1. Given $y = x^3$ find the surface area and volume of the given object when rotated around the x-axis. $1 \le x \le 2$

$$V = \int_a^b \pi r^2 \, dx \qquad SA = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$V = \int_1^2 \pi (x^3)^2 \, dx$$

$$V = \pi \int_1^2 x^6 \, dx$$

$$\pi \left[\frac{x^7}{7}\right]_1^2 = \frac{128}{7} - \frac{1}{7} = \boxed{\frac{127}{7}\pi}$$

1. Given $y = x^3$ find the surface area and volume of the given object when rotated around the x-axis. $1 \leq x \leq 2$

$$V = \int_a^b \pi r^2 \, dx$$

$$SA = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$\frac{dy}{dx} = 3x^2$$

$$\int_1^2 2\pi (x^3) \sqrt{1 + 9x^4} \, dx =$$

199.5

2. Evaluate $\int \frac{x}{1+x^2}\,dx$

$u = 1 + x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln|u| + C$

$\boxed{\frac{1}{2}\ln|1+x^2| + C}$

3. Evaluate $\int \frac{1}{1+x^2}\,dx$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

x^2+1

$a=1$

$= \tan^{-1}(x) + C$

4. Evaluate $\int \frac{1}{1-x^2}\,dx = \int \frac{A}{(1-x)} + \frac{B}{(1+x)}$

$1 = A(1+x) + B(1-x)$

$x = 1 \rightarrow 1 = A(2) \rightarrow A = 1/2$

$x = -1 \rightarrow 1 = B(2) \rightarrow B = 1/2$

$$\int \frac{1/2}{1-x} + \frac{1/2}{1+x}\,dx$$

$$\int \frac{1/2}{1-x} + \frac{1/2}{1+x}\,dx$$

$$\frac{1}{2}\int \frac{1}{1-x}\,dx + \frac{1}{2}\int \frac{1}{1+x}\,dx$$

$$\boxed{\frac{1}{2}\ln|1-x| + \frac{1}{2}\ln|1+x| + C}$$

5. Does $\sum_{n=1}^{\infty}\left(\frac{1}{n^3}+\frac{1}{3^n}\right)$ converge or diverge?

$\sum \frac{1}{n^3}$ p-series $3 > 1$ Converges

≤ 1 diverges

$\sum \frac{1}{3^n}$ Geometric $|r| < 1$ Converges

ar^{n-1} $|r| \geq 1$ diverges

$\left(\frac{1}{3}\right)^n = \frac{1}{3}\left(\frac{1}{3}\right)^{n-1}$

$r = 1/3 < 1$ Converges

6. A tank contains 1000L of brine with 15kg of dissolved salt. Pure water enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank after 20 minutes?

$y(t)$ $y(0) = 15$

concentration $\frac{y(t)}{1000}$

$\frac{dy}{dt} = \frac{y(t)}{1000}(-10) = \frac{y(t)}{100}$

$\frac{dy}{dt} = \frac{y}{100} \rightarrow \int \frac{dy}{y} = \int \frac{dt}{100}$ $\int \frac{1}{y} dy = \int \frac{1}{100} dt$

$\ln|y| = \frac{1}{100}t + C$

$\ln|y| = \frac{-1}{100}t + C$

$y(0) = 15$

$t \quad y$

$\ln|15| = C$

$\ln|y| = \frac{-1}{100}t + \ln|15|$

$\ln|y| = -1/5 + \ln|15|$

$-\ln|15| \qquad -\ln|15|$

$\ln|y| - \ln|15| = -1/5$

$\ln|y/15| = 1/5$

$e \qquad e$

$(y/15 = e^{-1/5}) 15$

$y = \boxed{12.28}$

7. Find the distance from the point P (2, 1, 4) to the plane through the points Q(1, 0, 0), R(0, 2, 0), and S(0, 0, 3). Use the formula $d = \frac{|a\cdot(b\times c)|}{|a\times b|}$ where $a = \overrightarrow{QR}, b = \overrightarrow{QS}$, and $c = \overrightarrow{QP}$.

$$d = \frac{|a\cdot(b\times c)|}{|a\times b| = 7}$$

$$|a\times b| = \begin{vmatrix} i & j & k \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix}$$

$$\vec{a} = \langle -1, 2, 0 \rangle$$

$$\vec{b} = \langle -1, 0, 3 \rangle \qquad i\begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} - j\begin{vmatrix} -1 & 0 \\ -1 & 3 \end{vmatrix} + k\begin{vmatrix} -1 & 2 \\ -1 & 0 \end{vmatrix}$$

$$\vec{c} = \langle 1, 1, 4 \rangle \qquad i(6-0) - j(-3-0) + k(0+2)$$

$$|6i + 3j + 2k| = 7$$

$$d = \frac{|a \cdot (b \times c)|}{7 = |a \times b|}$$

$$|b \times c| = \begin{vmatrix} i & j & k \\ -1 & 0 & 3 \\ 1 & 1 & 4 \end{vmatrix}$$

$$\vec{a} = \langle -1, 2, 0 \rangle$$

$$\vec{b} = \langle -1, 0, 3 \rangle$$

$$\vec{c} = \langle 1, 1, 4 \rangle$$

$$i \begin{vmatrix} 0 & 3 \\ 1 & 4 \end{vmatrix} - j \begin{vmatrix} -1 & 3 \\ 1 & 4 \end{vmatrix} + k \begin{vmatrix} -1 & 0 \\ 1 & 1 \end{vmatrix}$$

$$i(0-3) - j(-4-3) + k(-1-0)$$

$$-3i + 7j - 1k$$

$$a \cdot (b \times c) = \langle -1, 2, 0 \rangle \cdot \langle -3, 7, -1 \rangle$$

$$= 3 + 14 + 0 = 17$$

$$\boxed{d = \frac{17}{7}}$$