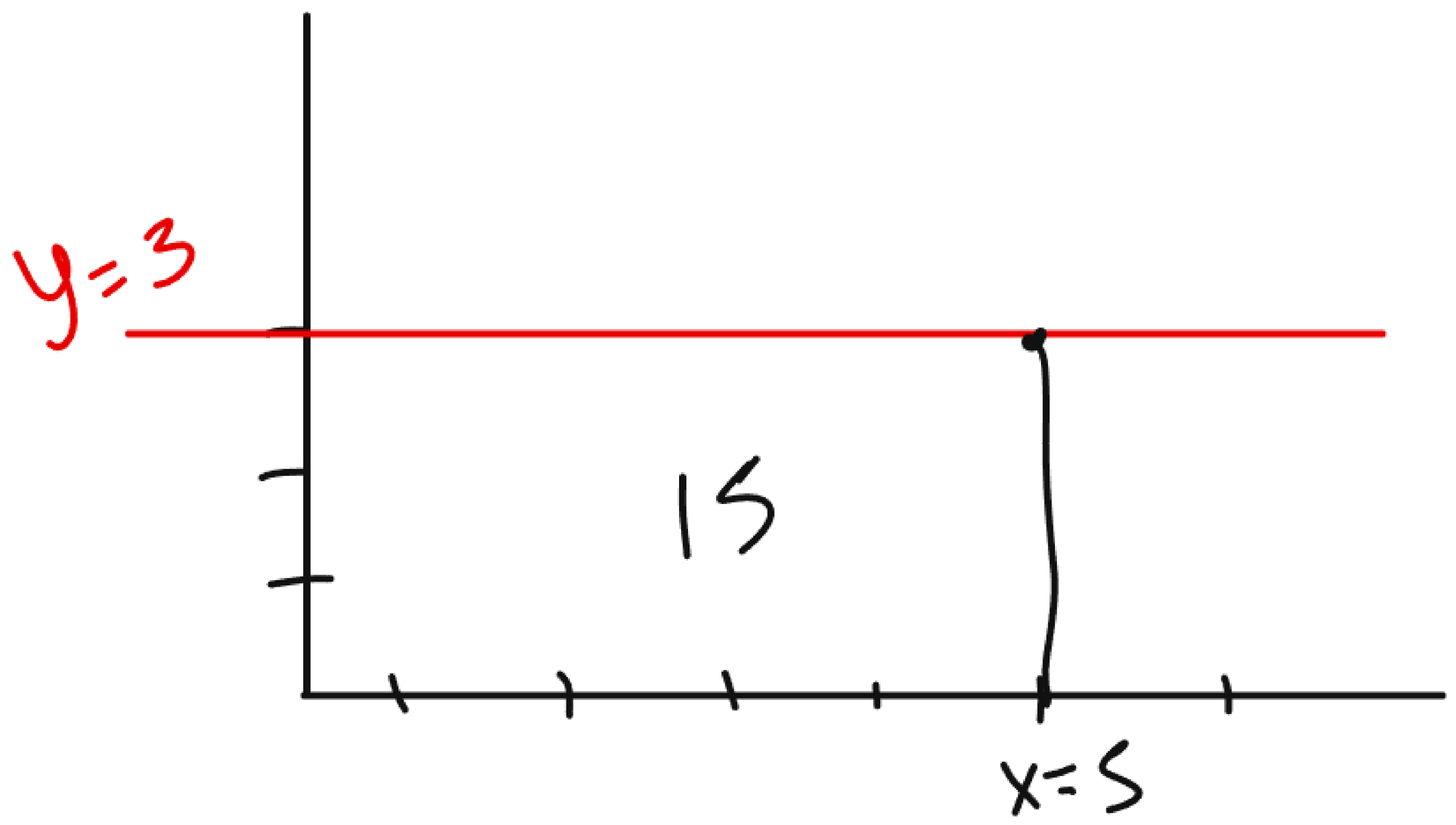
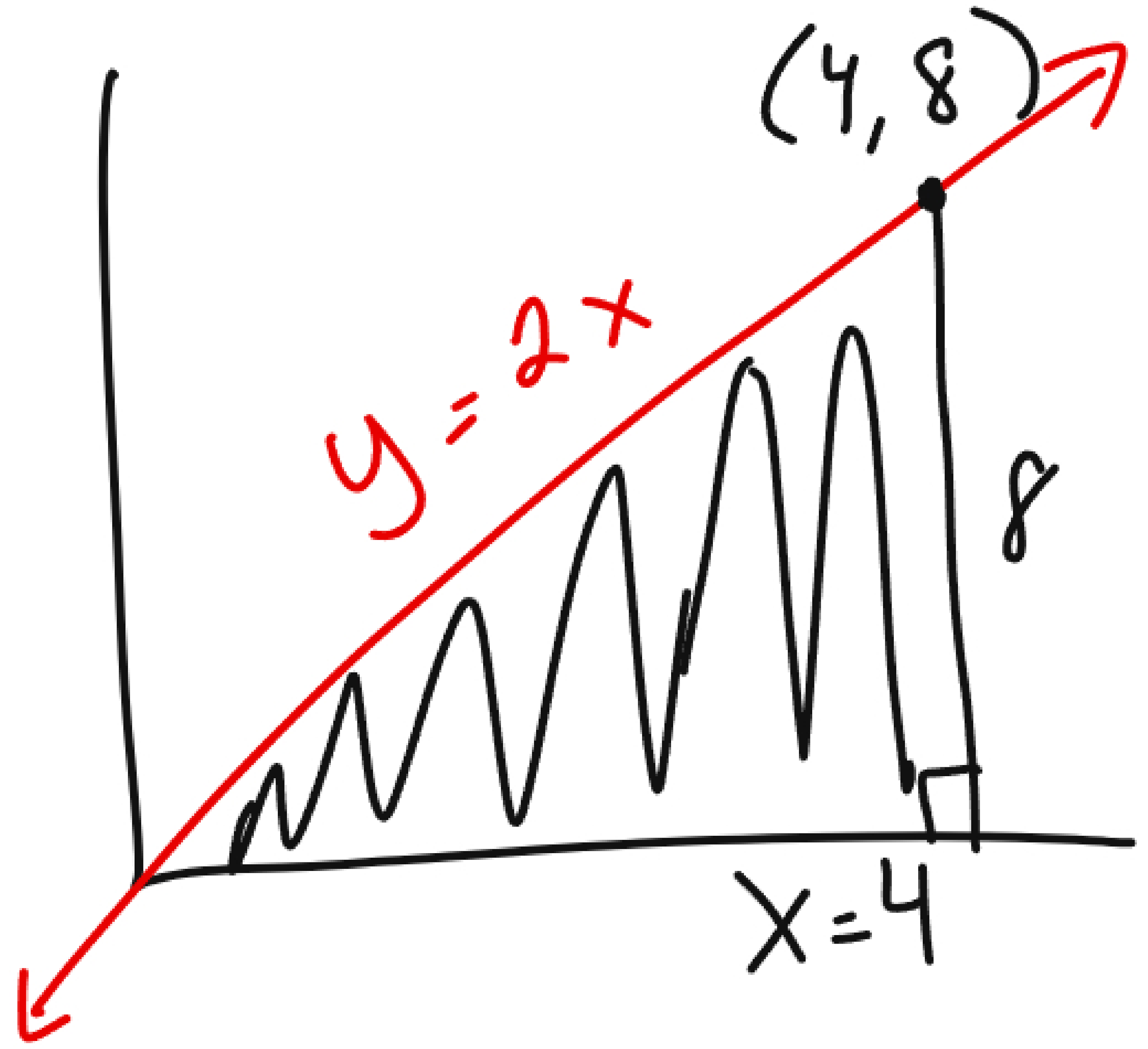
Antiderivative to Integral

$A = lw$



$f'(x) = 3$ $\qquad$ $f(x) = 3x + c$

$x = 5$ $\qquad$ $f(5) = 15$



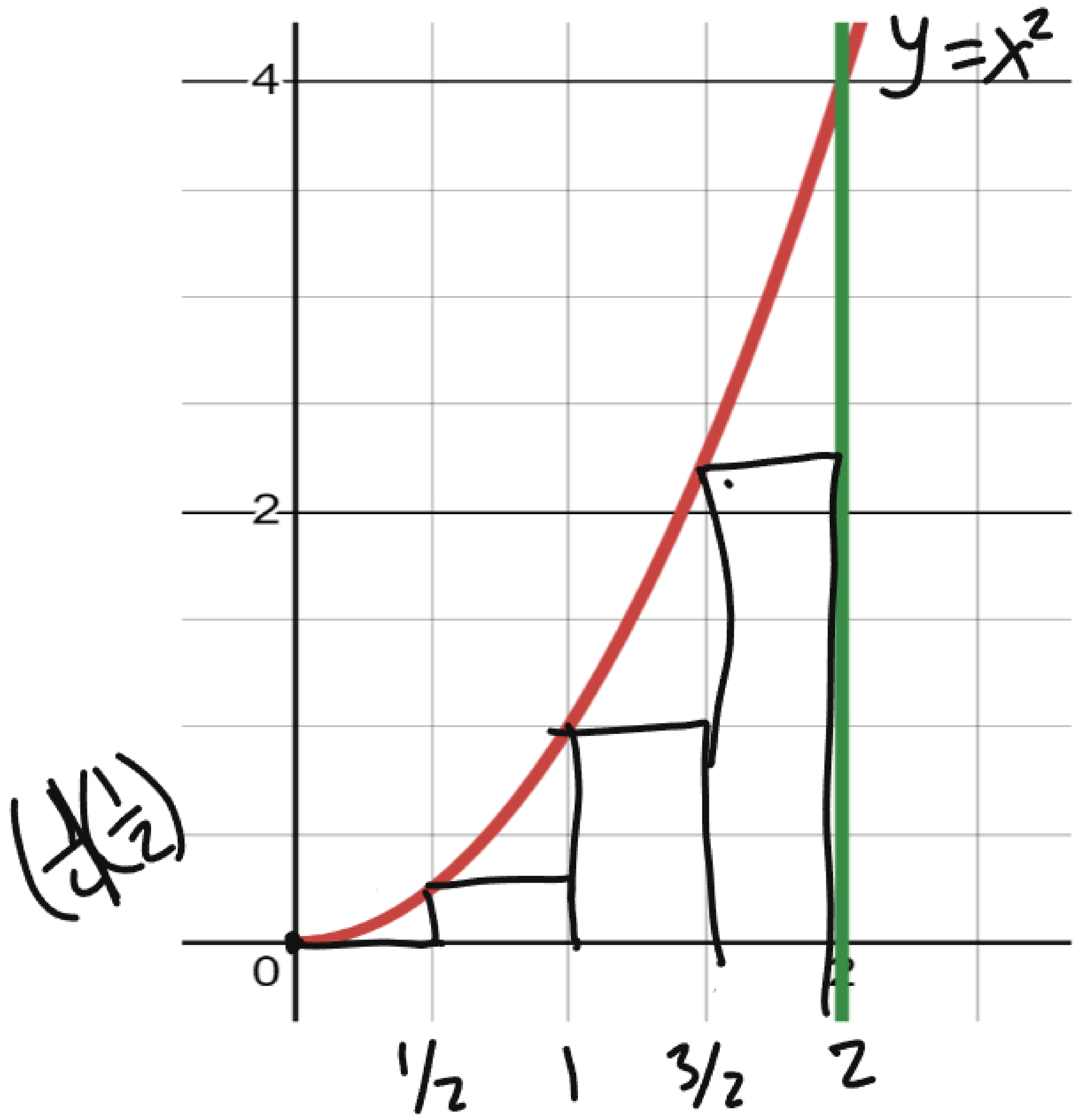
$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(4)(8)$$

$$= 16$$

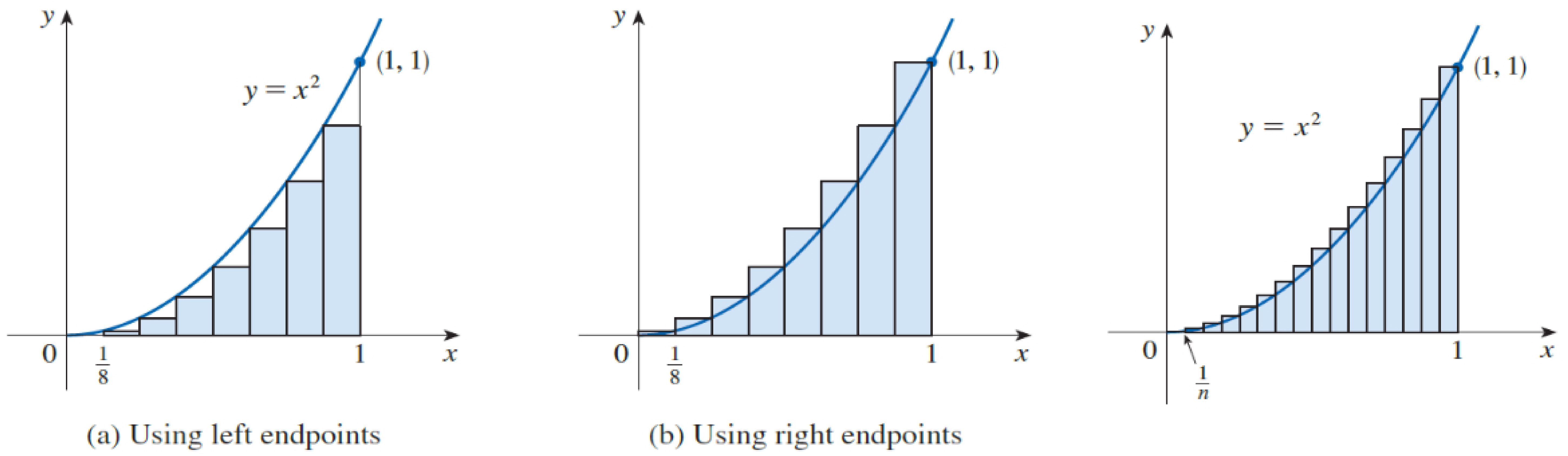
$$f'(x) = 2x \quad \rightarrow \quad f(x) = x^2$$

$$x = 4 \quad f(4) = 16$$



$A = 5$

$A = \frac{30}{8} = 3.75$



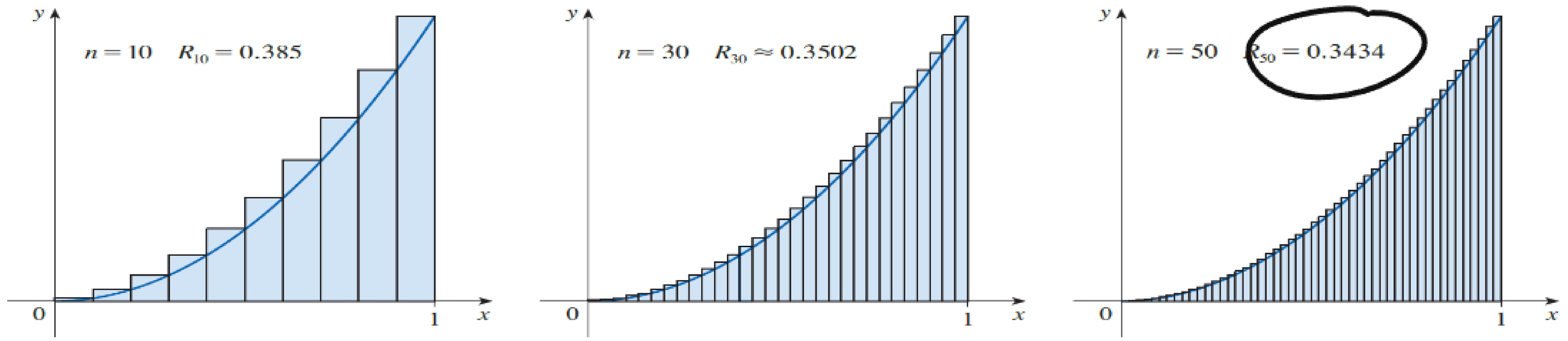


FIGURE 8 Right endpoints produce upper estimates because $f(x) = x^2$ is increasing.

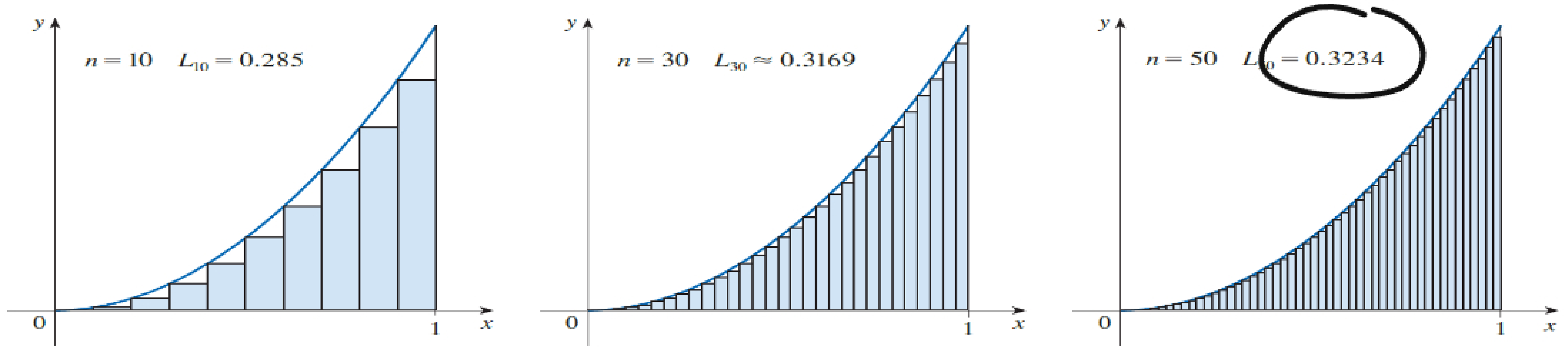


FIGURE 9 Left endpoints produce lower estimates because $f(x) = x^2$ is increasing.

$$f'(x) = x^2 \longrightarrow f(x) = \frac{x^3}{3}$$

$$\boxed{f(1) = \frac{1}{3}}$$

$$\sum_{n=0}^{\infty} n$$

function

Change in x

$$\int_a^b f(x) \, dx$$

Interval

$$\int_0^1 x^2\,dx$$ Definate Integral

$$\int_0^1 x^2\,dx = \frac{1}{3}x^3\Big]_{0 \leftarrow \text{start}}^{1 \leftarrow \text{end}}$$

$$= \frac{1}{3}(1)^3 - \frac{1}{3}(0)^3$$

$$\frac{1}{3} - 0 = \boxed{\frac{1}{3}}$$

$$\int_0^2 x^2\,dx = \frac{1}{3}x^3\Big]_0^2 = \frac{1}{3}(2)^3 - \frac{1}{3}(0)^3$$

$$= \boxed{\frac{8}{3}}$$

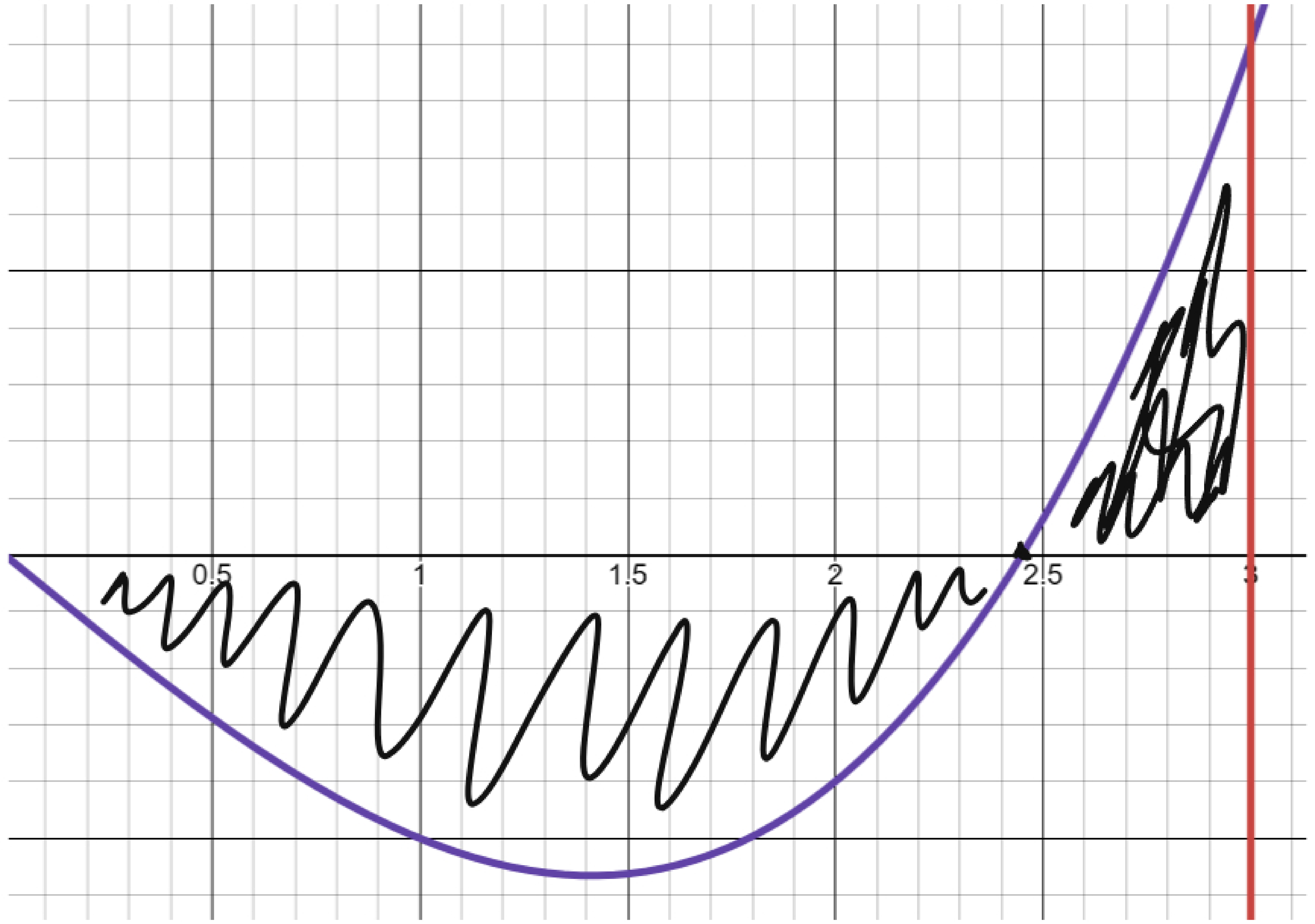
EXAMPLE 3 Evaluate $\int_0^3 (x^3 - 6x)\,dx$.

$$\left.\frac{1}{4}x^4 - 3x^2\right]_0^3$$

$$\left[\frac{1}{4}(3)^4 - 3(3)^2\right] - \left[\frac{1}{4}(0)^4 - 3(0)^2\right]$$

$$\frac{81}{4} - 27 = \frac{-27}{4}$$

$$\begin{aligned}
\int_0^3 (x^3 - 6x)\,dx &= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{3i}{n}\right)\frac{3}{n} \\
&= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left[\left(\frac{3i}{n}\right)^3 - 6\left(\frac{3i}{n}\right)\right] \\
&= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left[\frac{27}{n^3} i^3 - \frac{18}{n} i\right] \\
&= \lim_{n\to\infty} \left[\frac{81}{n^4} \sum_{i=1}^{n} i^3 - \frac{54}{n^2} \sum_{i=1}^{n} i\right] \\
&= \lim_{n\to\infty} \left\{\frac{81}{n^4}\left[\frac{n(n+1)}{2}\right]^2 - \frac{54}{n^2}\,\frac{n(n+1)}{2}\right\} \\
&= \lim_{n\to\infty} \left[\frac{81}{4}\left(1 + \frac{1}{n}\right)^2 - 27\left(1 + \frac{1}{n}\right)\right] \\
&\boxed{= \frac{81}{4} - 27 = -\frac{27}{4} = -6.75}
\end{aligned}$$



0.5
1
1.5
2
2.5
3

$$\int_1^3 e^x \, dx$$

$$e^x \Big]_1^3 = \boxed{e^3 - e^1}$$

$$\int_0^3 e^x \, dx = e^x \Big]_0^3 = e^3 - e^0 = \boxed{e^3 - 1}$$

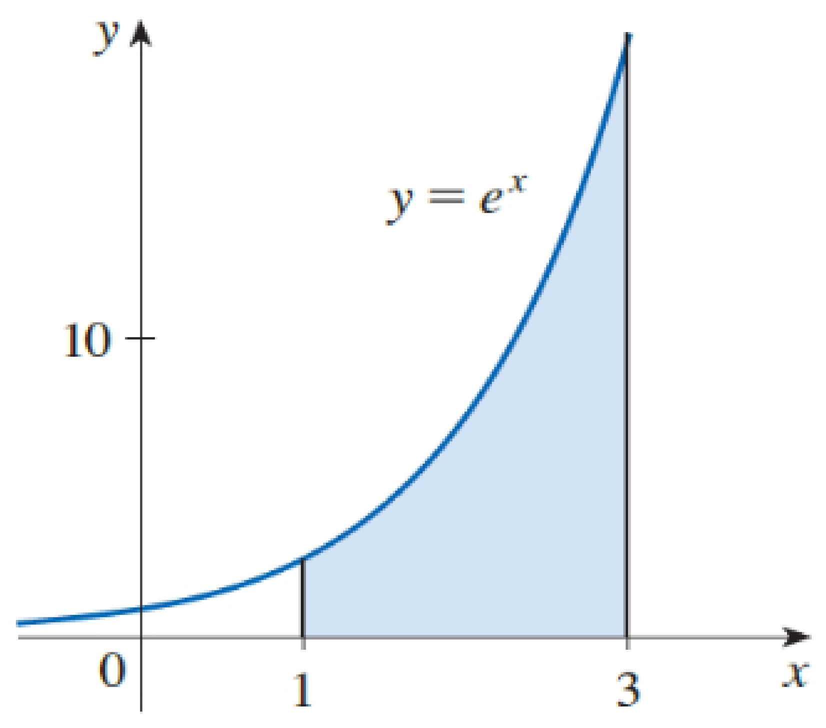


FIGURE 8

Properties of the Integral

1. $\int_a^b c\,dx = c(b - a)$, where c is any constant

2. $\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

3. $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$, where c is any constant

4. $\int_a^b [f(x) - g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

$$\int_{-1}^{2} (4x^2 + x + 2)\,dx$$

$$\left.\frac{4x^3}{3} + \frac{x^2}{2} + 2x\right]_{-1}^{2}$$

$$\left[\frac{4(2)^3}{3} + \frac{(2)^2}{2} + 2(2)\right] - \left[\frac{4(-1)^3}{3} + \frac{(-1)^2}{2} + 2(-1)\right]$$

$$\left(\frac{32}{3} + 2 + 4\right) - \left(-\frac{4}{3} + \frac{1}{2} - 2\right) = \frac{36}{3} + \frac{15}{2}$$

$$+\frac{4}{3} - \frac{1}{2} + 2 \qquad 12 + \frac{15}{2} = \frac{39}{2}$$

The Fundamental Theorem of Calculus, Part 1 If f is continuous on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t)\,dt \qquad a \leqslant x \leqslant b$$

FTC I

is continuous on $[a, b]$ and differentiable on (a, b), and $g'(x) = f(x)$.

The Fundamental Theorem of Calculus, Part 2 If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

FTC II

where F is any antiderivative of f, that is, a function F such that $F' = f$.

$$\int_0^{2\pi} \cos x \; dx = \sin x \Big]_0^{2\pi}$$

$$\sin(2\pi) - \sin(0) =$$

$$0 - 0 = 0$$

$$\int_0^{\pi/2} \cos x \, dx = \sin x \Big]_0^{\pi/2} = (1) - (0) = 1$$

1 Table of Indefinite Integrals

$$\int cf(x)\,dx = c\int f(x)\,dx \qquad \int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int k\,dx = kx + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1) \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C \qquad \int b^x\,dx = \frac{b^x}{\ln b} + C$$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C \qquad \boxed{\int \csc x \cot x\,dx = -\csc x + C}$$

$$\int \frac{1}{x^2+1}\,dx = \tan^{-1}x + C \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$$

$$\int \sinh x\,dx = \cosh x + C \qquad \int \cosh x\,dx = \sinh x + C$$

$$\int (10x^4 - 2\sec^2 x)\,dx$$

$$\boxed{2x^5 - 2\tan x + C}$$

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Evaluate $\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$.

$$\cancel{\frac{\int \cos\theta}{\int \sin\theta \int \sin\theta}}$$

$$\frac{-\sin\theta}{\cos^2\theta} + C$$

$$-\csc\theta + C$$

$$\int \frac{\cos}{\sin} \cdot \frac{1}{\sin}$$

$$\int \cot(x)\csc(x)\,dx =$$

$$-\csc x + C$$

$$\int \left(2x^3 - 6x + \frac{3}{x^2+1}\right) dx$$

$$\frac{1}{2}x^4 - 3x^2 + 3\tan^{-1}(x) + C$$

(a) Find the displacement of the particle during the time period $1 \leqslant t \leqslant 4$.
(b) Find the distance traveled during this time period.

$$v(t) = p'(t)$$

$$\int v(t) = p(t)$$

$$\int t^2 - t - 6 \, dt = \frac{1}{3}t^3 - \frac{1}{2}t^2 - 6t = p(t)$$

a) $\frac{1}{3}t^3 - \frac{1}{2}t^2 - 6t\Big]_1^4 = \frac{-9}{2}$

b) $v(t) = t^2 - t - 6 = (t-3)(t+2)$

$t = 3 \quad t = -2$

$\frac{22}{3}$ $1 \to 3$

$17/6$ $3 \to 4$

$$\left| \frac{1}{3}t^3 - \frac{1}{2}t^2 - 6t\Big]_1^3 \right| = \frac{-27}{2} - \left(\frac{-37}{6}\right) = \frac{-22}{3}$$

$$\left| \frac{1}{3}t^3 - \frac{1}{2}t^2 - 6t\Big]_3^4 \right| = \frac{-32}{3} - \left(\frac{-27}{2}\right) = \frac{17}{6}$$

$$\frac{22}{3} + \frac{17}{6} = \boxed{\frac{61}{6}}$$