Find the derivative of

$$f(x) = 5x^3 + e^x + \ln|x| + 6$$

$$f'(x) = 15x^2 + e^x + \frac{1}{x}$$

$$F(x) = \frac{15}{3}x^3 + e^x + \ln|x| + C$$

Antiderivative

or

Integral $\rightarrow \int$

	$5x^{3}$	$\rightarrow$	$15x^{2}$
f'	x^{n}	$\rightarrow$	nx^{n-1}
$\int$	$\dfrac{x^{n+1}}{n+1}$	$\leftarrow$	x^{n}

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$$\int \quad \int x$$

$$\int \frac{1}{x^2} = \int x^{-2} \rightarrow \frac{x^{-1}}{-1} = \frac{-1}{x} = -\frac{1}{x}$$

$$\int \frac{1}{x} = \int x^{-1} \rightarrow \cancel{\frac{x^0}{0}} = \ln|x|$$

$$= x^0 = 1$$

2 Table of Antidifferentiation Formulas

Function	Particular antiderivative	Function	Particular antiderivative
$cf(x)$	$cF(x)$	$\sin x$	$-\cos x$
$f(x) + g(x)$	$F(x) + G(x)$	$\sec^2 x$	$\tan x$
$x^n \quad (n \neq -1)$	$\dfrac{x^{n+1}}{n+1}$	$\sec x \tan x$	$\sec x$
$\dfrac{1}{x}$	$\ln \lvert x \rvert$	$\dfrac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x$
e^x	e^x	$\dfrac{1}{1+x^2}$	$\tan^{-1} x$
b^x	$\dfrac{b^x}{\ln b}$	$\cosh x$	$\sinh x$
$\cos x$	$\sin x$	$\sinh x$	$\cosh x$

$\sin x$

$\cos x$

$-\sin x$

$-\cos x$

EXAMPLE 2 Find all functions g such that

$$g'(x) = 4\sin x + \boxed{\frac{2x^5 - \sqrt{x}}{x}}$$

$$\frac{2x^5}{x} - \frac{\sqrt{x}}{x} = 2x^4 - x^{-1/2}$$

$$\sqrt{x} \to x^{1/2}$$

$$G(x) = -4\cos x + \frac{2}{5}x^5 - \frac{x^{1/2}}{1/2}$$

$$G(x) = -4\cos x + \frac{2}{5}x^5 - 2\sqrt{x} + C$$

EXAMPLE 3 Find f if $f'(x) = e^x + 20(1 + x^2)^{-1}$ and $f(0) = -2$.

$$f'(x) = e^x + \frac{20}{1+x^2}$$

$$\boxed{F(x) = e^x + 20\tan^{-1}(x) - 3}$$

$$f(0) = e^0 + 20\tan^{-1}(0) + C = -2$$

$$= 1 + 20(0) + C = -2$$

$$1 + C = -2$$

$$C = -3$$

$$f(x) = \frac{4x^4}{4} + \frac{3x^3}{3} - \frac{4x^2}{2} + Cx + D$$

$f(0) = 4$

$f(1) = 1$

$$f(x) = x^4 + x^3 - 2x^2 + Cx + D$$

$$4 = D$$

$$f(x) = x^4 + x^3 - 2x^2 + Cx + 4$$

$$1 = 1 + 1 - 2 + C + 4 \rightarrow 1 = C + 4$$

$$-3 = C$$

$$\boxed{f(x) = x^4 + x^3 - 2x^2 - 3x + 4}$$

EXAMPLE 6 A particle moves in a straight line and has acceleration given by $a(t) = 6t + 4$. Its initial velocity is $\boxed{v(0) = -6 \text{ cm/s}}$ and its initial displacement is $s(0) = 9$ cm. Find its position function $s(t)$.

$$a(t) = s''(t) = 6t + 4$$

$$v(t) = s'(t) = 3t^2 + 4t + C \qquad (0, -6)$$

$$= 3t^2 + 4t - 6$$

$$s(t) = t^3 + 2t^2 - 6t + D \qquad (0, 9)$$

$$\boxed{s(t) = t^3 + 2t^2 - 6t + 9}$$

reach its maximum height? When does it hit the ground?

$a(t) = -32 \text{ ft/sec}^2$

$v(t) = -32t + C \rightarrow -32t + 48$

$s(t) = -16t^2 + 48t + D \rightarrow -16t^2 + 48t + 432$

$b^2 - 4ac$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$-16(t^2 - 3t - 27)$

$9 - 4(1)(-27)$

$\frac{3 \pm \sqrt{117}}{2}$

$\boxed{t = \frac{3 + \sqrt{117}}{2}}$

47. $f''(\theta) = \sin\theta + \cos\theta, \quad f(0) = 3, \quad f'(0) = 4$

$$f'(\theta) = -\cos\theta + \sin\theta + C \qquad (0,4)$$

$$4 = -1 + 0 + C \Rightarrow C = 5$$

$$f'(\theta) = -\cos\theta + \sin\theta + 5$$

$$f(\theta) = -\sin\theta - \cos\theta + 5\theta + D \qquad (0,3)$$

$$3 = 0 - 1 + 0 + D \Rightarrow D = 4$$

$$\boxed{f(\theta) = -\sin\theta - \cos\theta + 5\theta + 4}$$

applied, producing a constant deceleration of $\boxed{22\ ft/s^2}$. What is the distance traveled before the car comes to a stop?

$a(t) = -22$

$\frac{50 \text{ miles}}{\text{hour}} = 73.3 \text{ ft/sec}$

$v(t) = -22t + 73.3$

$s(t) = -11t^2 + 73.3t$

$0 = -22t + 73.3 \rightarrow \frac{73.3}{22} = t = 3.3$

$s(3.3) = -11(3.3)^2 + 73.3(3.3) = 122 \text{ ft}$

EXAMPLE 3 Find $\int t^2 e^t \, dt$.

SOLUTION Notice that e^t is unchanged when differentiated or integrated whereas t^2 becomes simpler when differentiated, so we choose

$$u = t^2 \qquad dv = e^t \, dt$$

Then $$du = 2t \, dt \qquad v = e^t$$

$$\int x^2 e^x \, dx$$

Integration by parts gives

$$\boxed{3} \qquad \int t^2 e^t \, dt = t^2 e^t - 2 \int t e^t \, dt$$

The integral that we obtained, $\int t e^t \, dt$, is simpler than the original integral but is still not obvious. Therefore we use integration by parts a second time, this time with $u = t$ and $dv = e^t \, dt$. Then $du = dt$, $v = e^t$, and

$$\int t e^t \, dt = t e^t - \int e^t \, dt$$

$$= t e^t - e^t + C$$

$$f(x) = x^2 e^x - 2 x e^x + 2 e^x$$

Putting this in Equation 3, we get

$$\int t^2 e^t \, dt = t^2 e^t - 2 \int t e^t \, dt$$

$$= t^2 e^t - 2(t e^t - e^t + C)$$

$$= t^2 e^t - 2t e^t + 2e^t + C_1 \qquad \text{where } C_1 = -2C$$

$$f(x) = e^x\left(x^2 - 2x + 2\right)$$

$$e^x \rightarrow e^x$$

$$x^2 - 2x + 2 \rightarrow 2x - 2$$

$$f'(x) = e^x\left(x^2 - 2x + 2\right) + (2x - 2)e^x$$

$$= e^x x^2 \cancel{- 2xe^x} \cancel{+ 2e^x} \cancel{+ 2xe^x} \cancel{- 2e^x}$$

$$= \boxed{e^x x^2}$$

Find $y'(x)$.

$y'(x) =$ $\boxed{\dfrac{2x+2}{2\sqrt{x^2+2x}} - 1}$

Find $y''(x)$.

$y''(x) =$ $\boxed{-\dfrac{1}{(x^2+2x)^{3/2}}}$

$$f(x) = \sqrt{x^2+5x} - x$$

$$\lim_{x\to\infty} \sqrt{x^2+5x} - x \quad \frac{\left(\sqrt{x^2+5x}+x\right)}{}$$

$(5x)$

$$\lim_{x\to\infty} \frac{(x^2+5x) - x^2}{\sqrt{x^2+5x} + x} \overset{H}{=} \frac{5}{\frac{x}{\sqrt{x^2+5x}} +}$$

$$2x\left(\tfrac{1}{2}(x^2+5x)^{-1/2}\right)$$

Find $y'(x)$.

$x = 0$

$y'(x) =$ $\dfrac{2x+2}{2\sqrt{x^2+2x}} - 1$

Find $y''(x)$.

$y''(x) =$ $-\dfrac{1}{(x^2+2x)^{3/2}}$

$$\frac{2x+2}{2\sqrt{x^2+2x}} - 1 = 0$$

$$2x+2 = 2\sqrt{x^2+2x}$$

$$4x^2 + 8x + 4 = 4(x^2+2x)$$

$$4x^2 + 8x$$

$$4 = 0 \quad \times$$

$(-\infty, 0)$ D

$(0, \infty)$ I

$$-\frac{\quad}{(x^2+2x)^{3/2}}$$

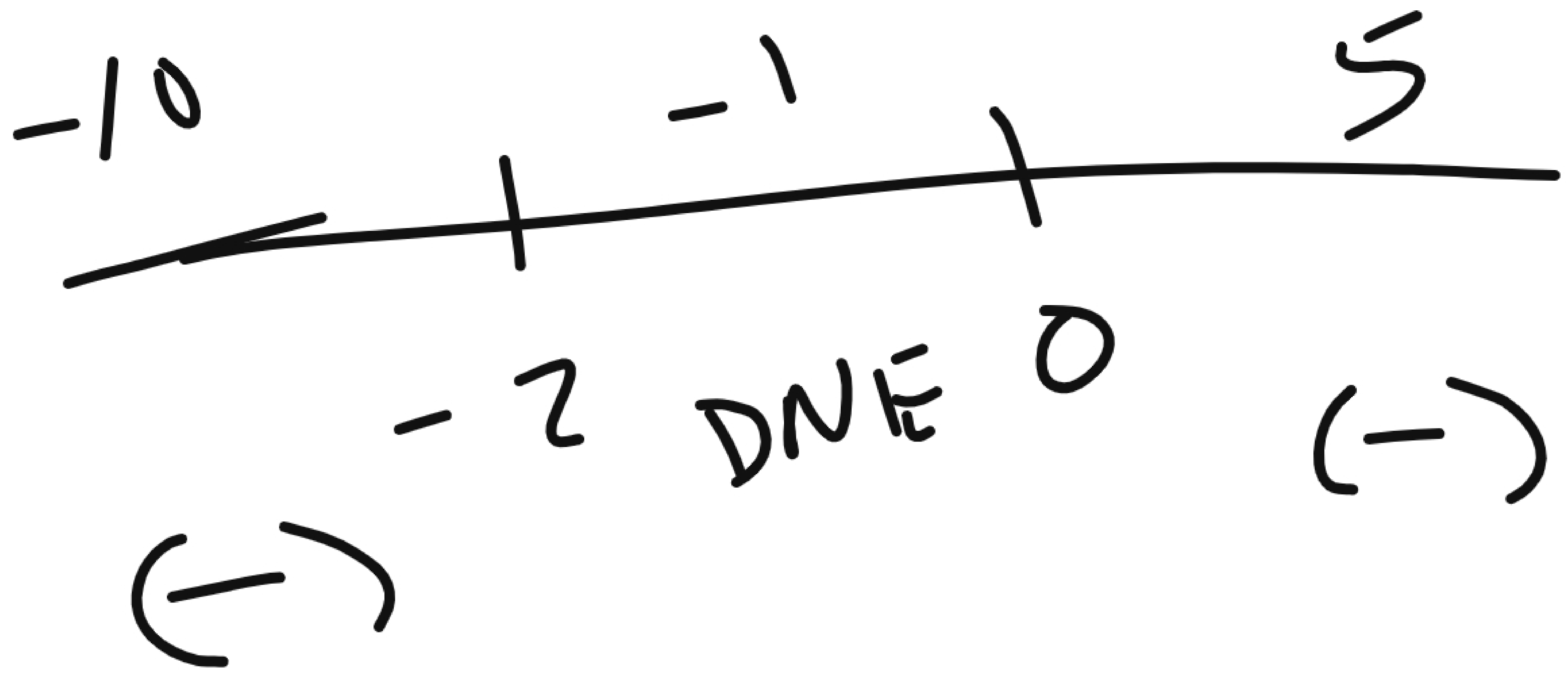
$(x^2+2x) \quad = 0$

$x^2+2x=0$

$x(x+2)=0$

$x=0$

$x=-2$



concave down $(-\infty,-2)\cup(0,\infty)$

Consider the equation below. (If an answer does not exist, enter DNE.)

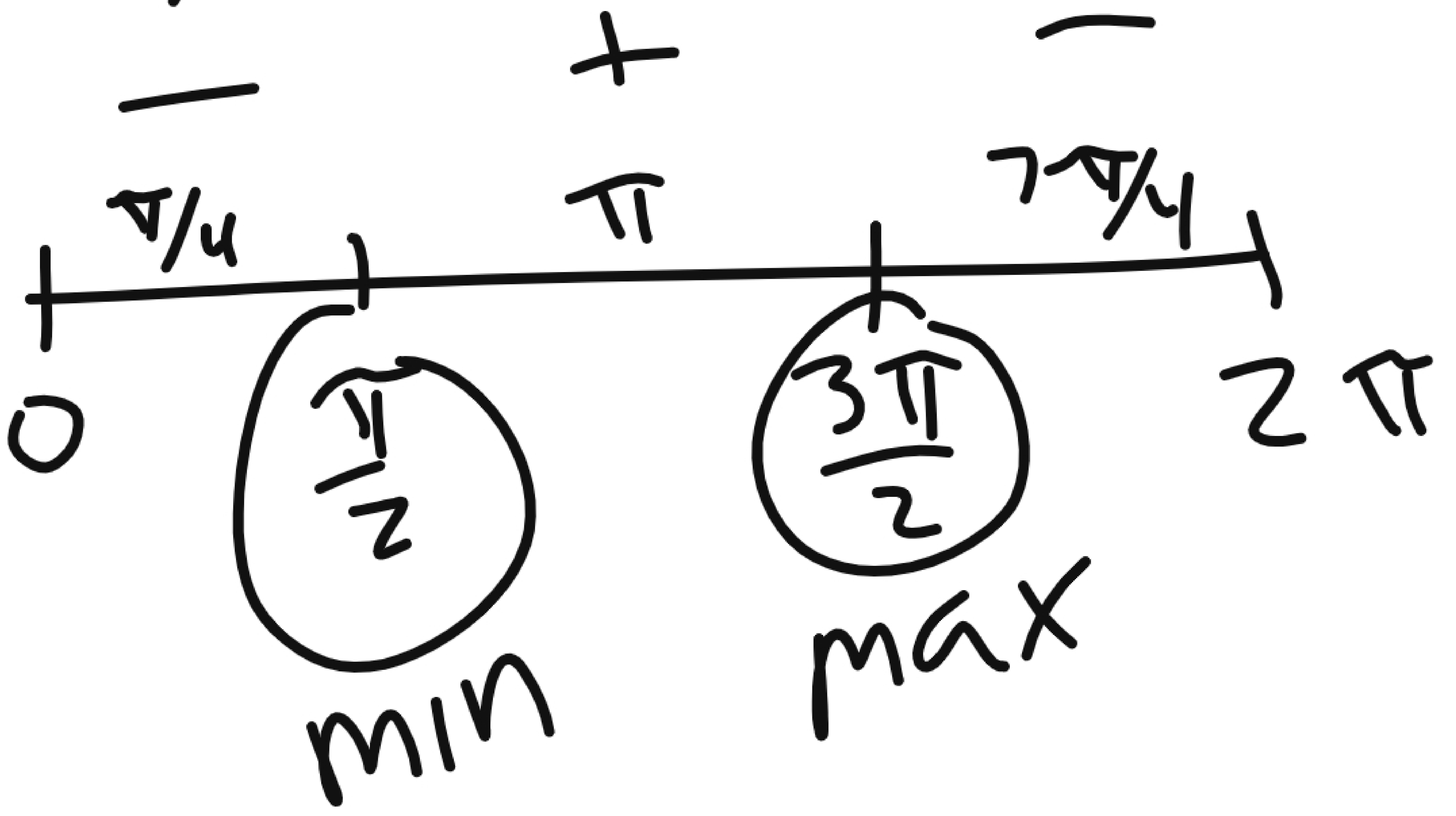
$$f(x) = \underline{6\cos^2(x)} - 12\sin(x), \qquad 0 \le x \le 2\pi$$

$$f'(x) = (12\cos x)(-\sin(x)) - 12\cos x$$

$$= -12\cos x \sin x - 12\cos x$$

$$0 = -12\cos x(\sin x + 1)$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$



$D \rightarrow \left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$

$I \rightarrow \left(\pi/2, 3\pi/2\right)$

$= 12(2\sin x - 1)(\sin x + 1)$

$2\sin x - 1 = 0$

$\sin x = 1/2$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\sin x + 1 = 0$

$\sin x = -1$

$x = \frac{3\pi}{2}$

Find the interval on which f is concave up. (Enter your an

$\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

Find the interval on which f is concave down. (Enter your

$\left(0, \frac{\pi}{6}\right), \left(\frac{5\pi}{6}, \frac{3\pi}{2}\right), \left(\frac{3\pi}{2}, 2\pi\right)$

$$\underline{1200b} - \boxed{\frac{b^3}{4}}$$

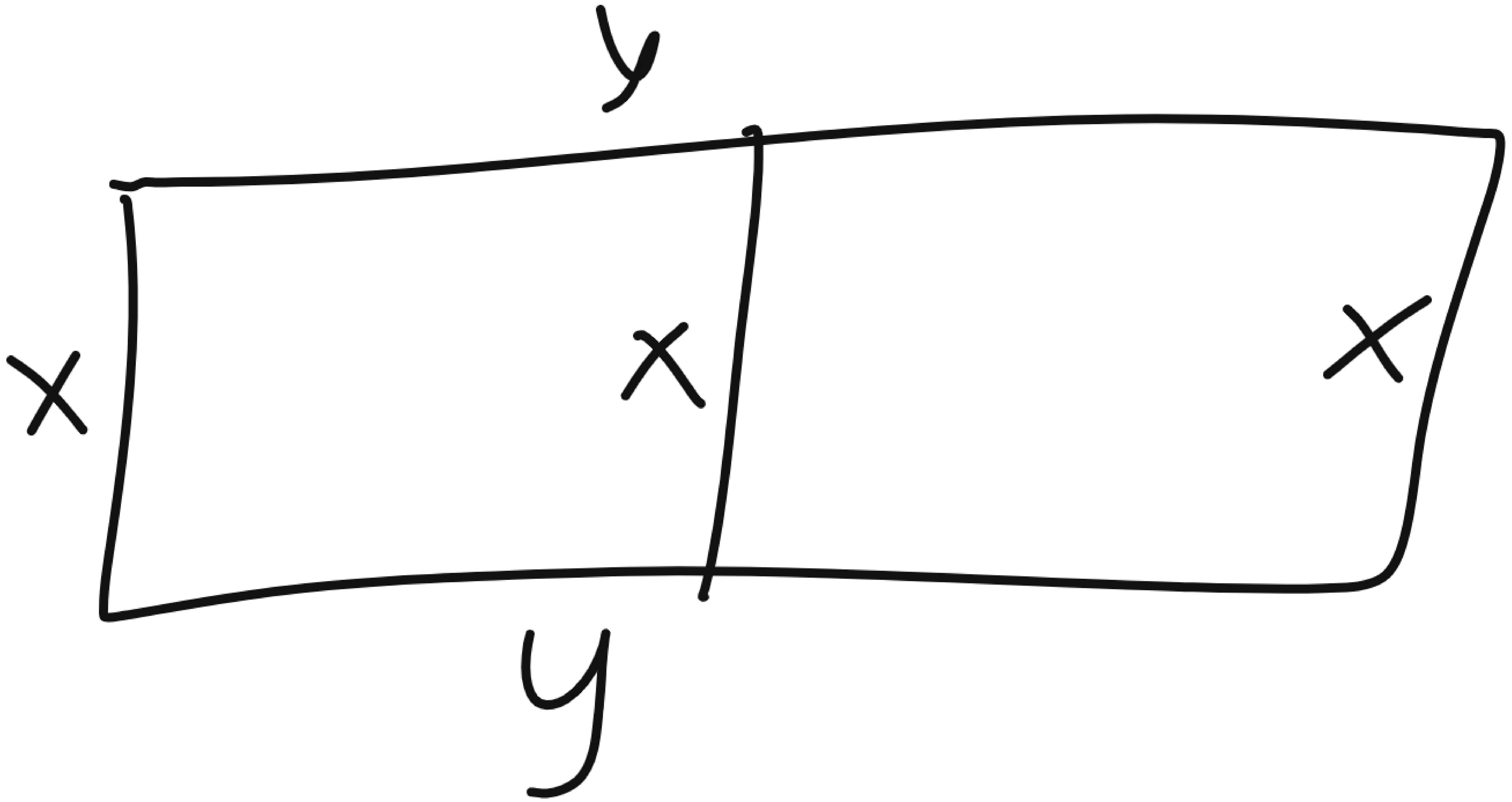
$$0 < b <$$

$$\frac{b^3}{4} = 1200b$$

$$b^3 = 4800b$$

$$\sqrt{b^2} = \sqrt{4800} = \sqrt{1600 \cdot 3} = 40\sqrt{3}$$

A farmer wants to fence an area of 6 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. Let y represent the length (in feet) of a side perpendicular to the dividing fence, and let x represent the length (in feet) of a side parallel to the dividing fence.



$$A = xy$$

$$\frac{37.5\times10^6}{x} = y$$

$$P = 3x + 2y = 3x + 2\left(\frac{37.5\times10^6}{x}\right)$$

$$P = 3x + \frac{75\times10^6}{x}$$

$$x^{-1} \to -1x^{-2}$$

$$P' = 3 + (75\times10^6)\left(\frac{-1}{x^2}\right)$$