Integration by Parts

The Product Rule states that if f and g are differentiable functions, then

$$\int \frac{d}{dx}[f(x)g(x)] = \int f(x)g'(x) + g(x)f'(x)$$

$$f(x)g(x) = \int f(x)g'(x) + \int g(x)f'(x)$$

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int g(x)f'(x)\,dx$$

$$\int u\,dv = uv - \int v\,du$$

<u>L</u>ogarithmic

<u>I</u>nverse trigonometric

<u>A</u>lgebraic

<u>T</u>rigonometric

<u>E</u>xponential

$u =$ $\quad\quad$ $v =$

$du =$ $\quad\quad$ $dv =$

$$\int x \sin x \, dx. \qquad \int u\,dv = uv - \int v\,du$$

$u = x \qquad v = -\cos x$

$du = 1\,dx \qquad dv = \sin x\,dx$

$$\int x \sin x\,dx = (x)(-\cos x) - \int -\cos x\,(1)\,dx$$

$$\int x\sin x\,dx = (x)(-\cos x) - \int -\cos x\,(1)\,dx$$

$$\int x\sin x\,dx = -x\cos x - (-\sin x) + C$$

$$= \boxed{-x\cos x + \sin x + C}$$

$$\int x \sin x \, dx.$$ $\int u\,dv = uv - \int v\,du$

$u = \sin x$ $\quad$ $v = \frac{x^2}{2}$

$du = \cos x\,dx$ $\quad$ $dv = x\,dx$

Not this way

$$\int x \sin x\,dx = (\sin x)\left(\frac{x^2}{2}\right) - \int \frac{x^2}{2} \cos x\,dx$$

$$\int \ln x \, dx. \Rightarrow \int \ln x \,(1)\, dx$$

$u = \ln x \qquad v = x$

$du = \frac{1}{x} dx \qquad dv = 1\, dx$

$$\int \ln x \, dx = (\ln x)(x) - \int x \left(\frac{1}{x} dx\right)$$

$$\int \ln x \, dx = (\ln x)(x) - \int x \left(\frac{1}{x} dx\right)$$

$$= x \ln x - \int 1 \, dx$$

$$= \boxed{x \ln x - x + C}$$

$$\int t^2 e^t \, dt.$$

$u = t^2 \qquad v = e^t$

$du = 2t\,dt \qquad dv = e^t\,dt$

$$\int t^2 e^t \, dt = t^2 e^t - \int 2t e^t \, dt$$

$$\int t^2 e^t \, dt = t^2 e^t - \boxed{\int 2te^t \, dt}$$

$$\int 2te^t \, dt$$

$u = 2t \qquad v = e^t$

$du = 2\,dt \qquad dv = e^t \, dt$

$$2te^t - \int 2e^t \, dt$$

$$2te^t - 2e^t + C$$

$$\boxed{t^2 e^t - 2te^t + 2e^t + C}$$

$$\int e^x \sin x \, dx.$$

$u = \sin x \qquad v = e^x$

$du = \cos x \, dx \qquad dv = e^x \, dx$

$$\int e^x \sin x \, dx = e^x \sin x - \int e^x \cos x \, dx$$

$$\int e^x \sin x\,dx = e^x \sin x - \boxed{\int e^x \cos x\,dx}$$

$u = \cos x \qquad v = e^x$

$du = -\sin x\,dx \qquad dv = e^x\,dx$

$$\int e^x \cos x = e^x \cos x - \int -e^x \sin x\,dx$$

$$e^x \cos x + \int e^x \sin x\,dx$$

$$\int e^x \sin x\,dx = e^x \sin x - \left(e^x \cos x + \int e^x \sin x\,dx\right)$$

$$\int e^x \sin x\,dx = e^x \sin x - e^x \cos x - \int e^x \sin x\,dx$$

$$2\int e^x \sin x\,dx = e^x \sin x - e^x \cos x$$

$$\boxed{\int e^x \sin x\,dx = \frac{1}{2}\left(e^x \sin x - e^x \cos x\right)}$$

$$\int_0^1 \tan^{-1} x \, dx. = \int_0^1 \tan^{-1} x \,(1)\, dx$$

$u = \tan^{-1} x \qquad v = x$

$du = \dfrac{1}{1+x^2}\, dx \qquad dv = 1\, dx$

$$= \underbrace{x \tan^{-1} x}_{\pi/4} \Big]_0^1 - \int_0^1 (x)\left(\frac{1}{1+x^2}\right) dx$$

$$x\tan^{-1}x\Big]_0^1$$

$$(1)(\tan^{-1}(1)) - (0)(\tan^{-1}(0))$$

$$\frac{\pi}{4}$$

$$\frac{\pi}{4} - \int_0^1 \frac{x}{1+x^2}\,dx$$

$u = 1 + x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\frac{1}{2}\int_0^1 \frac{1}{u}\,du = \frac{1}{2}\ln|u|$$

$$\frac{1}{2}\ln|1+x^2|\Big]_0^1 = \frac{1}{2}\ln(2) - \frac{1}{2}\ln(1)$$

$$\boxed{\frac{\pi}{4} - \frac{1}{2}\ln(2)}$$

$$\int t^4 \ln t \, dt$$

$$\int t^4 \ln t \, dt = \frac{1}{5}t^5 \ln t - \int \frac{1}{5}t^5 \cdot \frac{1}{t} \, dt = \frac{1}{5}t^5 \ln t - \int \frac{1}{5}t^4 \, dt = \frac{1}{5}t^5 \ln t - \frac{1}{25}t^5 + C.$$

$u = \ln t \qquad v = \dfrac{t^5}{5}$

$du = \dfrac{1}{t}\,dt \qquad dv = t^4\,dt$

$\displaystyle\int \frac{t^5}{5}\left(\frac{1}{t}\right) dt$

$\displaystyle\frac{1}{5}\int t^4\,dt$

$$\int_0^1 x\,3^x\,dx$$

$$\int a^x = \frac{a^x}{\ln(a)}$$

$$\int_0^1 x3^x\,dx = \left[\frac{1}{\ln 3}\,x\,3^x\right]_0^1 - \frac{1}{\ln 3}\int_0^1 3^x\,dx$$

$$\left(\frac{1}{\ln 3}\right)\left(\frac{3^x}{\ln(3)}\right) \boxed{= \frac{3^x}{(\ln(3))^2}}$$

$$\left.\frac{x3^x}{\ln 3} - \frac{3^x}{(\ln 3)^2}\right]_0^1$$

$$\left.\frac{x3^x}{\ln 3} - \frac{3^x}{(\ln 3)^2}\right]_0^1 = \left[\frac{1(3)}{\ln 3} - \frac{3}{(\ln 3)^2}\right] - \left(-\frac{1}{(\ln 3)^2}\right)$$

$$= \frac{3}{\ln 3} - \frac{2}{(\ln 3)^2}$$