Evaluate the integral.

$$\int_0^1 \frac{3x}{(2x+1)^3}\,dx$$

$$u = 2x + 1 \rightarrow x = \frac{u-1}{2}$$

$$du = 2\,dx \rightarrow \frac{1}{2}du = dx$$

$$u(0) = 1 \qquad u(1) = 3$$

$$\int_1^3 \frac{3x}{u^3}\,\frac{du}{2}$$

$$\int_1^3 \frac{3\left(\frac{u-1}{2}\right)}{u^3}\,\frac{du}{2} = \frac{3}{4}\int_1^3 \frac{u-1}{u^3}\,du$$

$$\frac{3}{4}\int_1^3 \frac{u}{u^3} - \frac{1}{u^3}\,du = \frac{3}{4}\int_1^3 \frac{1}{u^2} - \frac{1}{u^3}\,du$$

$$\downarrow \qquad \downarrow$$

$$u^{-2} \qquad u^{-3}$$

$$\frac{3}{4}\left[\frac{-1}{u} + \frac{+1}{2u^2}\right]_1^3$$

$$\frac{3}{4}\left[\left(\overset{-6}{\frac{-1}{3}} + \overset{+1}{\frac{1}{18}}\right) - \left(\overset{+18}{-1} + \overset{-9}{\frac{1}{2}}\right)\right] = \frac{3}{4}\left(\frac{4}{18}\right) = \boxed{\frac{1}{6}}$$

Evaluate the integral. (Remember the constant of integration.)

$$\int 3t \sin(t)\cos(t)\,dt$$

$\sin(t)\cos(t)$

Double Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

LIATE

$$\frac{1}{2}\left(2\sin(t)\cos(t)\right)$$

$$\frac{1}{2}\sin(2t)$$

$$\int \frac{3}{2}t\sin(2t)\,dt$$

$u = t \qquad v = -\frac{1}{2}\cos(2t)$

$du = dt \qquad dv = \sin(2t)\,dt$

$$(t)\left(-\frac{1}{2}\cos(2t)\right) - \int -\frac{1}{2}\cos(2t)\,dt$$

$$\int u v\,dt = uv - \int v\,du$$

$$-\frac{1}{2}t\cos(2t) + \frac{1}{2}\int\cos(2t)\,dt$$

$$-\frac{1}{2}t\cos(2t) + \frac{1}{2}\left(\frac{1}{2}\sin(2t)\right) + C$$

$$\frac{3}{2}\left[-\frac{1}{2}t\cos(2t) + \frac{1}{4}\sin(2t) + C\right]$$

$$\boxed{-\frac{3}{4}t\cos(2t) + \frac{3}{8}\sin(2t) + C}$$

$$\int \frac{2x-3}{x^3+3x}\,dx$$

Partial Fractions

$$\int \frac{2x-3}{x(x^2+3)}\,dx = \int \frac{A}{x} + \frac{Bx+C}{x^2+3}\,dx$$

$$2x-3 = A(x^2+3) + x(Bx+C)$$

$x=0$ $\quad -3 = 3A \rightarrow A = -1$

$$2x-3 = Ax^2+3A + Bx^2 + Cx$$

$$0 = A + B\,(x^2) \rightarrow B = 1$$

$$2 = C\,(x) \rightarrow C = 2$$

$$-3 = 3A\,(1) \rightarrow A = -1$$

$$\int \frac{-1}{x} + \frac{1x+2}{x^2+3}\,dx = \int -\frac{1}{x} + \int \frac{x}{x^2+3} + \int \frac{2}{x^2+3}$$

$$\boxed{-\ln|x| + \frac{1}{2}\ln|x^2+3| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C}$$

$U = x^2+3$

$du = 2x\,dx$

$\frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u|$

$$\frac{a}{x^2+b^2} = \frac{a}{b}\tan^{-1}\left(\frac{x}{b}\right)$$

Evaluate the integral.

$$\int_0^{\pi/4} \tan^3(\theta)\sec^2(\theta)\,d\theta$$

$$\int_0^1 u^3\,du$$

$$\left.\frac{u^4}{4}\right]_0^1 = \frac{1}{4}$$

$u = \tan(\theta$

$du = \sec^2\theta\,d\theta$

$u(0) = 0$

$u(\frac{\pi}{4}) = 1$

Evaluate the integral.

$$\int \frac{1}{x\sqrt{4x + 49}}\,dx$$

$$u = \sqrt{4x+49}$$

$$u^2 = 4x+49 \rightarrow \frac{u^2 - 49}{4} = x$$

$$2u\,du = 4\,dx$$

$$\frac{1}{2}u\,du = dx$$

$$\boxed{\int \frac{dx}{x^2 - a^2}} = \frac{1}{2a}\ln\left(\left|\frac{x-a}{x+a}\right|\right)]$$

$$a = 7$$

$$\int \frac{1}{\left(\frac{u^2-49}{4}\right)\cancel{u}} \cdot \frac{1}{2}\cancel{u}\,du$$

$$= \int \frac{2}{u^2-49}\,du$$

$$= 2\left(\frac{1}{14}\right)\ln\left(\left|\frac{u-7}{u+7}\right|\right)$$

$$\boxed{\frac{1}{7}\ln\left|\frac{\sqrt{4x+49}-7}{\sqrt{4x+49}+7}\right| + C}$$

Consider the solid obtained by rotating the region bounded by the given curves about the specified line.

$y = \sqrt{x-1}$, $y = 0$, $x = 6$ about the x-axis

$$\int_a^b A(x)\,dx$$

$$\boxed{A(x) = \pi r^2}$$

$$= \pi\left(\sqrt{x-1}\right)^2 = \pi(x-1)$$

$$0 = \sqrt{x-1} \rightarrow x = 1$$

$$\int_1^6 \pi(x-1)\,dx = \pi\left[\frac{x^2}{2} - x\right]_1^6$$

$$\pi\left((18-6) - \left(\tfrac{1}{2} - 1\right)\right)$$

$$12 + \tfrac{1}{2}$$

$$\frac{25\pi}{2}$$

Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the y-axis.

$y = 9e^{-x^2}$, $y = 0$, $x = 0$, $x = 1$

$$\int_a^b 2\pi x(h)\,dx \quad y\text{-axis}$$

$$\int_a^b 2\pi y(h)\,dy \quad x\text{-axis}$$

$$\int_0^1 2\pi (x)(9e^{-x^2})\,dx$$

$u = -x^2$

$du = -2x\,dx$

$u(0) = 0$

$u(1) = -1$

$$-9\pi \int_0^{-1} e^u\,du$$

$$-9\pi (e^u)\Big|_0^{-1} \rightarrow -9\pi(e^{-1} - 1)$$

$$\boxed{9\pi\left(1 - \frac{1}{e}\right)}$$

Find the exact length of the curve.

$$x = \frac{1}{3}\sqrt{y}\,(y - 3),\quad 16 \le y \le 25$$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$x = \frac{1}{3}\left(y^{3/2} - 3y^{1/2}\right)$$

$$\frac{dx}{dy} = \frac{1}{3}\left(\frac{3}{2}y^{1/2} - \frac{3}{2}y^{-1/2}\right)$$

$$= \left[\frac{1}{2}\left(y^{1/2} - y^{-1/2}\right)\right]^2 = \frac{1}{4}\left(y - 2 + y^{-1}\right) + 1$$

$$= \frac{1}{4}y - \frac{1}{2} + \frac{1}{4}y^{-1} + 1$$

$$= \sqrt{\frac{1}{4}y + \frac{1}{2} + \frac{1}{4}y^{-1}}$$

$$= \sqrt{\left(\frac{1}{2}y^{1/2} + \frac{1}{2}y^{-1/2}\right)^2}$$

$$\int_{16}^{25} \frac{1}{2}y^{1/2} + \frac{1}{2}y^{-1/2}\, dy = \frac{1}{2}\left[\frac{2}{3}y^{3/2} + 2y^{1/2}\right]_{16}^{25}$$

$$\frac{1}{2}\left[\left(\frac{2}{3}(125) + 10\right) - \left(\frac{2}{3}(64) + 4\right)\right]$$

$$y = \cos\left(\frac{1}{6}x\right), 0 \le x \le 3\pi$$

$$S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\left(\frac{dy}{dx} = -\frac{1}{6}\sin\left(\frac{1}{6}x\right)\right)^2$$

$$\frac{1}{36}\sin^2\left(\frac{1}{6}x\right) + 1 = \sqrt{\frac{36 + \sin^2(1/6\, x)}{36}}$$

$$S = \int_0^{3\pi} 2\pi \left(\cos \frac{1}{6}x\right) \frac{\sqrt{36 + \sin^2(1/6\, x)}}{6}$$

$$= \frac{\pi}{3} \int_0^{3\pi} \cos(1/6\, x)\sqrt{36 + \sin^2(1/6\, x)}\, dx$$

$$u = \sin(1/6\, x)$$

$$du = \frac{1}{6}\cos(1/6\, x)\, dx$$

$$u(0) = 0$$

$$u(3\pi) = 1$$

$$\frac{\pi}{18}\int_0^1 \sqrt{36 + u^2}\, du$$

$\boxed{\text{use desmos}}$

$\boxed{1.05}$

10) A hot, wet summer is causing a mosquito population explosion in a lake resort area. The number of mosquitos is increasing at an estimated rate of $n(t) = 1500 + 10e^{0.9t}$ per week. By how much does the mosquito population increase between the fifth and ninth weeks of summer?

$$\int_5^9 1500 + 10e^{0.9t} = 41{,}605$$

$$1500t + \frac{10}{0.9}e^{0.9t}\Big]_5^9$$

$$\left(1500(9) + \frac{10}{0.9}e^{8.1}\right) - \left(1500(5) + \frac{10}{0.9}e^{4.5}\right)$$

Test the series for convergence or divergence.

$$\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{3n}}{(2n)!}$$

Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} \frac{\sin(6n)}{1 + 8^n}$$

Test the series for convergence or divergence.

$$\sum_{k=1}^{\infty} \frac{k \ln(k)}{(k+5)^3}$$

Find the radius of convergence, R, of the series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n\, 3^n}{\sqrt{n}} x^n$$

Evaluate the indefinite integral as an infinite series.

$$\int \frac{\cos(x) - 1}{x}\, dx$$