1) $x = \frac{y^3}{3} + \frac{1}{4y}$ $\qquad 1 \le y \le 2$

$$\int_a^b \sqrt{1 + f'(y)^2}\, dy$$

$$f'(y) = y^2 + \frac{-2}{4y^3}$$

$$= y^2 - \frac{1}{2y^3}$$

$$\int_1^2 \sqrt{1 + \left(y^2 - \frac{1}{2y^3}\right)^2}\, dy$$

$$\frac{y^{-2}}{4} \qquad \frac{-2y^{-3}}{4}$$

$$\int_1^2 \sqrt{1+\left(y^2-\frac{1}{2y^3}\right)^2}\,dy \qquad y^4-\frac{1}{2y}-\frac{1}{2y}+\frac{1}{4y^6}$$

$$1+y^4-\frac{1}{\phantom{y}}+\frac{1}{4y^6}$$

$$\int_1^2 \sqrt{\left(1+y^4-\frac{1}{y}+\frac{1}{4y^6}\right)}$$

$$\int_1^2 \sqrt{1+\left(y^2-\frac{1}{2y^3}\right)^2}\,dy$$

$$\int_1^2 \sqrt{\left(y^2+\frac{1}{2y^3}\right)^2} = \int_1^2 y^2+\frac{1}{2y^3}\,dy$$

$$\left.\frac{y^3}{3}+\frac{1}{-6y^2}\right]_1^2 \qquad \frac{y^{-3}}{2}=\frac{y^{-2}}{-6}$$

$$\left[\frac{y^3}{3} + \frac{1}{-6y^2}\right]_1^2 = \left(\frac{8}{3} - \frac{1}{24}\right) - \left(\frac{1}{3} - \frac{1}{6}\right)$$

$$= \frac{64 - 1 - 8 + 4}{24} = \boxed{\frac{59}{24}}$$

$$y = \ln(\cos x) \qquad 0 \le x \le \frac{\pi}{3}$$

$$y' = \frac{-\sin x}{\cos x} = -\tan x$$

$$\int_0^{\pi/3} \sqrt{1 + (-\tan x)^2} \leftarrow 1 + \tan^2 x = \sec^2 x$$

$$\int_0^{\pi/3} \sec x \, dx = \ln|\sec x + \tan x|$$

$$\ln\left|\sec x + \tan x\right|\Big]_0^{\pi/3}$$

$$\ln\left|\sec\frac{\pi}{3} + \tan\frac{\pi}{3}\right| - \ln\left|\sec 0 + \tan 0\right|$$

$$2 \qquad + \quad \sqrt{3}$$

$$\boxed{\ln(2+\sqrt{3})} - \ln|1|$$

A manufacture of corrugated metal roofing wants to produce panels that are 28 inches wide and 2 inches high by processing flat sheets of metal as shown below. The profile of the roofing takes the shape of a sine wave modeled by the curve $y = \sin\left(\frac{\pi x}{7}\right)$. What is the width of the flat panels they need to order?

$$y = \sin\left(\frac{\pi}{7}x\right) \qquad 0 \le x \le 28$$

$$y' = \frac{\pi}{7}\cos\left(\frac{\pi}{7}x\right)$$

$$\int_0^{28} \sqrt{1 + \frac{\pi^2}{49}\cos^2\left(\frac{\pi}{7}x\right)}\, dx$$

$$- \; 29.360726622$$

$$\int_0^{28} \sqrt{1 + \left(\frac{\pi}{7}\cos\left(\frac{\pi}{7}x\right)\right)^2} \rightarrow \sqrt{1 + \frac{\pi^2}{49}\cos^2\left(\frac{\pi}{7}x\right)}$$

Find the surface area by rotating $y^2 = x + 1, 3 \leq x \leq 15$ around the x-axis.

$$\int_a^b 2\pi y\sqrt{1+(y')^2}\,dx$$

$$y^2 = x+1 \rightarrow y = \sqrt{x+1} \quad y' = \frac{1}{2\sqrt{x+1}}$$

$$2\pi\int_3^{15}\left(\sqrt{x+1}\right)\left(\sqrt{1+\left(\frac{1}{2\sqrt{x+1}}\right)^2}\right)dx$$

$$2\pi\int_3^{15}\left(\sqrt{x+1}\right)\left(\sqrt{1+\left(\frac{1}{2\sqrt{x+1}}\right)^2}\right)dx$$

$$(x+1)\left(1+\frac{1}{4(x+1)}\right)$$

$$\cancel{(x+1)}\left(\frac{4(x+1)+1}{4\cancel{(x+1)}}\right)$$

$$2\pi\int_3^{15}\sqrt{\frac{4x+5}{4}}\,dx$$

$$2\pi \int_3^{15} \sqrt{\frac{4x+5}{4}}\, dx$$

$$U = \frac{4x+5}{4}$$

$$U = x + \frac{5}{4}$$

$$du = 1\, dx$$

$$2\pi \int_3^{15} \sqrt{U}\, dU \qquad U^{1/2}$$

$$2\pi \left[ \frac{2U^{3/2}}{3} \right] = 2\pi \left[ \frac{2\left(x + 5/4\right)^{3/2}}{3} \right]_3^{15}$$

$$= 237.6897366$$

Find the surface area by rotating $x^{\frac{2}{3}} + y^{\frac{2}{3}}, 0 \leq y \leq 1$ around the y-axis.

$$x^{2/3} + y^{2/3} = 1 \rightarrow y^{2/3} = 1 - x^{2/3}$$

$$y = \left(1 - x^{2/3}\right)^{3/2}$$

$$0 \leq x \leq 1$$

$$2\pi \int_0^1 x \sqrt{1 + (y')^2}\, dx$$

$$y = \left(1 - x^{2/3}\right)^{3/2}$$

$$y' = \left(-\frac{2}{3} x^{-1/3}\right)\left(\frac{3}{2}\left(1 - x^{2/3}\right)^{1/2}\right)$$

$$y' = -\frac{\sqrt{1 - x^{2/3}}}{x^{1/3}}$$

$$y' = \left( -\frac{\sqrt{1 - x^{2/3}}}{x^{1/3}} \right)^2$$

$$\sqrt{1 + \frac{1 - x^{2/3}}{x^{2/3}}} = \sqrt{\frac{x^{2/3} + 1 - x^{2/3}}{x^{2/3}}} = \sqrt{\frac{1}{x^{2/3}}}$$

$$= \frac{1}{x^{1/3}}$$

$$2\pi\int_0^1 x\left(\frac{1}{x^{1/3}}\right)dx \rightarrow 2\pi\int_0^1 x^{2/3}\,dx$$

$$2\pi\left[\frac{3}{5}x^{5/3}\right]_0^1 = 2\pi\left(3/5\right) = \boxed{6\pi/5}$$

8) $xyy' = x^2 + 1$

$y' = \frac{dy}{dx}$

$xy\frac{dy}{dx} = x^2 + 1$

$y\,dy = \frac{x^2+1}{x}\,dx$

$\int y\,dy = \int x + \frac{1}{x}\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + \ln|x| + C$

$y^2 = x^2 + 2\ln|x| + C$

$\boxed{y = \pm\sqrt{x^2 + 2\ln|x| + C}}$

$$\frac{dx}{dy} = \frac{2y+\sec^2 y}{2x}, x(0) = -5$$

$x = -5 \quad y = 0$

$$\int 2x \, dx = \int 2y + \sec^2 y \, dy$$

$$x^2 = y^2 + \tan y + C$$

$$x = \sqrt{y^2 + \tan y + C}$$

$$x = \sqrt{y^2 + \tan y + C}$$ $x = -5 \quad y = 0$

$$-5 = \sqrt{C}$$

$$25 = C$$

$$\boxed{x = \sqrt{y^2 + \tan y + 25}}$$

The air in a room with a volume of 200 $m^3$ contains 0.5% carbon dioxide initially. Fresher air with only 0.05% carbon flows into the room at a rate of $4\,\frac{m^3}{min}$ and the mixed air flows out at the same rate. Find the percentage of carbon dioxide in the room as a function of time? What is the % of carbon dioxide in the room after 2 hours?

$$y(0) = 0.005(200) = 1\ m^3$$

$$\frac{dy}{dx} = \text{rate in} - \text{rate out}$$

$$= 0.0005\left(4\frac{m^3}{min}\right) - \frac{y}{200m^3}\left(4\frac{m^3}{min}\right)$$

$$\frac{dy}{dx} = 0.002 - y/50$$

$$\frac{dy}{dx} = 0.002 - y/50$$

$$\frac{dy}{dx} \cancel{=} \frac{0.1 - y}{50}$$

$$\int \frac{dy}{0.1 - y} = \int \frac{dx}{50}$$

$$-\ln|0.1 - y| = \frac{1}{50}x + C$$

$$\ln|0.1 - y| = -\frac{1}{50}x + C$$

$$e \qquad e^{-1/50x + C}$$

$$0.1 - y = e$$

$$-0.1 \qquad -0.1$$

$$y = 0.1 - e^{-1/50x}$$

$$0.1 - Ae^{-1/50x}$$

$$e^{-1/50\,x + C} = e^{-1/50\,x} \cdot e^{C}$$

$$\boxed{y = 0.1 + 0.9e^{-x/50}}$$

$$= Ae^{-1/50\,x}$$

$$x = 0 \quad y = 1$$

$$y = 0.1 - Ae^{-1/50\,x}$$

$$0.9 = -A$$

$$A = -0.9$$