$$f(x) = \boxed{(\boxed{3x^2+4})^5}$$

$$f'(x) = 6x\,(5)\,(3x^2+4)^4$$

$$f'(x) = \boxed{30x}\,(3x^2+4)^4$$

Chain Rule $\leftrightarrow$ u-substitution

d/dx $\qquad\qquad$ $\int dx$

$$f'(x) = 30x(\boxed{3x^2+4})^4$$

$u = 3x^2 + 4$

$du = 6x\,dx$

$$\int 30x(3x^2+4)^4\,dx$$

$$\int 5(3x^2+4)^4\,6x\,dx$$

$$\int 5(u)^4\,du \rightarrow 5\frac{u^5}{5} = u^5$$

$$\boxed{(3x^2+4)^5}$$

4 The Substitution Rule If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then

$$\int f(g(x))\, g'(x)\, dx = \int f(u)\, du$$

Find $\int x^3 \cos(\underline{x^4 + 2})\,dx$

$u = x^4 + 2$

$du = 4x^3\,dx$

$\frac{du}{4x^3} = dx$

$\int \cancel{x^3} \cos(u) \frac{du}{4\cancel{x^3}}$

$\int \frac{1}{4} \cos(u)\,du$

$\frac{1}{4} \int \cos(u)\,du = \frac{1}{4} \sin(u) \rightarrow \boxed{\frac{1}{4} \sin(x^4 + 2) + C}$

Evaluate $\int \sqrt{2x+1}\,dx$. $\rightarrow \int (2x+1)^{1/2}\,dx$

$u = 2x+1 \quad \rightarrow \quad du = 2\,dx$

$\frac{1}{2}du = dx$

$\int u^{1/2} \cdot \frac{1}{2}du = \frac{1}{2}\int u^{1/2}\,du$

$= \frac{1}{2}\left[\frac{u^{3/2}}{3/2}\right] = \boxed{\frac{1}{3}(2x+1)^{3/2} + C}$

Evaluate $\int e^{5x}\,dx$

$u = 5x$

$du = 5\,dx$

$\frac{1}{5}du = dx$

$\int e^u \cdot \frac{1}{5}du$

$\frac{1}{5}\int e^u\,du = \frac{1}{5}e^u \rightarrow \boxed{\frac{1}{5}e^{5x} + C}$

$$\int \frac{x^3}{x^4 - 5}\, dx$$

$u = x^4 - 5$

$du = 4x^3\, dx \rightarrow \frac{du}{4x^3} = dx$

$$\int \frac{\cancel{x^3}}{u} \cdot \frac{1}{4\cancel{x^3}}\, du \rightarrow \frac{1}{4} \int \frac{1}{u}\, du$$

$$= \frac{1}{4} \ln|u| \rightarrow \boxed{\frac{1}{4} \ln|x^4 - 5| + C}$$

$\cancel{u^{-1}}$

Evaluate $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$

$u = \cos x \qquad du = -\sin x\,dx \rightarrow \frac{-1}{\sin x}\,du = dx$

$\int \frac{\cancel{\sin x}}{u} \cdot \frac{-1}{\cancel{\sin x}}\,du = -\int \frac{1}{u}\,du$

$= -\ln|u| \rightarrow \boxed{-\ln|\cos x| + C}$

Evaluate $\int_1^e \frac{\ln x}{x}\,dx$

$$u = \ln x$$

$$du = \frac{1}{x}\,dx$$

$$x\,du = dx$$

$$\int \frac{u}{\cancel{x}} \cdot \cancel{x}\,du = \int u\,du = \frac{1}{2}u^2$$

$$\frac{1}{2}[\ln(x)]^2 \Big|_1^e = \frac{1}{2}(\ln(e)^2 - \ln(1)^2)$$

$$= \frac{1}{2}(1 - 0) = \boxed{\frac{1}{2}}$$

$$\int_0^1 \sqrt[3]{1 + 7x}\, dx$$

$$u = 1 + 7x$$

$$du = 7\,dx \rightarrow \frac{1}{7}du = dx$$

$$\int_0^1 u^{1/3} \cdot \frac{1}{7} du$$

$$\frac{1}{7}\int_0^1 u^{1/3} du = \frac{1}{7}\left(\frac{3}{4}u^{4/3}\right) = \frac{3}{28}\left[u^{4/3}\right]_0^1$$

$$= \frac{3}{28}(1-0) = \boxed{\frac{3}{28}}$$

$$\int \frac{dx}{\sqrt{1-x^2}\,\sin^{-1}x}$$

$$\int \frac{\cancel{\sqrt{1-x^2}}\,du}{\cancel{\sqrt{1-x^2}}\,u}$$

$$\int \frac{1}{u}\,du = \ln|u|$$

$$\boxed{\ln|\sin^{-1}(x)| + C}$$

$$\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$u = \sin^{-1}(x)$$

$$du = \frac{1}{\sqrt{1-x^2}}\,dx$$

$$\sqrt{1-x^2}\,du = dx$$

Find $\int \sqrt{1+x^2}\, x^5\, dx$

$$\boxed{u = 1 + x^2}$$

$$du = 2x\, dx$$

$$\frac{1}{2x}\, du = dx$$

$$\int (u^{1/2})(x^5)\left(\frac{1}{2x}\right) du$$

$$\frac{1}{2}\int u^{1/2}\, \boxed{x^4}\, du \quad ?$$

$$u = 1 + x^2$$

$$\sqrt{u-1} = x$$

$$(u-1)^2 = x^4$$

$$\frac{1}{2}\int u^{1/2}(u-1)^2\, du$$

$$(u-1)^2 = u^2 - 2u + 1$$

$$u^{1/2}$$

$$= u^{5/2} - 2u^{3/2} + u^{1/2}$$

$$\frac{1}{2}\int u^{5/2} - 2u^{3/2} + u^{1/2}\, du$$

$$\frac{1}{2}\left[\frac{2}{7}u^{7/2} - 2\left(\frac{2}{5}\right)u^{5/2} + \frac{2}{3}u^{3/2}\right]$$

$$\boxed{\frac{1}{7}(1+x^2)^{7/2} - \frac{2}{5}(1+x^2)^{5/2} + \frac{1}{3}(1+x^2)^{3/2} + C}$$