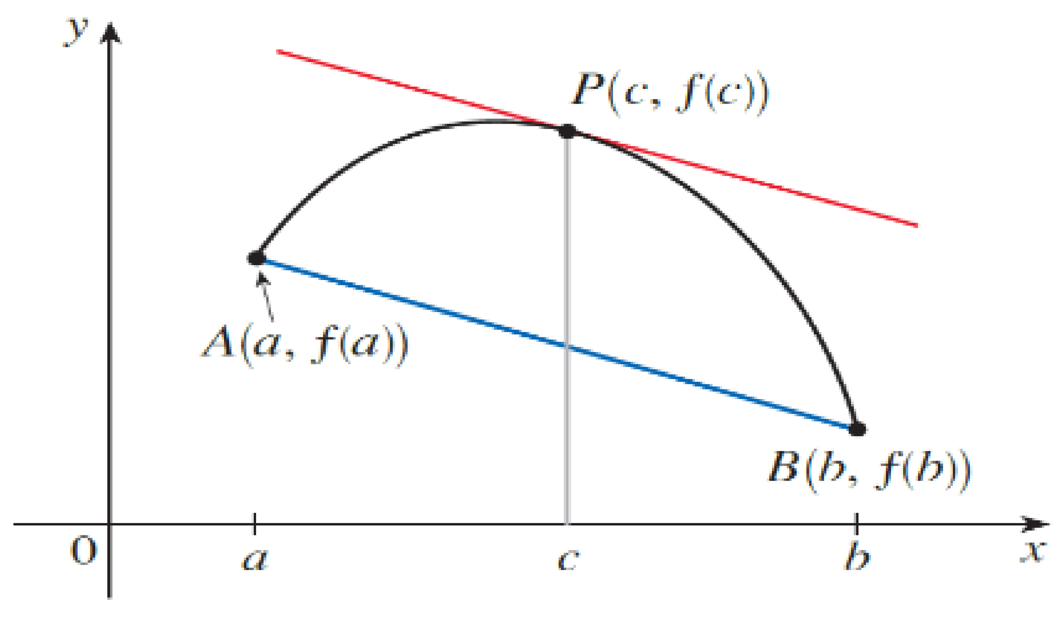


FIGURE 3

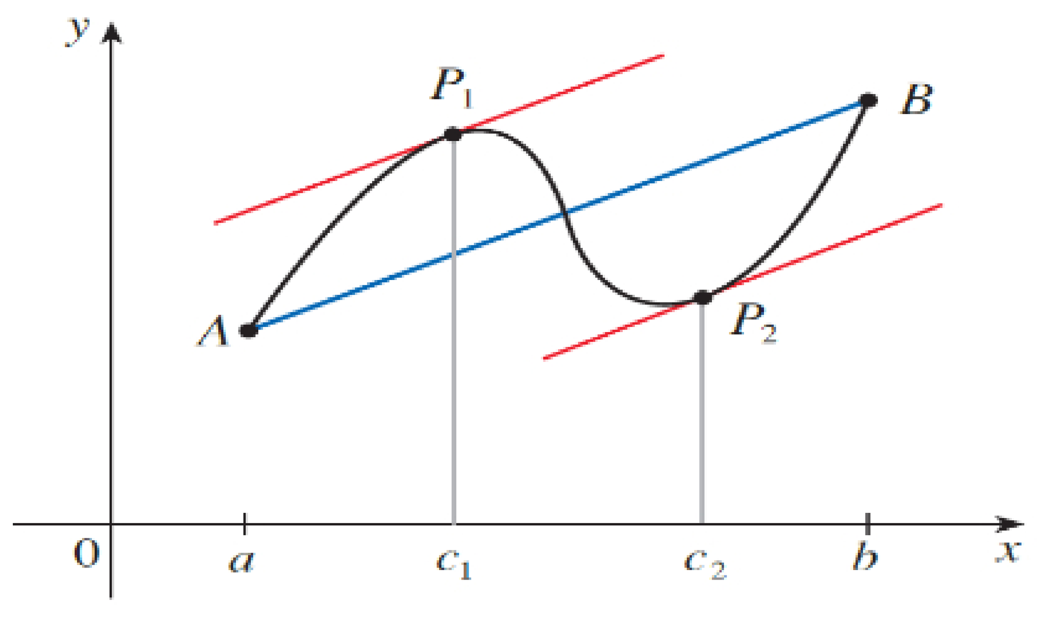


FIGURE 4

EXAMPLE 4 If an object moves in a straight line with position function $s = f(t)$, then the average velocity between $t = a$ and $t = b$ is

$$\frac{f(b) - f(a)}{b - a}$$

and the velocity at $t = c$ is $f'(c)$. Thus the Mean Value Theorem (in the form of Equation 1) tells us that at some time $t = c$ between a and b the instantaneous velocity $f'(c)$ is equal to that average velocity. For instance, if a car traveled 180 km in 2 hours, then the speedometer must have read 90 km/h at least once.

$$f(x) = x^3 - 3x + 2, \quad [-2, 2]$$

a b

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$f(b) - f(a) = f'(c)(b - a)$$

$f(-2) = -8 + 6 + 2 = 0$

$f(2) = 8 - 6 + 2 = 4$

$f'(c) = \dfrac{4 - 0}{2 - (-2)} = \dfrac{4}{4} = 1$

$f'(c) = 1$

$f'(x) = 3x^2 - 3$

$f'(c) = 1 = 3c^2 - 3$

$\rightarrow 4 = 3c^2$

$\frac{4}{3} = c^2$

$\pm\frac{2}{\sqrt{3}} = c = \pm\frac{2\sqrt{3}}{3}$

$\sqrt{x^2 - 4}$

$\sqrt{(0^2) - 4} \neq$

$\sqrt{-4}$

$\cancel{x = 0}, \pm 2$