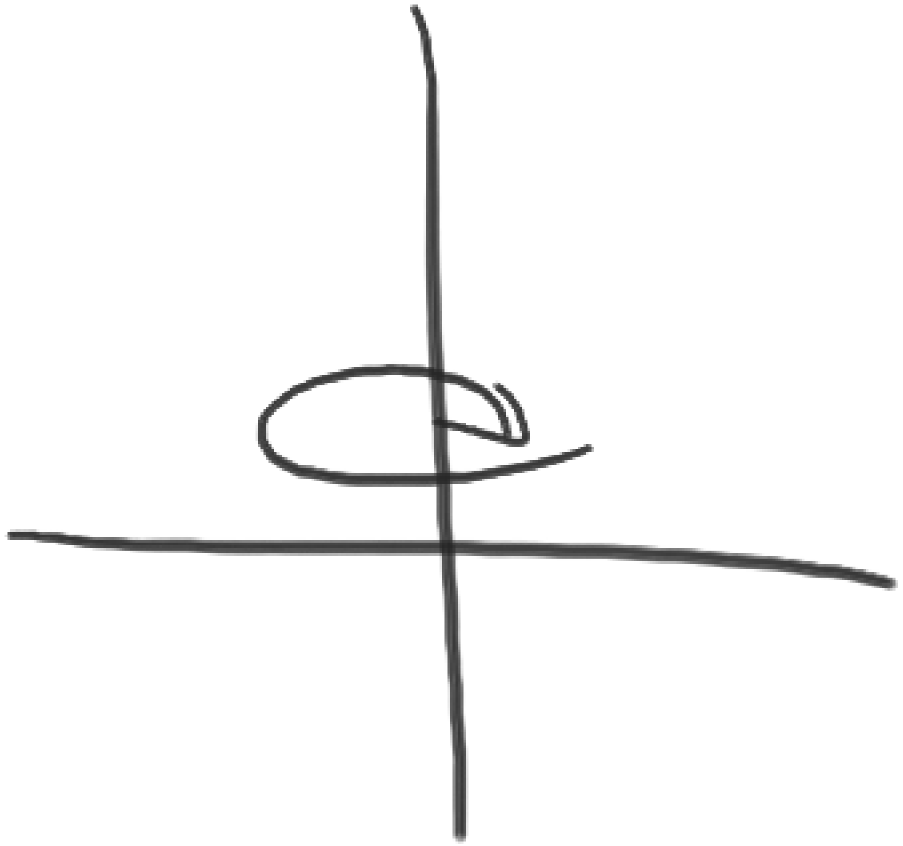
Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = 2x^2 - x^3$ and $y = 0$.

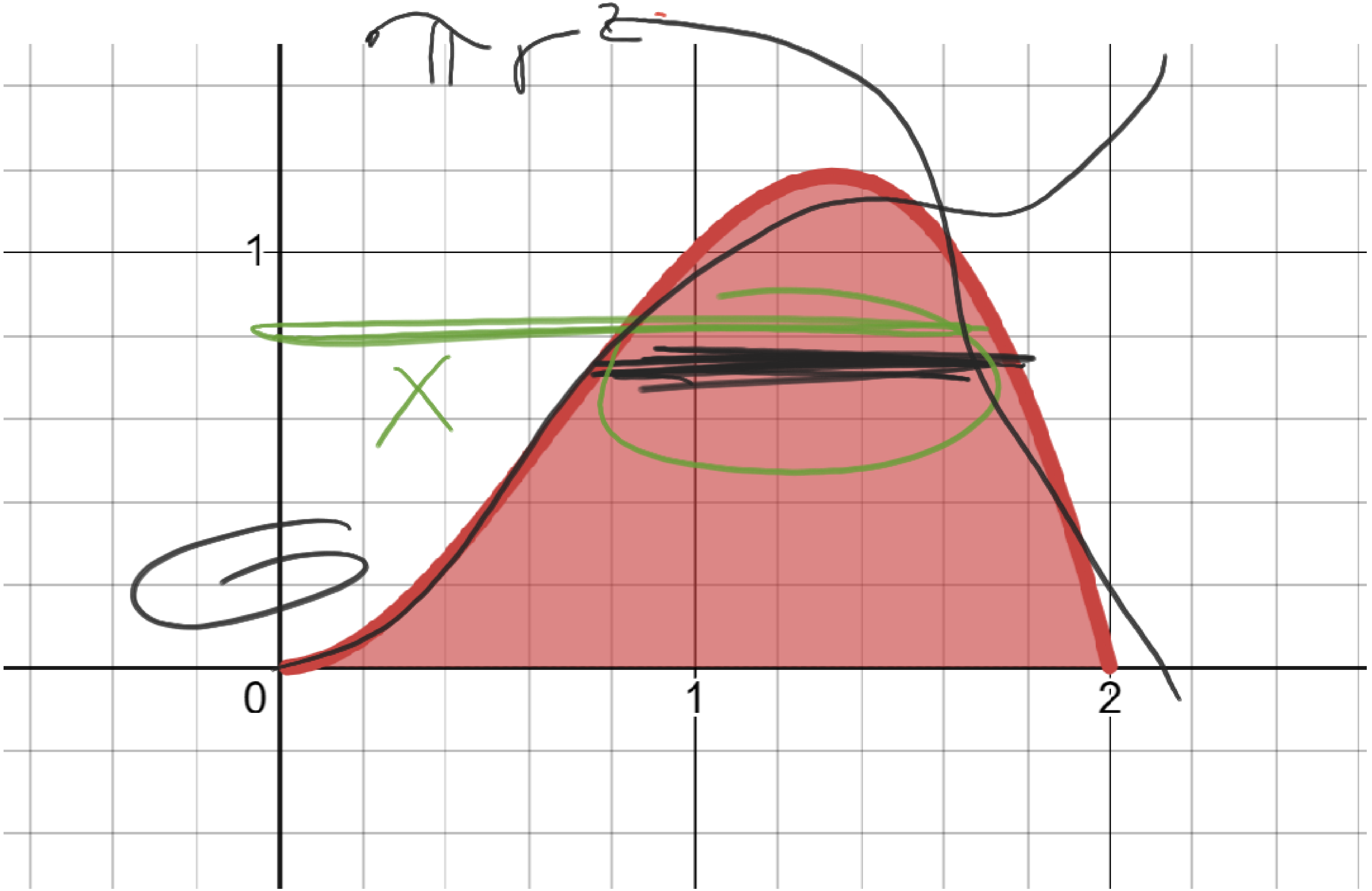
$$y = 2x^2 - x^3$$

$$y = x^2(2 - x)$$

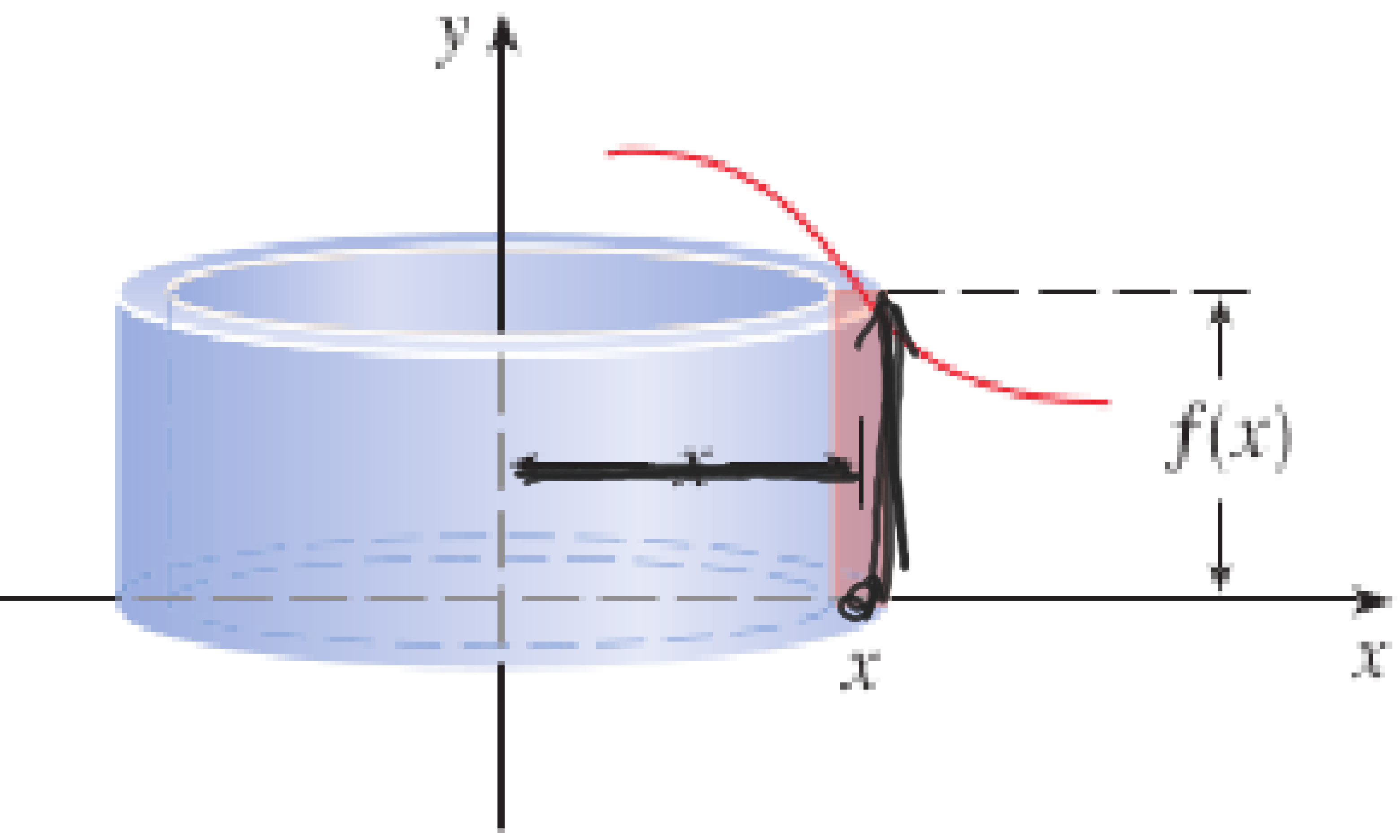


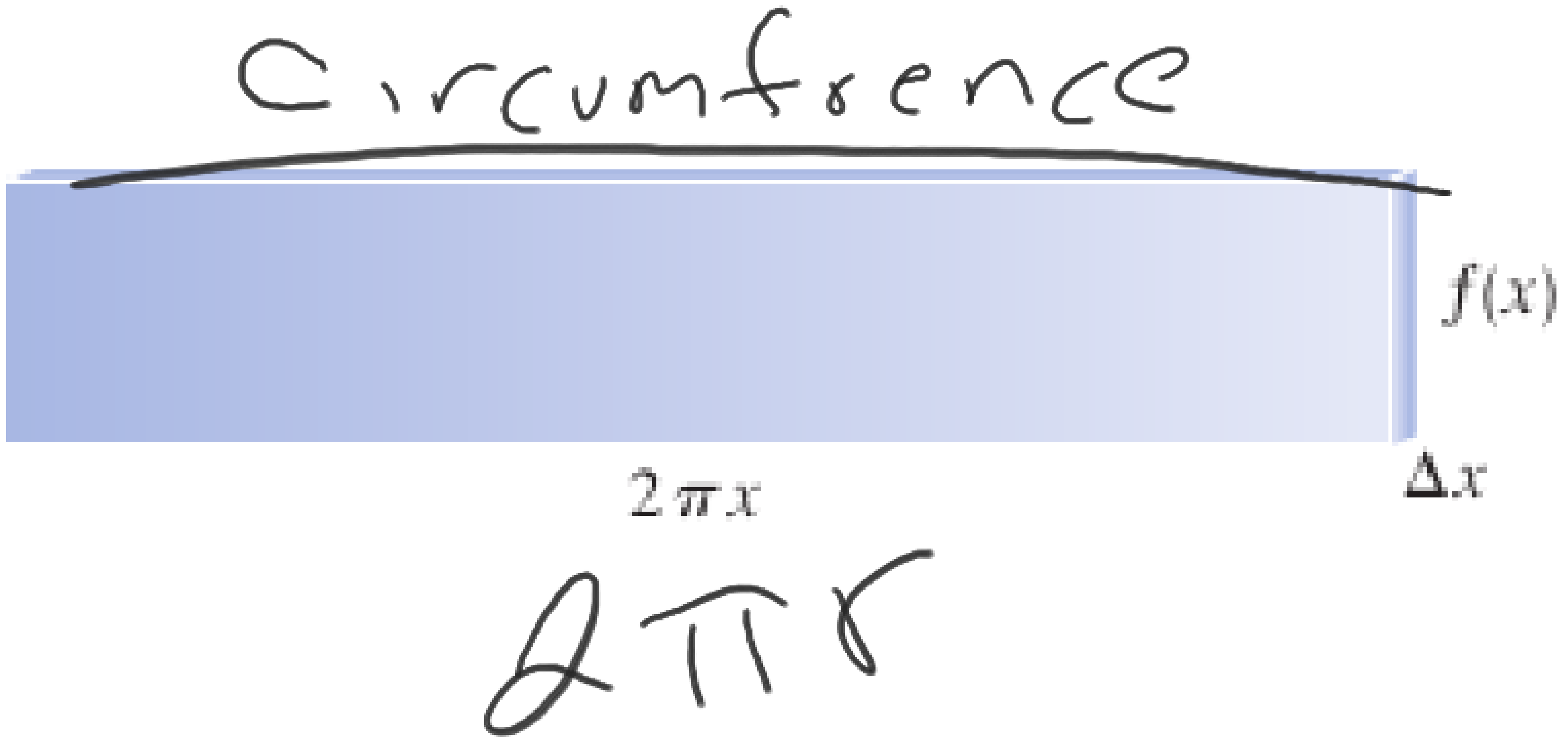
$r =$

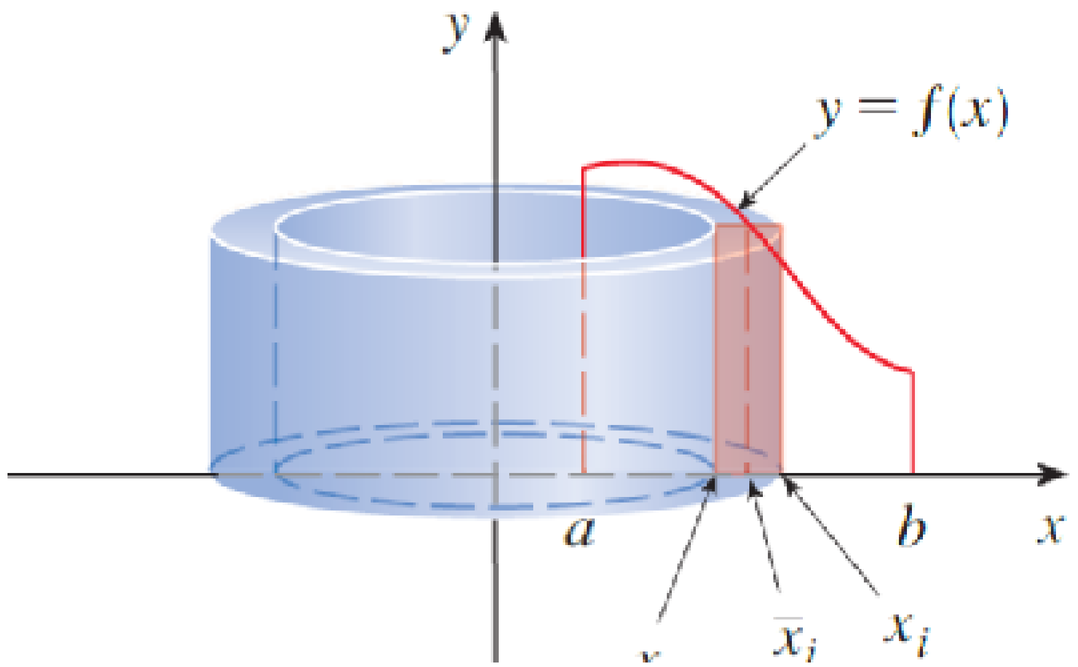
Intercection


πr²
1
0
1
2
x

$$V = \int_a^b \underbrace{(2\pi x)}_{\text{circumference}} \underbrace{[f(x)]}_{\text{height}} \underbrace{dx}_{\text{thickness}} .$$

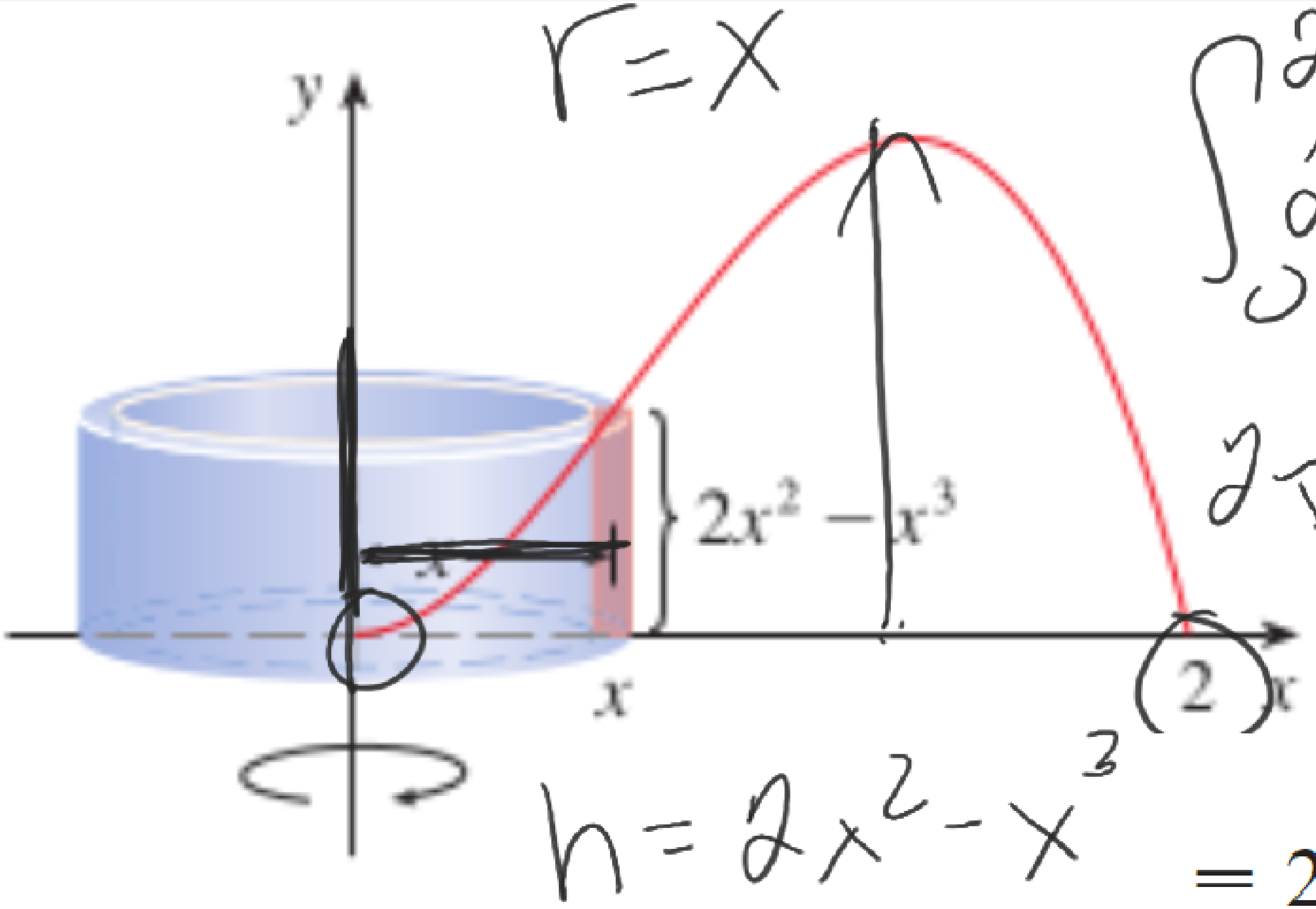






y
y = f(x)
a
b
x
$\bar{x}_i$
x_i
y
y = f(x)
a
b
x
y
y = f(x)
a
b
x

Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = 2x^2 - x^3$ and $y = 0$.



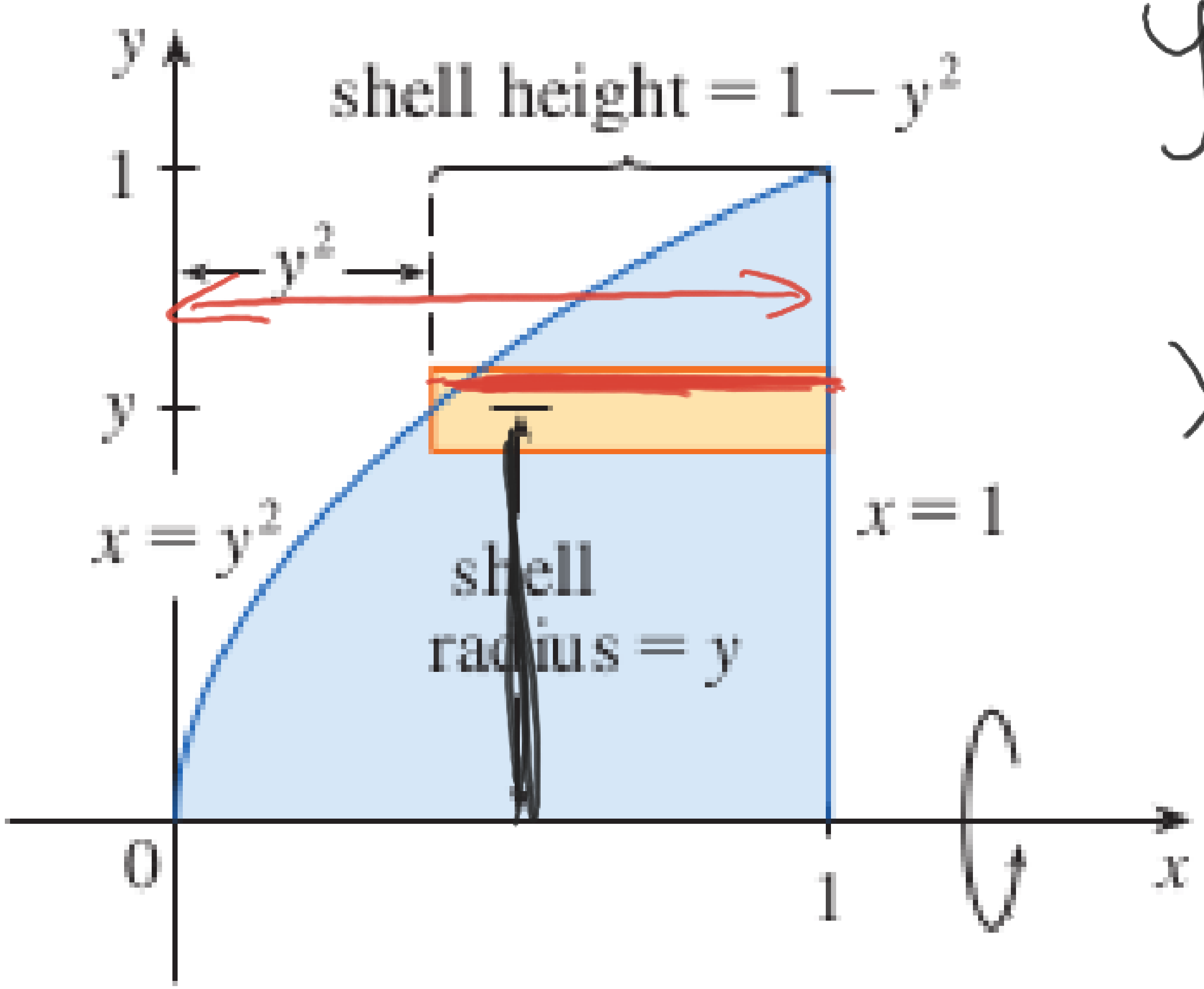
$$\int_0^2 2\pi x\left(2x^2 - x^3\right)dx$$

$$2\pi\int_0^2 2x^3 - x^4\,dx$$

$$= 2\pi\int_0^2 (2x^3 - x^4)\,dx = 2\pi\left[\tfrac{1}{2}x^4 - \tfrac{1}{5}x^5\right]_0^2$$

$$= 2\pi\left(8 - \tfrac{32}{5}\right) = \tfrac{16}{5}\pi$$

Use cylindrical shells to find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \sqrt{x}$ from 0 to 1.



$y = \sqrt{x} \qquad 0 \le y \le 1$

rotate around x-axis

$x = y^2 \qquad r = y$

$h = 1 - y^2$

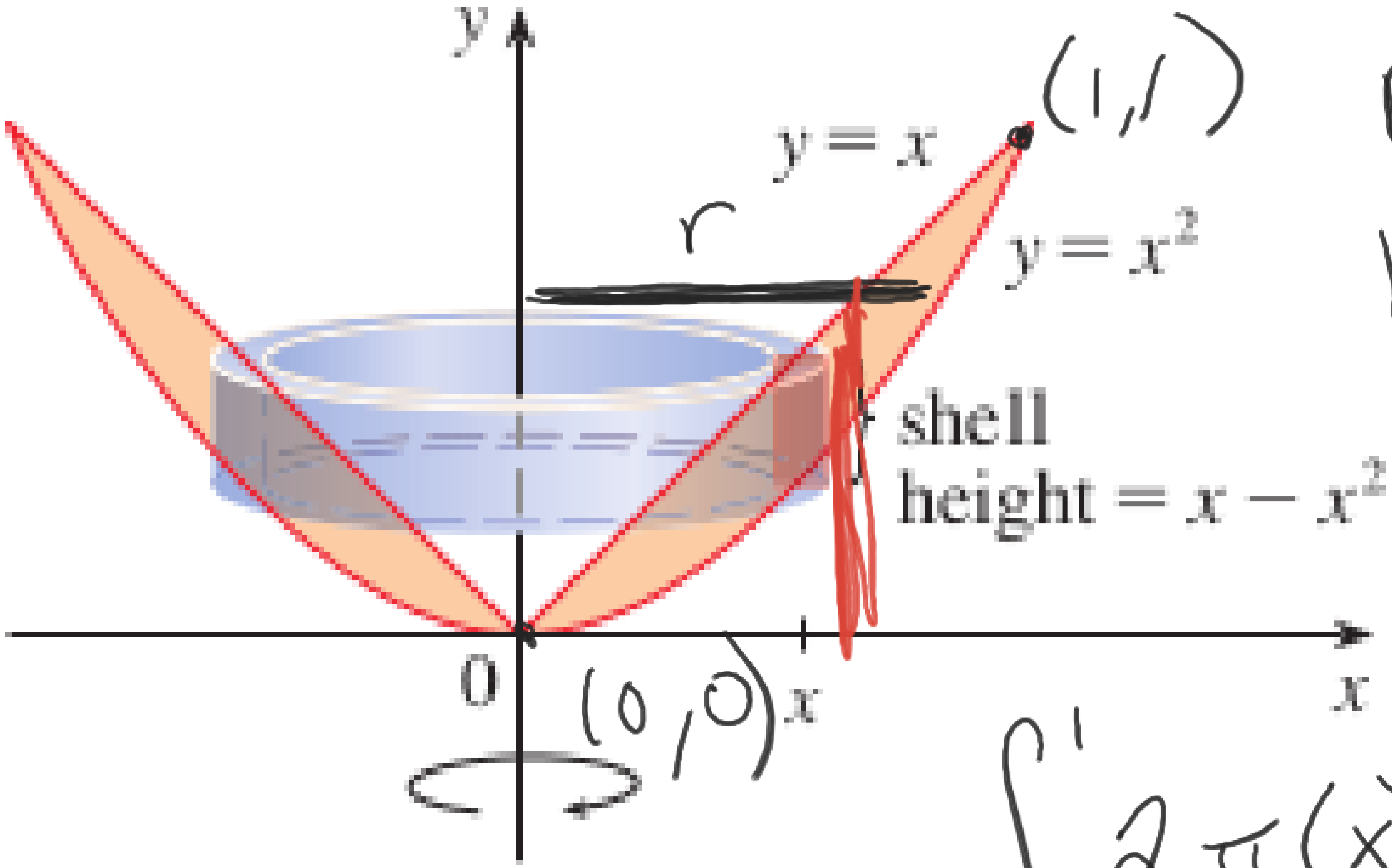
$$\int_0^1 2\pi y (1 - y^2)\, dy$$

$$V = \int_0^1 (2\pi y)(1 - y^2)\,dy = 2\pi \int_0^1 (y - y^3)\,dy$$

$$= 2\pi \left[\frac{y^2}{2} - \frac{y^4}{4} \right]_0^1 = \frac{\pi}{2}$$

Find the volume of the solid obtained by rotating about the y-axis the region between $y = x$ and $y = x^2$.

$0 \leq x \leq 1$



$r = x$

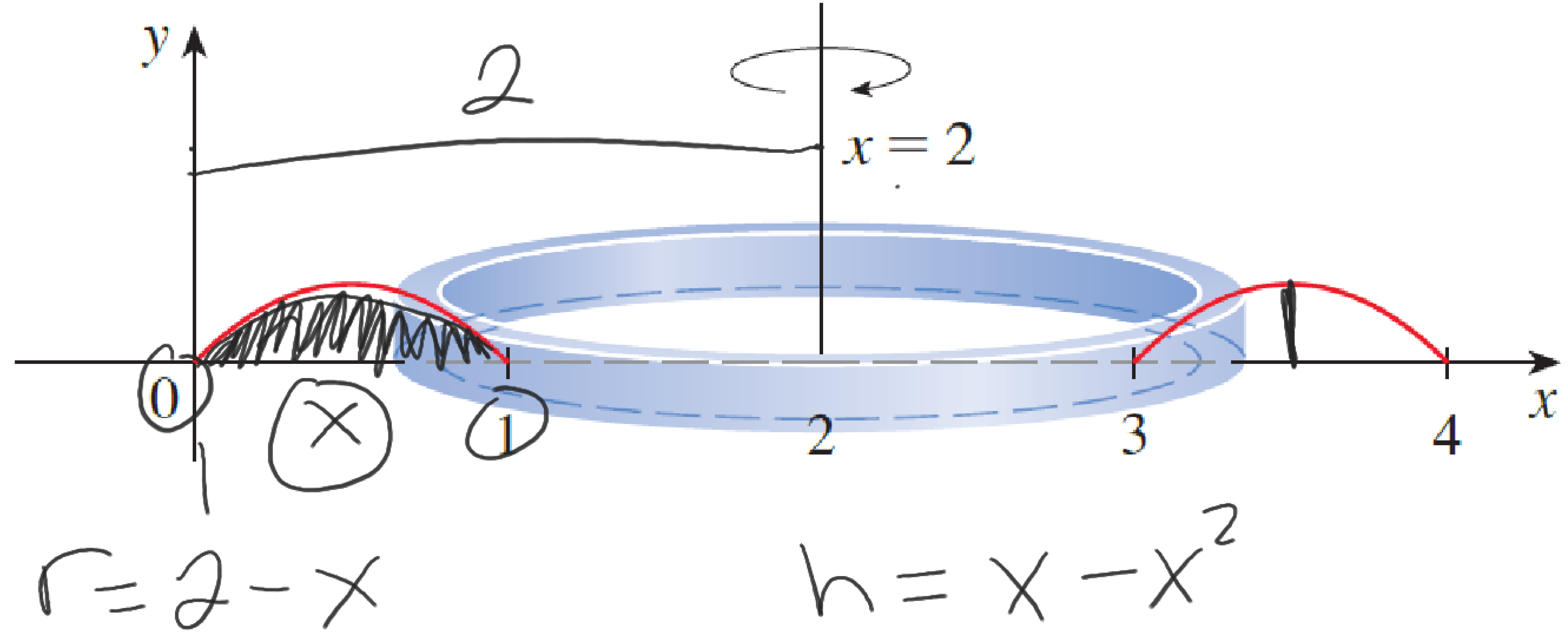
$h = x - x^2$

$$\int_0^1 2\pi(x)(x-x^2)\,dx$$

$$V = \int_0^1 (2\pi x)(x - x^2)\,dx = 2\pi \int_0^1 (x^2 - x^3)\,dx$$

$$= 2\pi \left[\frac{x^3}{3} - \frac{x^4}{4} \right]_0^1 = \frac{\pi}{6}$$

EXAMPLE 4 Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$.



$$V = \int_0^1 2\pi(2 - x)(x - x^2)\,dx$$

$$= 2\pi \int_0^1 (x^3 - 3x^2 + 2x)\,dx$$

$$= 2\pi \left[\frac{x^4}{4} - x^3 + x^2 \right]_0^1 = \frac{\pi}{2}$$

Disk method

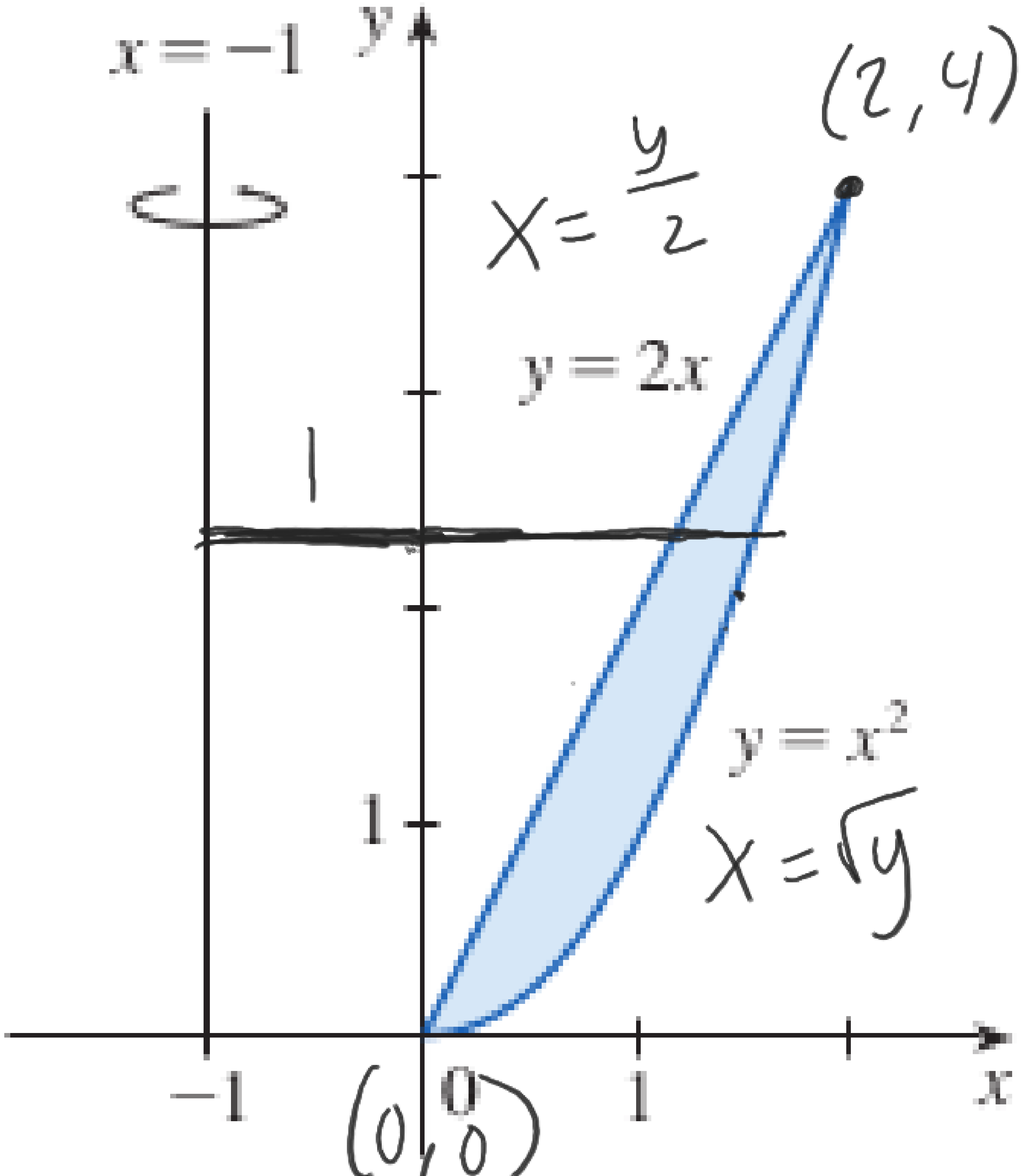
If the function is rotated around the x-axis it must be in the form y =

If the function is rotated around the y-axis it must be in the form x =

Shell Method

If the function is rotated around the x-axis it must be in the form x =

If the function is rotated around the y-axis it must be in the form y =



$y =$ y-axis shell

$x =$ y-axis disk

$$V = \int_0^2 2\pi(x + 1)(2x - x^2)\,dx = 2\pi \int_0^2 (x^2 + 2x - x^3)\,dx$$

$$= 2\pi\left[\frac{x^3}{3} + x^2 - \frac{x^4}{4}\right]_0^2 = \frac{16\pi}{3}$$

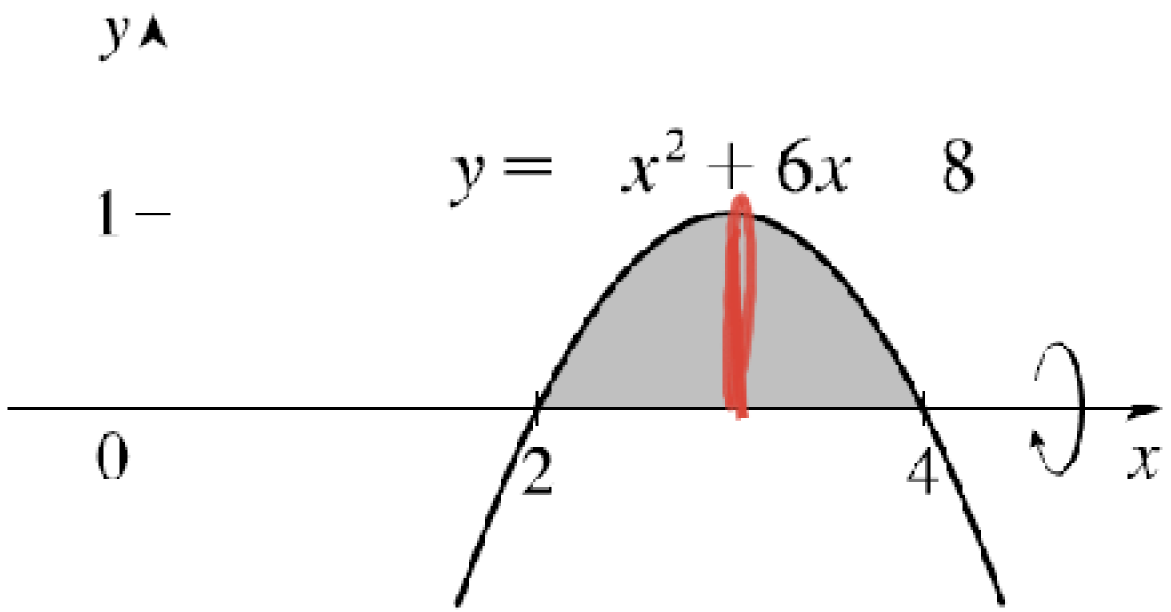
Shell

$$V = \int_0^4 \left[\pi\left(\sqrt{y} + 1\right)^2 - \pi\left(\tfrac{1}{2}y + 1\right)^2\right] dy = \pi \int_0^4 \left(2\sqrt{y} - \tfrac{1}{4}y^2\right) dy$$

$$= \pi\left[\tfrac{4}{3}y^{3/2} - \tfrac{1}{12}y^3\right]_0^4 = \frac{16\pi}{3}$$

Disk

$y = -x^2 + 6x - 8$, $y = 0$; about the x-axis



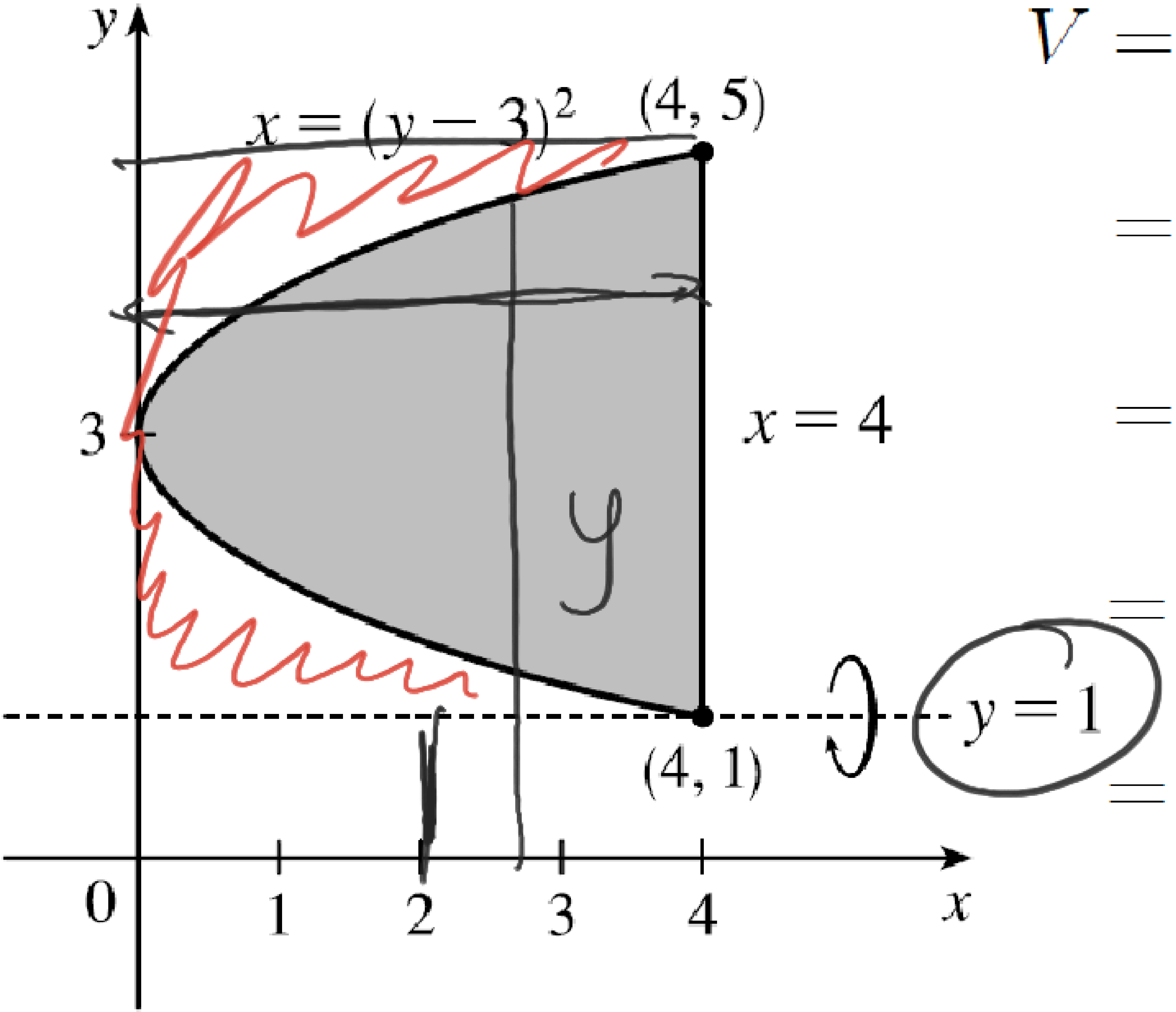
$$V = \int_2^4 \pi(-x^2 + 6x - 8)^2\, dx$$

$$= \pi \int_2^4 (x^4 - 12x^3 + 52x^2 - 96x + 64)\, dx$$

$$= \pi \left[\frac{1}{5}x^5 - 3x^4 + \frac{52}{3}x^3 - 48x^2 + 64x\right]_2^4$$

$$= \pi\left(\frac{512}{15} - \frac{496}{15}\right) = \frac{16}{15}\pi$$

$x = (y-3)^2,\ x = 4;\quad \text{about } y = 1$



$$V = \int_1^5 2\pi(y-1)[4-(y-3)^2]\,dy$$

$$= 2\pi \int_1^5 (y-1)(-y^2+6y-5)\,dy$$

$$= 2\pi \int_1^5 (-y^3+7y^2-11y+5)\,dy$$

$$= 2\pi\left[-\tfrac{1}{4}y^4+\tfrac{7}{3}y^3-\tfrac{11}{2}y^2+5y\right]_1^5$$

$$= 2\pi\left(\tfrac{275}{12}-\tfrac{19}{12}\right) = \tfrac{128}{3}\pi$$