1) Given $x = t^2$ and $y = t^3, 0 \le t \le 1$ find the following:

a. Tangent line at $\left(\frac{1}{4}, \frac{1}{8}\right)$

b. Area under the curve

c. Surface Area rotated around the x-axis

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2}{2t}$$

x y

$$\left(\frac{1}{4}, \frac{1}{8}\right) \rightarrow t = \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{3}{2}t = \frac{3}{4}$$

a) $\boxed{y - \frac{1}{8} = \frac{3}{4}\left(x - \frac{1}{4}\right)}$

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$$\int y \, dx$$

$$\int_0^1 (t^3)(2t)\, dt$$

$$\int_0^1 2t^4 \, dt = \frac{2}{5}t^5 \Big]_0^1 = \boxed{\frac{2}{5}}$$

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$$S = \int_{\alpha}^{\beta} 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$\int_0^1 2\pi(t^3)\sqrt{(2t)^2 + (3t^2)^2}\, dt$$

$$\int_0^1 2\pi t^3 \sqrt{4t^2 + 9t^4}$$

$$\int_0^1 2\pi x^3 \sqrt{4x^2 + 9x^4}\, dx$$

$= 4.03913679135$

$$\boxed{4.04}$$

Given $r = 2\sin\theta$, $0 \le \theta \le \pi$ find the following:

a. Tangent line at $\theta = \frac{\pi}{2}$

b. Area under the curve

c. Arc Length

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

$r = 2\sin\theta$

$dr/d\theta = 2\cos\theta$

$$\boxed{y = 2}$$

$$\frac{dy}{dx} = \frac{2\cos\theta\sin\theta + 2\sin\theta\cos\theta}{2\cos\theta\cos\theta - 2\sin\theta\sin\theta} = \frac{0}{-2} = 0$$

0 1 1 0

0 0 1 1

Given $r = 2\sin\theta, 0 \le \theta \le \pi$ find the following:

a. Tangent line at $\theta = \frac{\pi}{2}$

b. Area under the curve

c. Arc Length

$$A = \int_a^b \frac{1}{2} r^2 \, d\theta$$

$$A = \frac{1}{2}\int_0^{\pi} \underbrace{(2\sin\theta)^2}_{4} \, d\theta \qquad \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

$$= 2\int_0^{\pi} \sin^2\theta \, d\theta$$

$$= \int_0^{\pi} 1 - \cos 2\theta \, d\theta \rightarrow \theta - \frac{1}{2}\sin 2\theta \Big]_0^{\pi} = \boxed{\pi}$$

Given $r = 2\sin\theta, 0 \le \theta \le \pi$ find the following:

a. Tangent line at $\theta = \frac{\pi}{2}$

b. Area under the curve

c. Arc Length

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$\int_0^{\pi} \sqrt{(2\sin\theta)^2 + (2\cos\theta)^2}\; d\theta$$

$4\sin^2\theta + 4\cos^2\theta$

$4(\sin^2\theta + \cos^2\theta)$

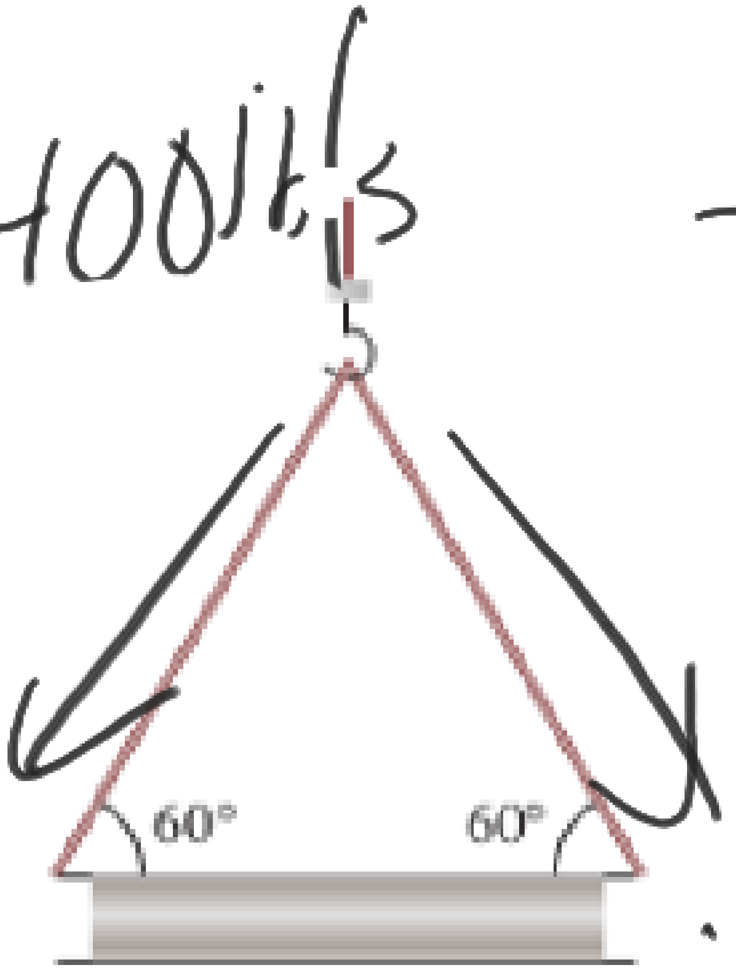
$\sqrt{4} = 2$

$$\int_0^{\pi} 2\, d\theta = 2\theta \Big]_0^{\pi} = \boxed{2\pi}$$

3) A crane suspends a 400-lb steel beam horizontally by support cables attached from a hook to each end of the beam. The support cables each make an angle of 60° with the beam. Find the tension vector in each support cable and the magnitude of each tension

$T_1 + T_2 = -w = 400\text{lbs}$

$400j$



$T_1 = |T_1| \cos 60° i - |T_1| \sin 60° j$

$T_2 = |T_2| \cos 60 i - |T_2| \sin 60 j$

$-i - j$

$i - j$

$$\boxed{-|T_1|\cos 60 + |T_2|\cos 60 = 0}$$

$$-|T_1|\sin 60 - |T_2|\sin 60 = 400$$

$$|T_2| = \frac{|T_1|\cos 60}{\cos 60} \qquad |T_2| = |T_1| = -230.9$$

$$-2|T_1|\sin 60 = 400$$

$$|T_1|\sin 60 = -200$$

$$\frac{-200}{\sin 60} = \frac{-200}{\sqrt{3}/2}$$

$$-400/\sqrt{3}$$

$\underline{T_1} = |T_1| \cos 60^\circ i - |T_1| \sin 60^\circ j$

230.9

$T_2 = |T_2| \cos 60 \, i - |T_2| \sin 60 \, j$

$$\boxed{\begin{aligned} T_1 &= -230.9\left(\tfrac{1}{2}\right) i - 230.9\left(\tfrac{\sqrt{3}}{2}\right) j \\ T_2 &= 230.9\left(\tfrac{1}{2}\right) i - 230.9\left(\tfrac{\sqrt{3}}{2}\right) j \end{aligned}}$$

4) Find the angle between the vectors **u = I – 3j + 2k** and **v = –2i + j + 4k.**

$$a \cdot b = (1)(-2) + (-3)(1) + (2)(4)$$

$$= -2 - 3 + 8 = \boxed{3}$$

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

$$|a| = \sqrt{1^2 + 3^2 + 2^2} = \sqrt{14}$$

$$|b| = \sqrt{2^2 + 1^2 + 4^2} = \sqrt{21}$$

$$\cos\theta = \frac{3}{\sqrt{14}\sqrt{21}}$$

$$\theta = 79.9^\circ$$

5) Use the scalar triple product to determine whether the points A(3, 0, 2), B(-1, 2, 5), C(5, 1, -1) and (0, 4, 2) lie in the same plane

D

$\overrightarrow{AB} = \langle -4, 2, 3 \rangle$

$\overrightarrow{AC} = \langle 2, 1, -3 \rangle$

$\overrightarrow{AD} = \langle -3, 4, 0 \rangle$

$$\begin{vmatrix} -4 & 2 & 3 \\ 2 & 1 & -3 \\ -3 & 4 & 0 \end{vmatrix}$$

$$-4\begin{vmatrix} 1 & -3 \\ 4 & 0 \end{vmatrix} - 2\begin{vmatrix} 2 & -3 \\ -3 & 0 \end{vmatrix} + 3\begin{vmatrix} 2 & 1 \\ -3 & 4 \end{vmatrix}$$

$$-4\begin{vmatrix} 1 & -3 \\ 4 & 0 \end{vmatrix} - 2\begin{vmatrix} 2 & -3 \\ -3 & 0 \end{vmatrix} + 3\begin{vmatrix} 2 & 1 \\ -3 & 4 \end{vmatrix}$$

$$-4(0-(-12)) - 2(0-9) + 3(8-(-3))$$

$$-48 + 18 + 33 \neq 0$$

$$-30+33 = 3$$