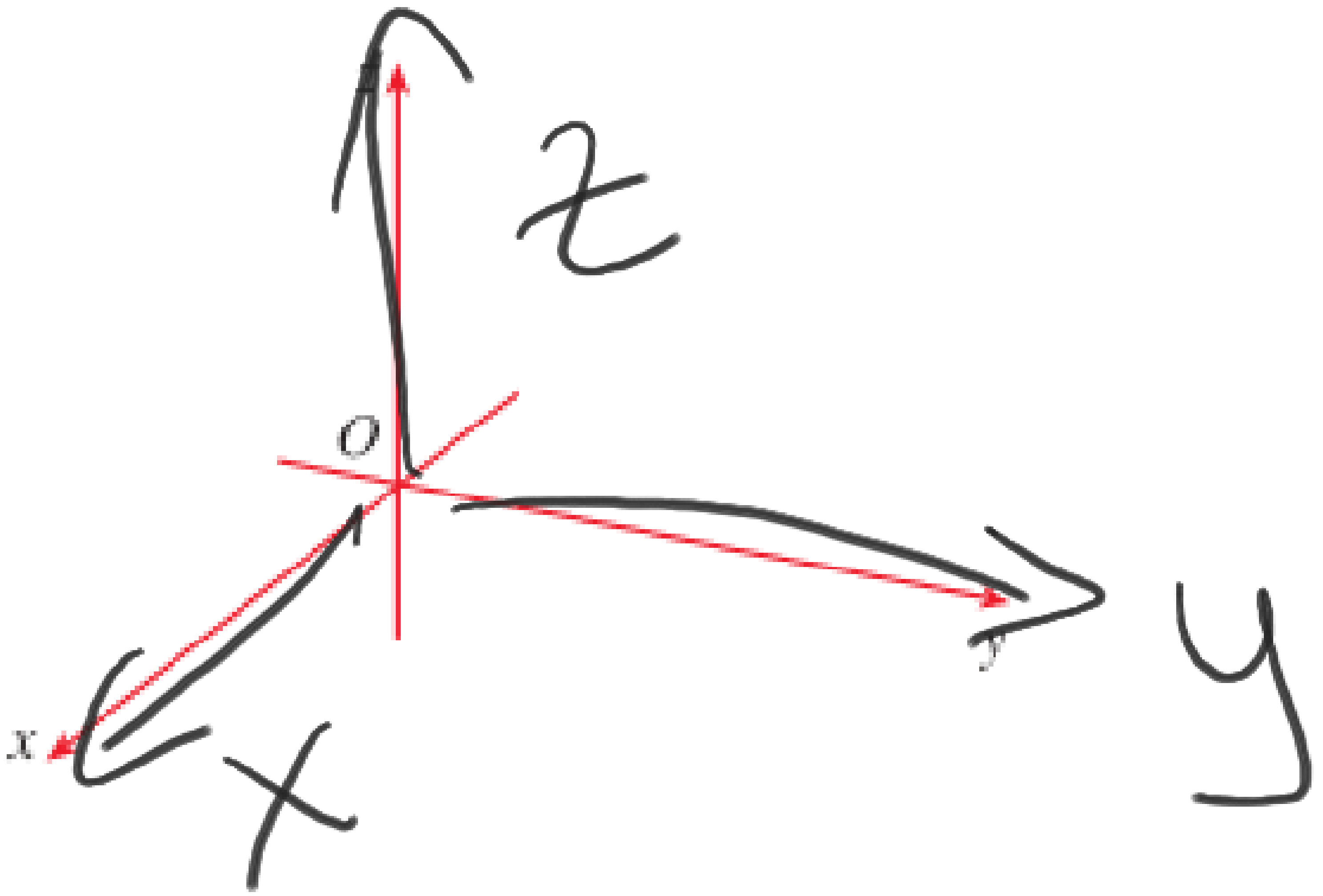
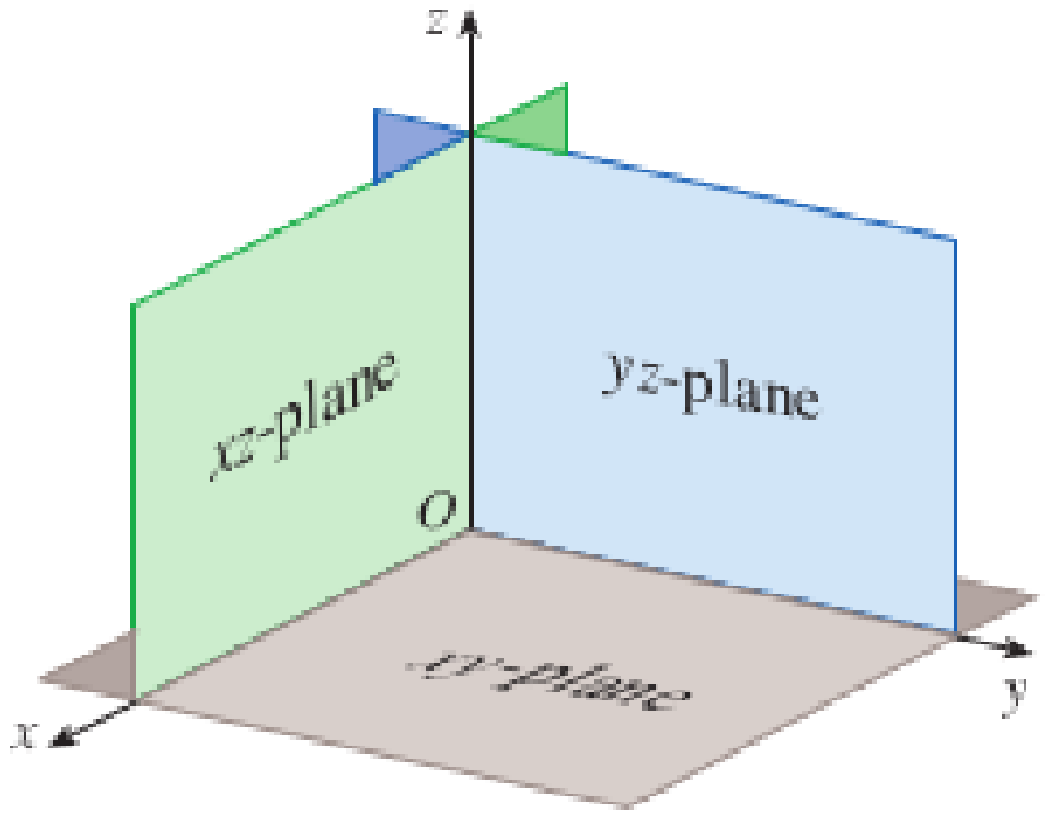


$$(x, y, z)$$



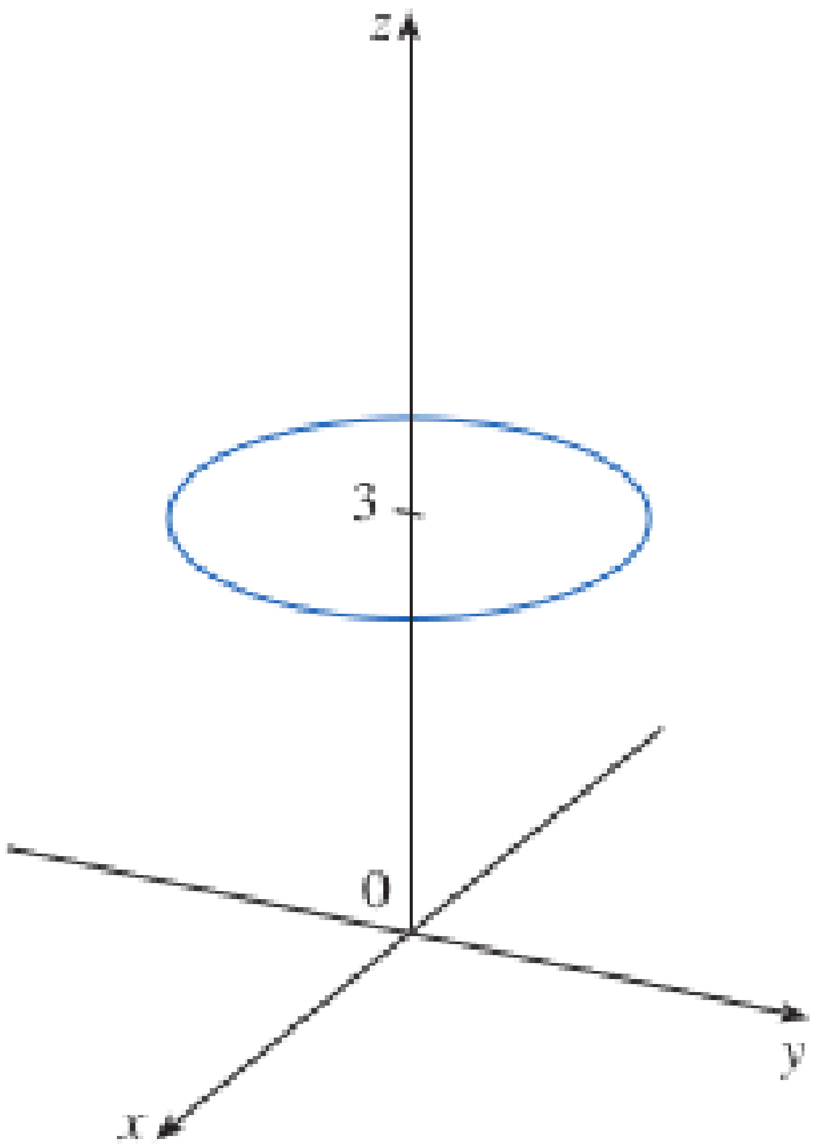
(a) Coordinate planes

Which points (x, y, z) satisfy the equations

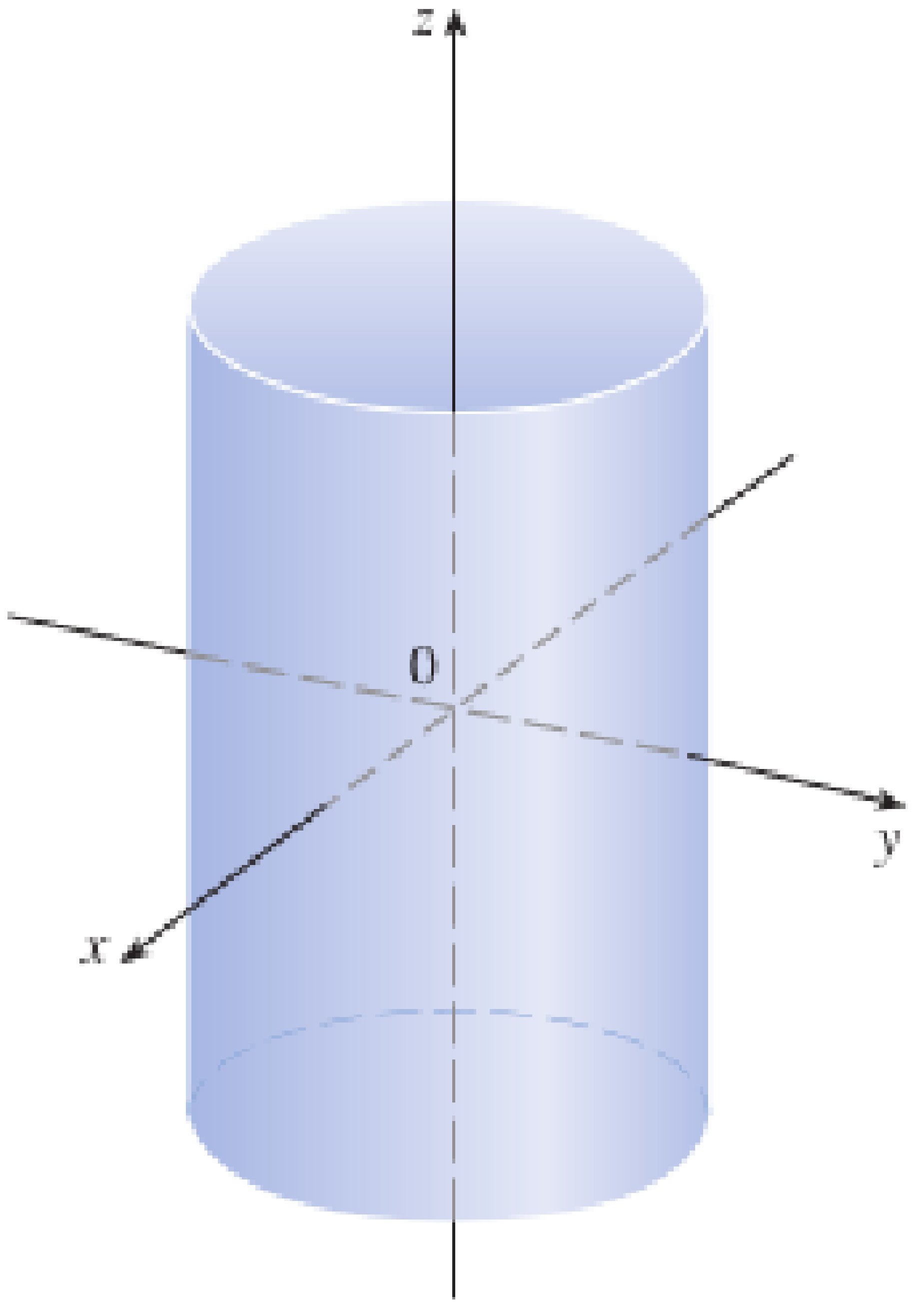
$$x^2 + y^2 = 1$$

and

$$z = 3$$



What does the equation $x^2 + y^2 = 1$ represent as a surface in $\mathbb{R}^3$?



Distance Formula in Three Dimensions

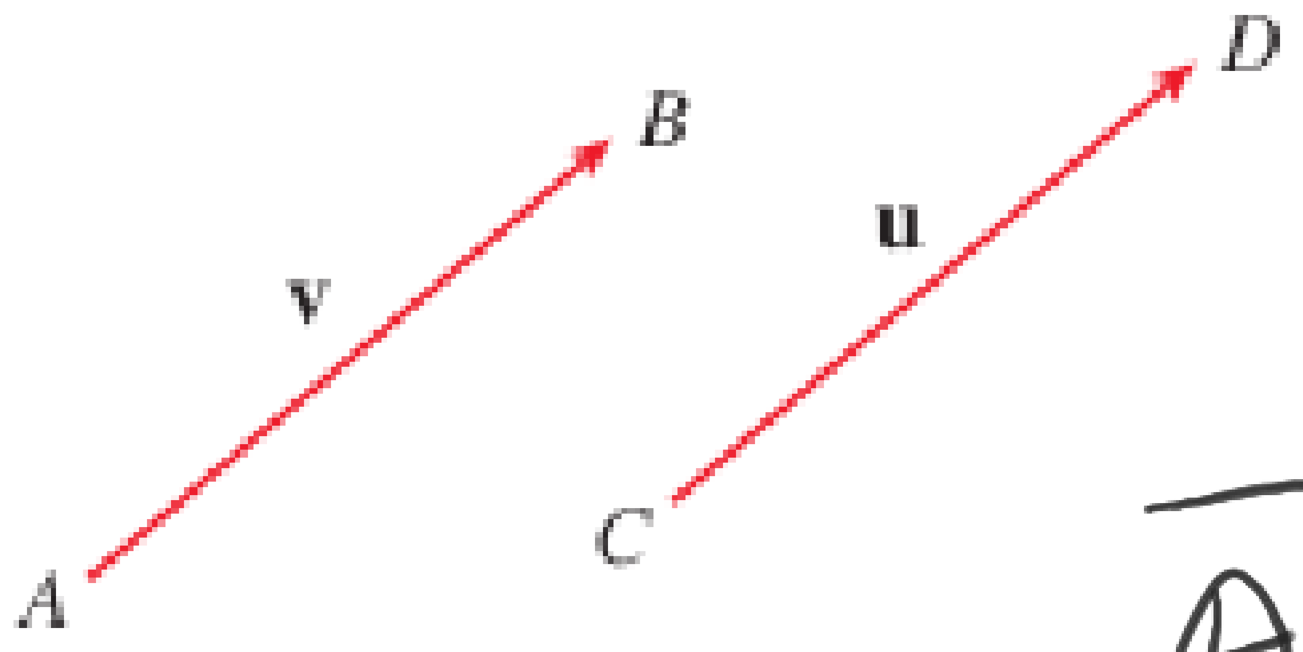
The distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

The distance from the point $P(2, -1, 7)$ to the point $Q(1, -3, 5)$ is

$$|PQ| = \sqrt{(1)^2 + (2)^2 + (2)^2} = \sqrt{9} \boxed{= 3}$$

Vectors

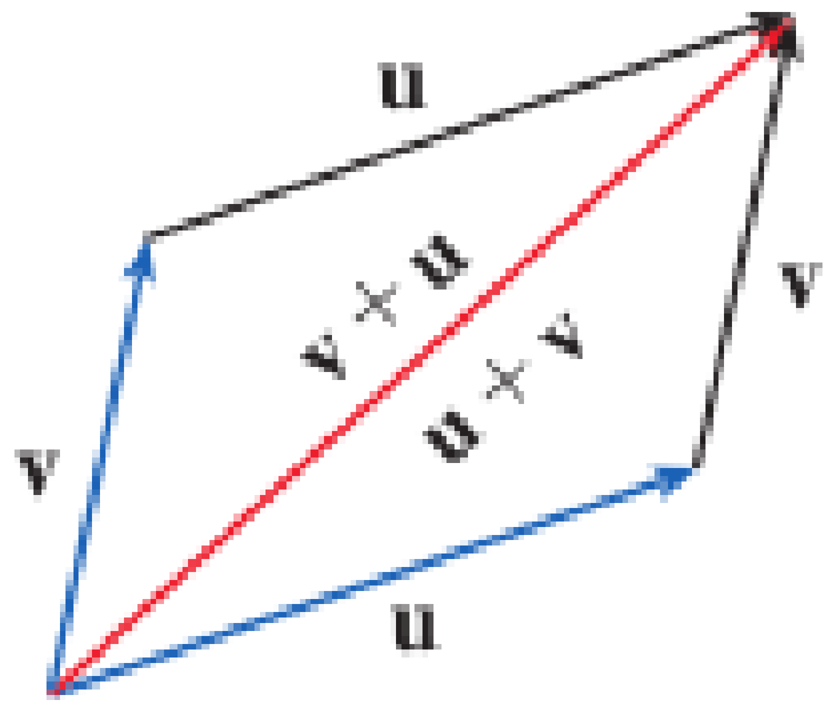


$\overrightarrow{AB} = \overrightarrow{CD}$

$\overrightarrow{AB}$ $\quad$ $\vec{v}$

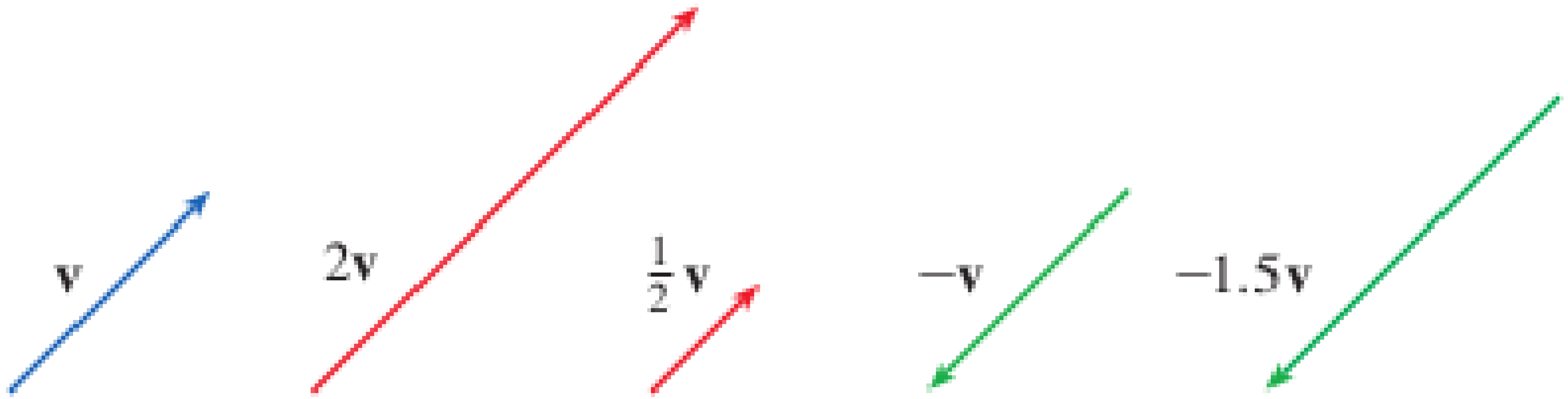
Definition of Vector Addition

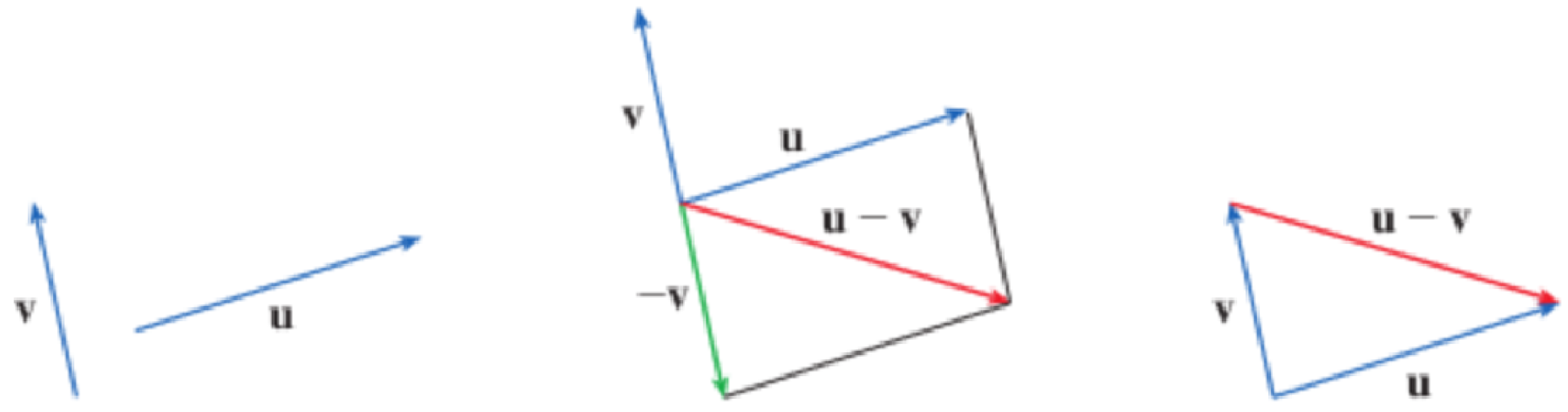
If $\mathbf{u}$ and $\mathbf{v}$ are vectors positioned so the initial point of $\mathbf{v}$ is at the terminal point of $\mathbf{u}$, then the **sum** $\mathbf{u}+\mathbf{v}$ is the vector from the initial point of $\mathbf{u}$ to the terminal point of $\mathbf{v}$.



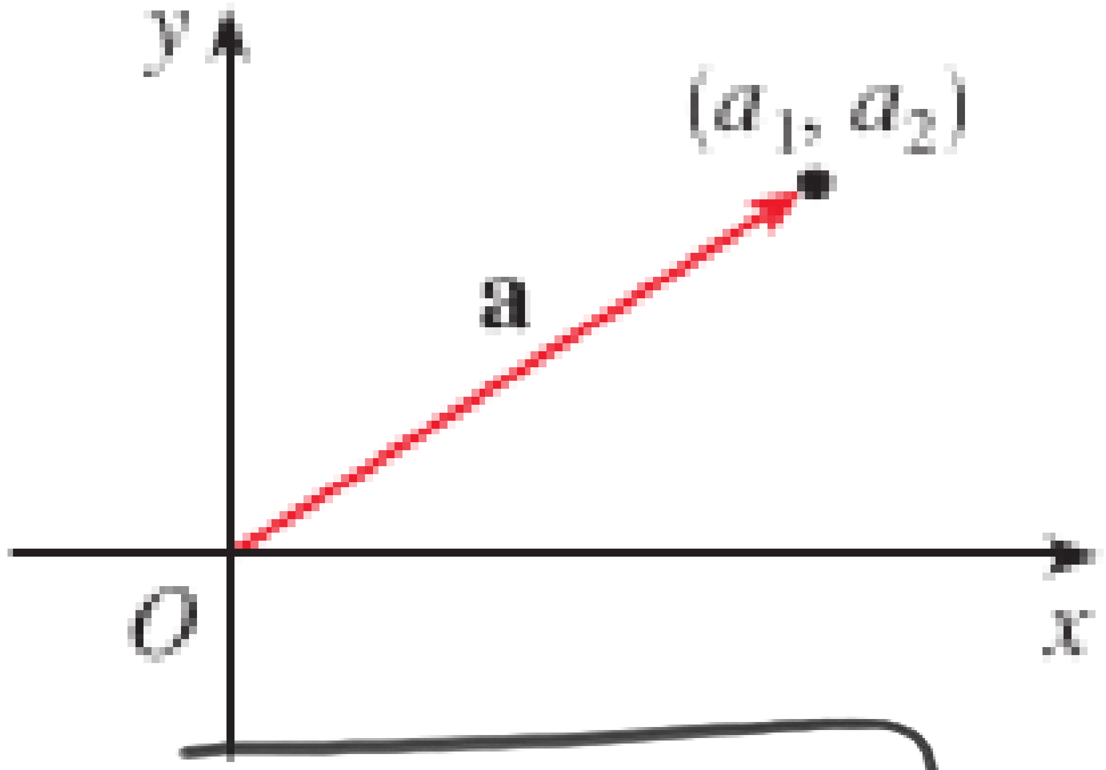
Definition of Scalar Multiplication

If c is a scalar and $\mathbf{v}$ is a vector, then the **scalar multiple** $c\mathbf{v}$ is the vector whose length is $|c|$ times the length of $\mathbf{v}$ and whose direction is the same as $\mathbf{v}$ if $c > 0$ and is opposite to $\mathbf{v}$ if $c < 0$. If $c = 0$ or $\mathbf{v} = \mathbf{0}$, then $c\mathbf{v} = \mathbf{0}$.

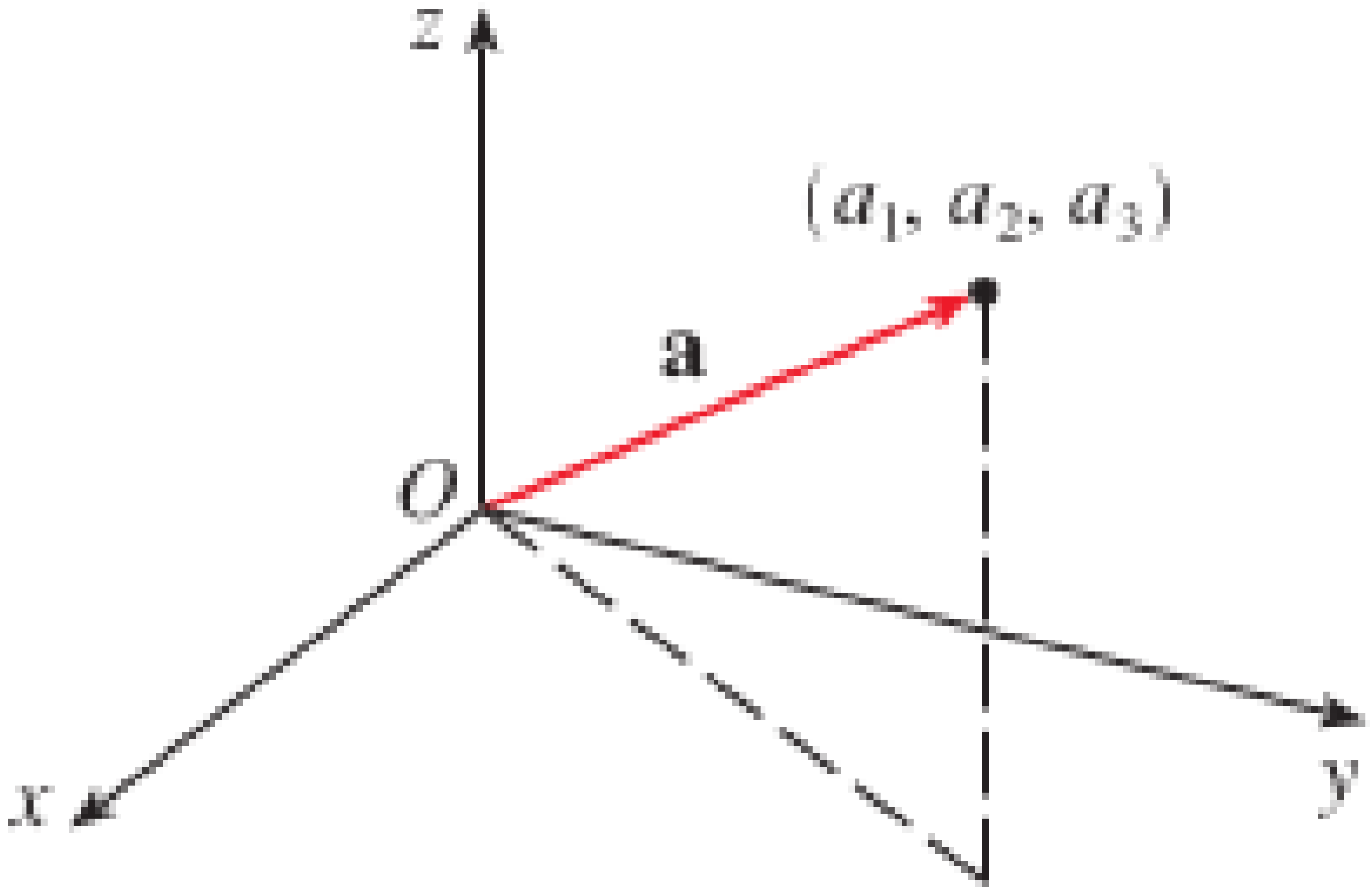




(a) (b) (c)



$\boxed{\mathbf{a} = \langle a_1, a_2 \rangle}$



$\mathbf{a} = \langle a_1, a_2, a_3 \rangle$

$\langle a_1, a_2 \rangle$

Find the vector represented by the directed line segment with initial point $A(2, -3, 4)$ and terminal point $B(-2, 1, 1)$.

$\langle 3, 2 \rangle$

A B

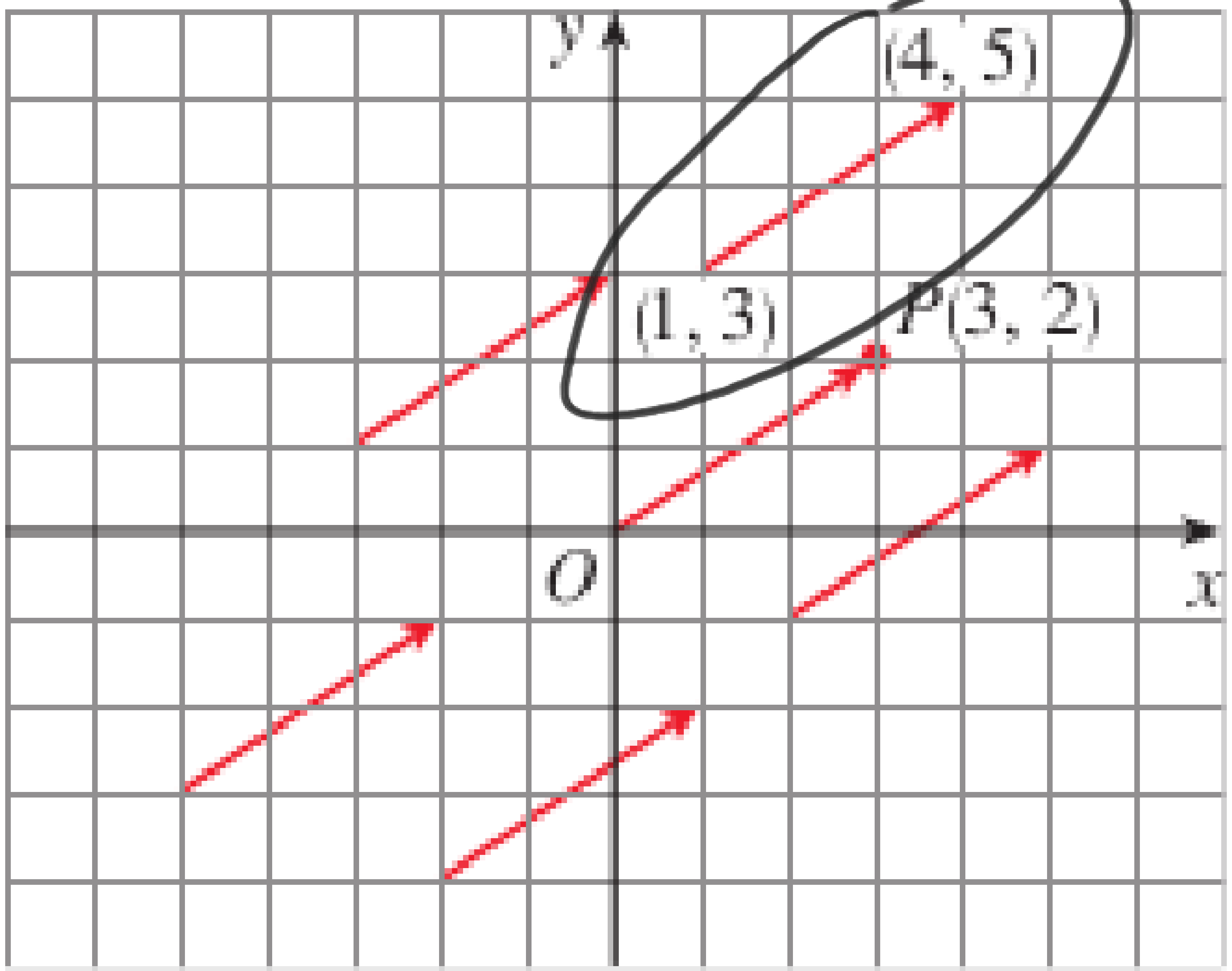
(2, -3, 4) (-2, 1, 1)

Intial Terminal

Vector

$\langle -4, 4, 3 \rangle$

$-2-2 \quad 1-{}^{-}3 \quad 1-4$



If $\mathbf{a} = \langle 4, 0, 3 \rangle$ and $\mathbf{b} = \langle -2, 1, 5 \rangle$, find $|\mathbf{a}|$ and the vectors $\mathbf{a} + \mathbf{b}$, $\mathbf{a} - \mathbf{b}$, $3\,\mathbf{b}$, and $2\mathbf{a} + 5\,\mathbf{b}$.

$a = \langle 4, 0, 3 \rangle \qquad b = \langle -2, 1, 5 \rangle$

$|a| \rightarrow$ distance of vector a

magnitude

$$|a| = \sqrt{4^2 + 0^2 + 3^2} = \sqrt{25} = 5$$

$$|b| = \sqrt{(-2)^2 + 1^2 + 5^2} = \sqrt{30}$$

If $\mathbf{a} = \langle 4, 0, 3 \rangle$ and $\mathbf{b} = \langle -2, 1, 5 \rangle$, find $|\mathbf{a}|$ and the vectors $\mathbf{a} + \mathbf{b}$, $\mathbf{a} - \mathbf{b}$, $3\mathbf{b}$, and $2\mathbf{a} + 5\mathbf{b}$.

$a = \langle 4, 0, 3 \rangle \qquad b = \langle -2, 1, 5 \rangle$

$a + b = \langle 2, 1, 8 \rangle$

$a - b = \langle 6, -1, -2 \rangle$

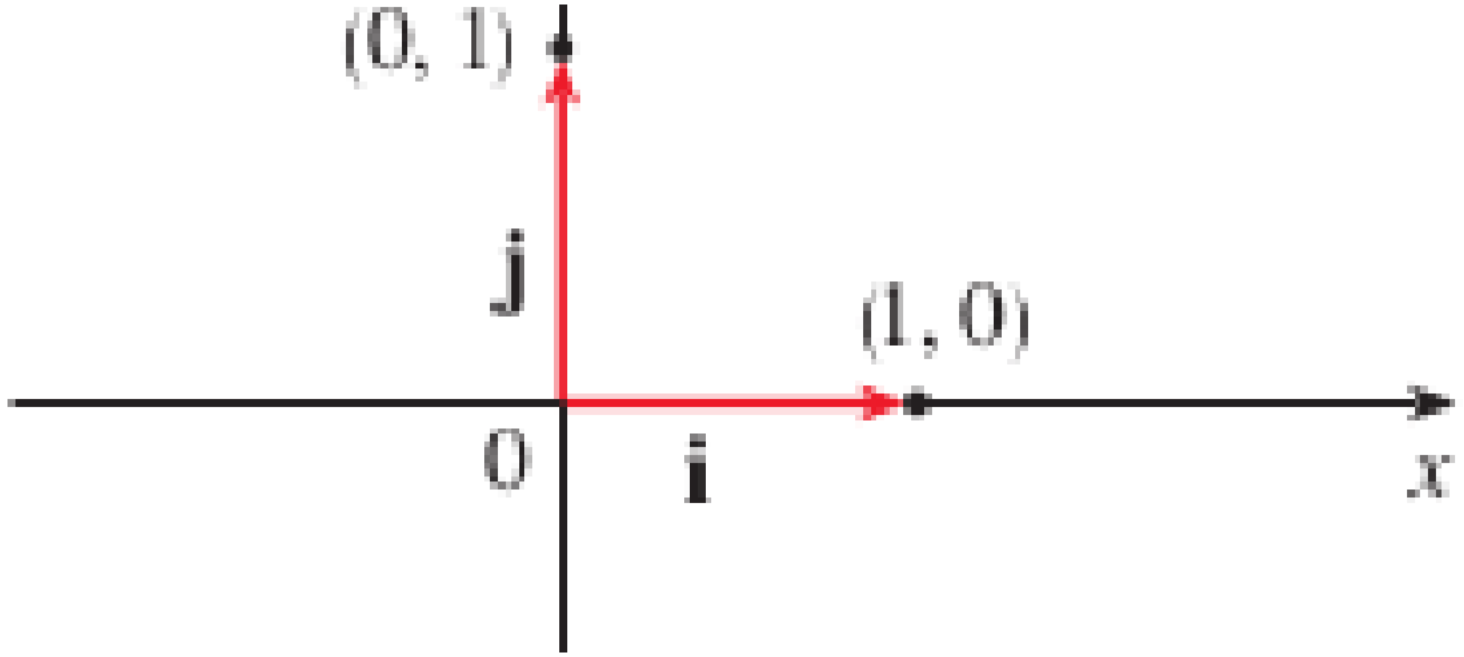
$3b = \langle -6, 3, 15 \rangle$

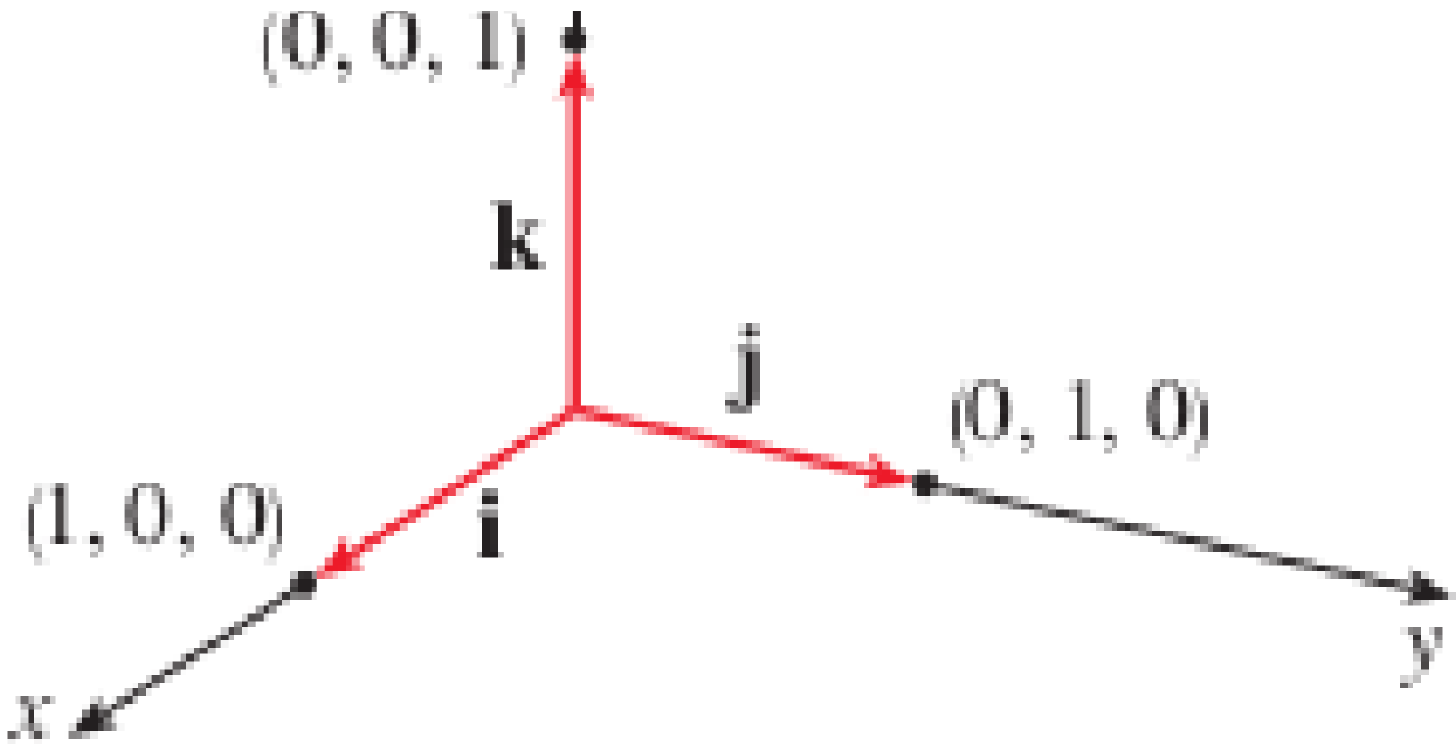
$2\mathbf{a} + 5\mathbf{b} = 2\langle 4, 0, 3 \rangle + 5\langle -2, 1, 5 \rangle$

$2a + 5b =$

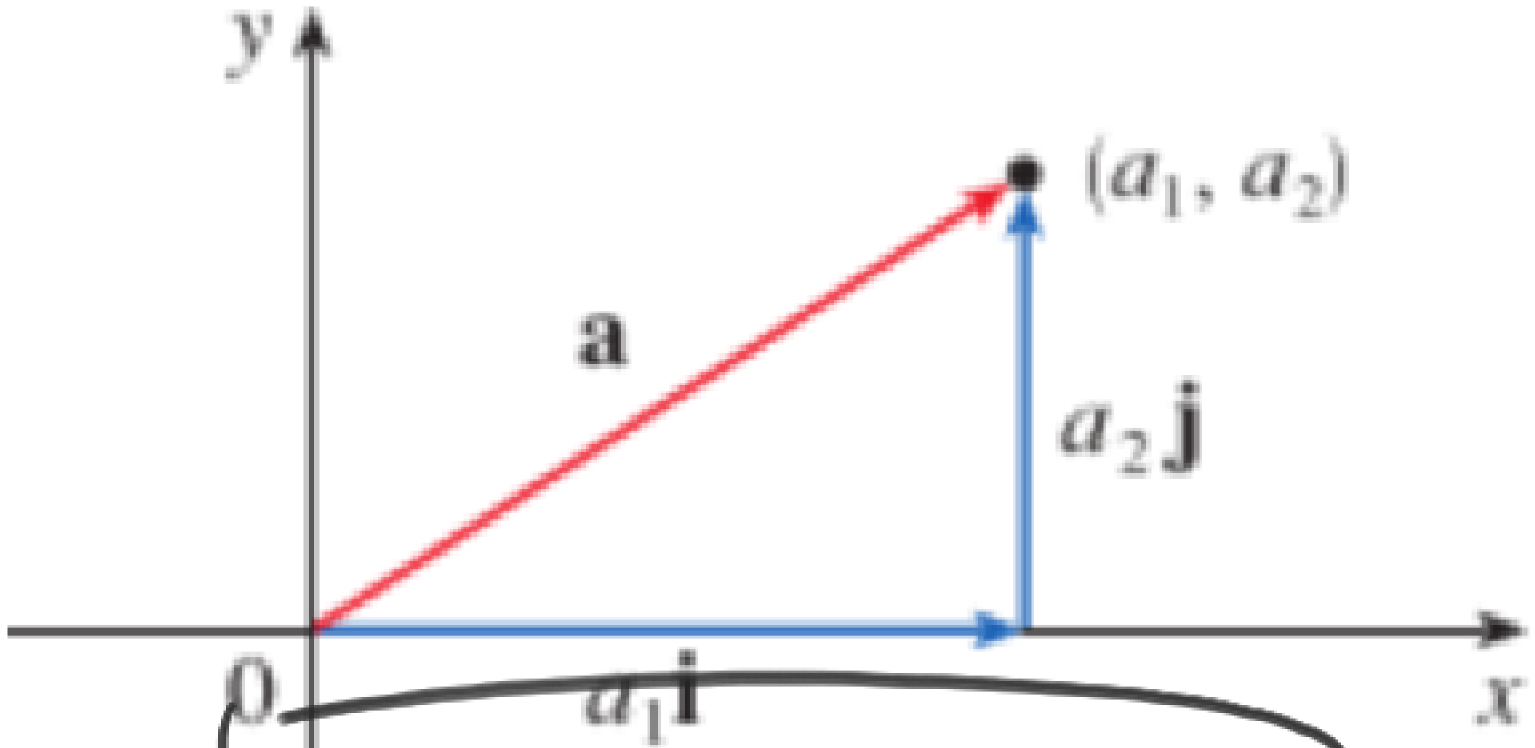
$= \langle 8, 0, 6 \rangle + \langle -10, 5, 25 \rangle = \langle -2, 5, 31 \rangle$

$$\mathbf{i} = \langle 1, 0, 0 \rangle \qquad \mathbf{j} = \langle 0, 1, 0 \rangle \qquad \mathbf{k} = \langle 0, 0, 1 \rangle$$





(b)



$\boxed{a = 5i - 2j}$

(a) $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j}$

Terminal Point

$\langle 5, -2 \rangle = (5, -2)$

$5\langle 1, 0 \rangle \quad + (-2)\langle 0, 1 \rangle$

$\langle 5, 0 \rangle + \langle 0, -2 \rangle$

If $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} + 7\mathbf{k}$, express the vector $2\mathbf{a} + 3\mathbf{b}$ in terms of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$.

$$2a = 2i + 4j - 6k$$

$$3b = 12i \qquad + 21k$$

$$2a + 3b = \boxed{14i + 4j + 15k}$$

19, 20, 21 and 22 Find $\cancel{\mathbf{a}+\mathbf{b}}$, $4\,\mathbf{a} + 2\,\mathbf{b}$, $\cancel{|\mathbf{a}|}$, and $|\mathbf{a} - \mathbf{b}|$.

$a-b = |\langle -12, 5\rangle| \to \sqrt{12^2+5^2} = \boxed{13}$

19. $\mathbf{a} = \langle -3, 4\rangle, \quad \mathbf{b} = \langle 9, -1\rangle$

$4a = \langle -12, 16\rangle \quad 2b = \langle 18, -2\rangle$

SHOW ANSWER

$4a+2b = \boxed{\langle 6, 14\rangle}$

20. $\mathbf{a} = 5\,\mathbf{i} + 3\,\mathbf{j}, \quad \mathbf{b} = -\mathbf{i} - 2\,\mathbf{j}$

$4a+2b = \boxed{18i+8j}$

$4a = 20i+12j \quad 2b = -2i-4j$

$a-b \to |6i+5j| = \sqrt{6^2+5^2} = \sqrt{61}$

A **unit vector** is a vector whose length is 1 .

$$\mathbf{u} = \frac{1}{|\mathbf{a}|}\mathbf{a} = \frac{\mathbf{a}}{|\mathbf{a}|}$$

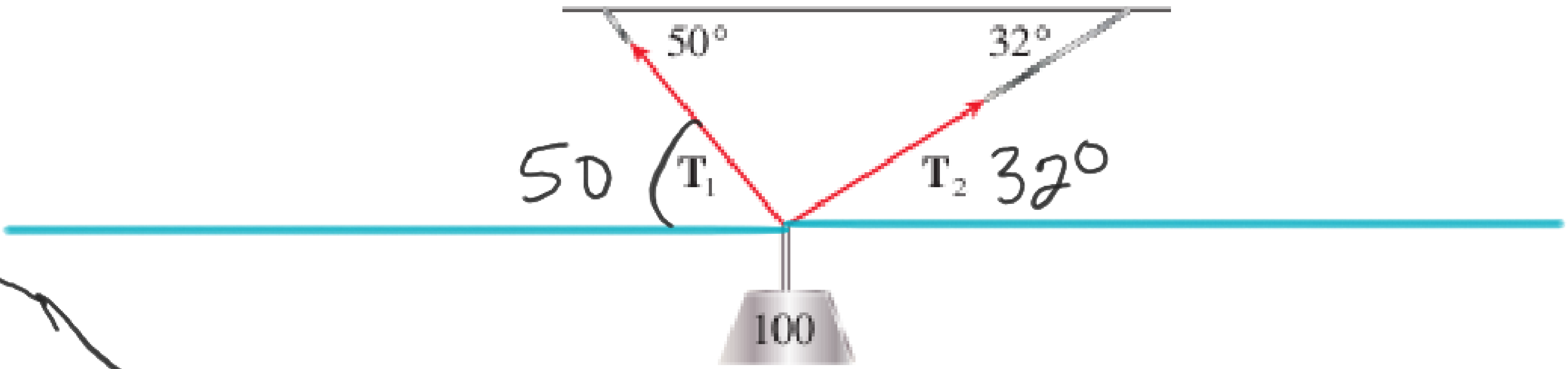
Find the unit vector in the direction of the vector $2\,\mathbf{i} - \mathbf{j} - 2\,\mathbf{k}$.

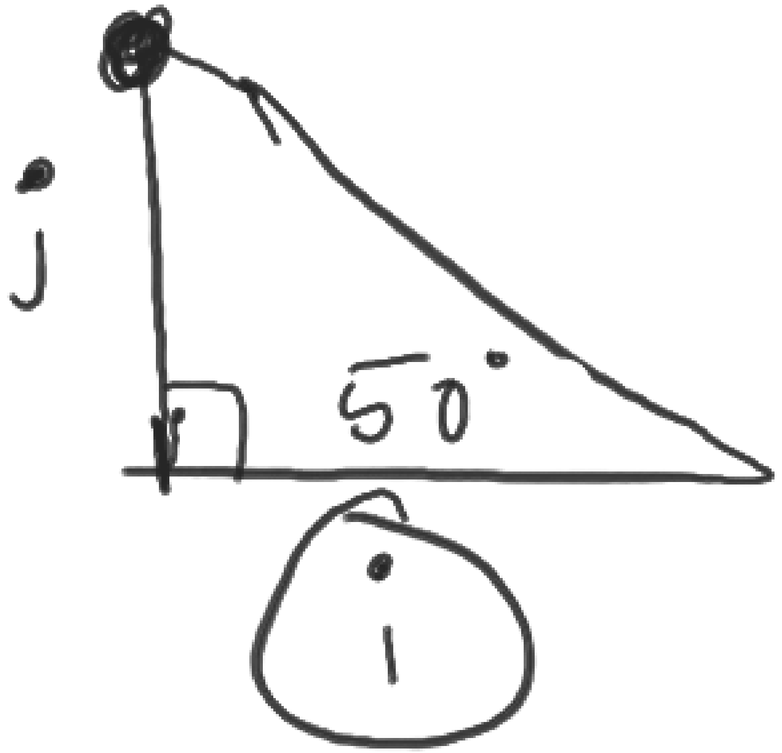
$$\sqrt{(2)^2 + (-1)^2 + (-2)^2} = \boxed{3}$$

unit vector $\rightarrow$ $\boxed{\frac{2}{3}i - \frac{1}{3}j - \frac{2}{3}k}$

A 100-lb weight hangs from two wires as shown in Figure 19. Find the tensions (forces) $\mathbf{T}_1$ and $\mathbf{T}_2$ in the wires and the magnitudes of these tensions.

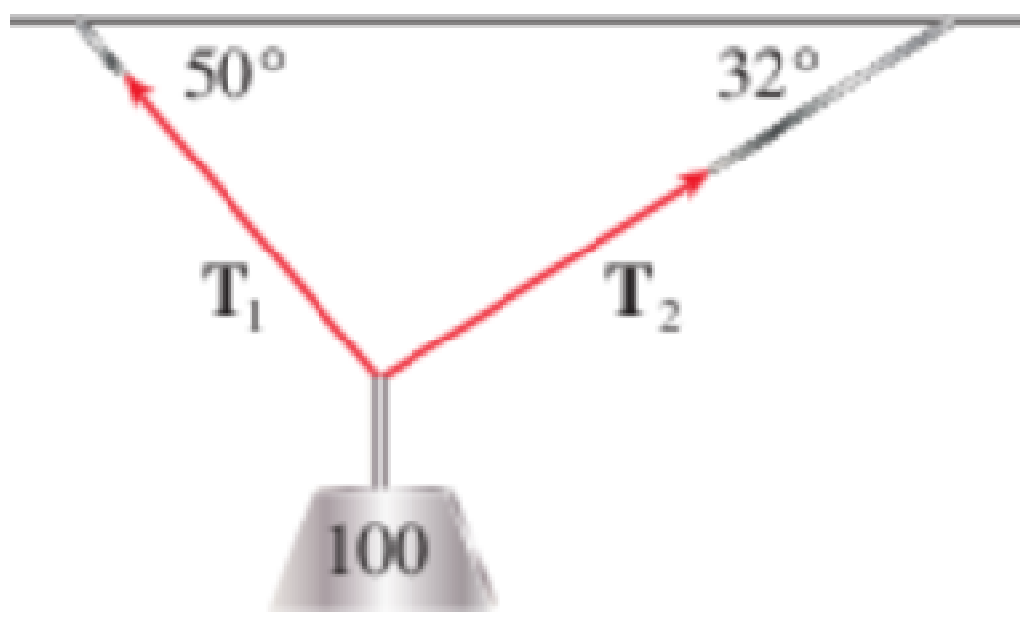
Figure 19





$$T_1 = -|T_1|\cos 50\, i + |T_1|\sin 50\, j$$

$$T_2 = |T_2|\cos 32\, i + |T_2|\sin 32\, j$$



$$T_1 = -|T_1|\cos 50\, i + |T_1|\sin 50\, j$$

$$T_2 = |T_2|\cos 32\, i + |T_2|\sin 32\, j$$

$$T_1 + T_2 = -w = 100j$$

$$(|T_2|\cos 32 - |T_1|\cos 50)\,i + (|T_1|\sin 50 + |T_2|\sin 32)\,j = 100j$$

$$|T_2|\cos 32 - |T_1|\cos 50 = 0$$

$$|T_2|\sin 32 + |T_1|\sin 50 = 100$$

$$|\mathbf{T}_1| \sin 50^\circ + \frac{|\mathbf{T}_1| \cos 50^\circ}{\cos 32^\circ} \sin 32^\circ = 100$$

$$|\mathbf{T}_1| \left(\sin 50^\circ + \cos 50^\circ \frac{\sin 32^\circ}{\cos 32^\circ} \right) = 100$$ $\rightarrow |T_1| = 85.64$

$$\sin 50 + (\cos 50)\left(\frac{\sin 32}{\cos 32}\right) \quad 1.16770272$$

$$|\mathbf{T}_2| = \frac{|\mathbf{T}_1| \cos 50^\circ}{\cos 32^\circ} \approx 64.91 \text{ lb}$$

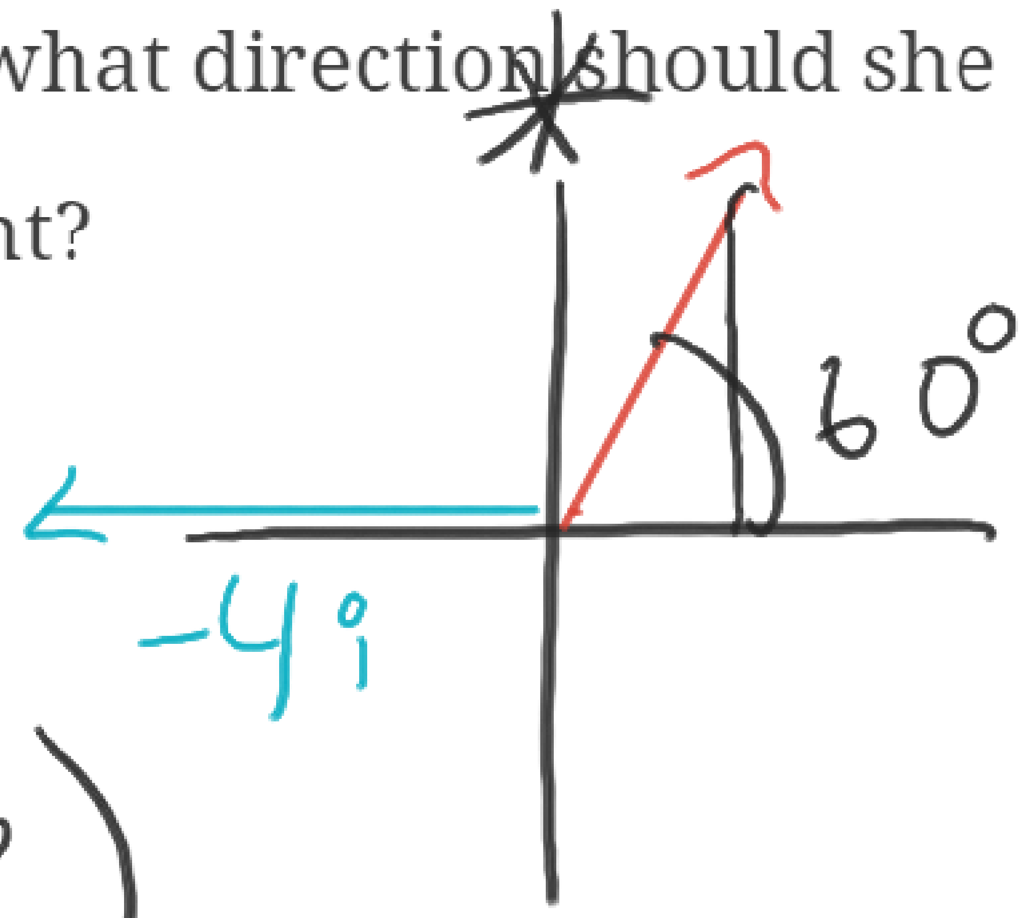
$$\mathbf{T}_1 = -|\mathbf{T}_1| \cos 50^\circ \ \mathbf{i} + |\mathbf{T}_1| \sin 50^\circ \ \mathbf{j}$$

$$\mathbf{T}_1 \approx -55.05\mathbf{i} + 65.60\mathbf{j}$$

$$\mathbf{T}_2 \approx 55.05\mathbf{i} + 34.40\mathbf{j}$$

$$\mathbf{T}_2 = |\mathbf{T}_2| \cos 32^\circ \ \mathbf{i} + |\mathbf{T}_2| \sin 32^\circ \ \mathbf{j}$$

A woman launches a boat from the south shore of a straight river that flows directly west at 4 mi/ h . She wants to land at the point directly across on the opposite shore. If the speed of the boat (relative to the water) is 8 mi/ h , in what direction should she steer the boat in order to arrive at the desired landing point?



$V_c = -4i$

$V_b = 8(\cos\theta\, i + \sin\theta\, j)$

$-4 + 8\cos\theta = 0 \rightarrow \cos\theta = 1/2$

$\theta = 60°$

N 30° E