Solve $\int x\sqrt{9-x^2}\,dx$

$u = 9 - x^2$

$\frac{du}{-2x} = \frac{-2x\,dx}{-2x}$

$\int \cancel{x}\sqrt{u}\,\frac{du}{-2\cancel{x}}$

$-\frac{1}{2}\int u^{1/2}\,du = -\frac{1}{2}\left(\frac{2}{3}u^{3/2}\right)$

$= -\frac{1}{3}(9-x^2)^{3/2} + C$

$$\int \sqrt{9-x^2}\,dx$$

$$u = 9 - x^2$$

$$\frac{du = -2x\,dx}{-2x}$$

$$\int \sqrt{u}\,\frac{du}{-2x}$$

Evaluate $\int \frac{\sqrt{9-x^2}}{x^2}\,dx$.

$$\int \frac{\sqrt{u}}{x^2} \cdot \frac{du}{-2x}$$

$$u = 9 - x^2$$

$$du = -2x\,dx$$

$$\frac{du}{-2x} = dx$$

Table of Trigonometric Substitutions

Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a\sin\theta,\quad \cancel{-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}}$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta,\quad \cancel{-\frac{\pi}{2} < \theta < \frac{\pi}{2}}$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta,\quad \cancel{0 \le \theta < \frac{\pi}{2} \text{ or } \pi \le \theta < \frac{3\pi}{2}}$	$\sec^2\theta - 1 = \tan^2\theta$

$$\sqrt{2^2 + x^2} = \sqrt{4 + x^2}$$

Evaluate $\int \frac{\boxed{\sqrt{9-x^2}}}{x^2}\,dx$.

$x = 3\sin\theta$

$\sqrt{9-x^2} \rightarrow \sqrt{a^2-x^2}$

$a = 3$

$x = x$

$dx = 3\cos\theta\, d\theta$

$\sqrt{9-(3\sin\theta)^2}$

$\sqrt{9-9\sin^2\theta}$

$\sqrt{9(1-\sin^2\theta)}$

$\sqrt{9\cos^2\theta} = 3\cos\theta$

Evaluate $\int \frac{\sqrt{9-x^2}}{x^2}dx.$

$x = 3\sin\theta$

$dx = 3\cos\theta \, d\theta$

$$\int \frac{3\cos\theta}{9\sin^2\theta} \cdot 3\cos\theta \, d\theta$$

$$\int \frac{\cos^2\theta}{\sin^2\theta} d\theta = \int \cot^2\theta \, d\theta$$

$$= \int \csc^2\theta - 1 \, d\theta$$

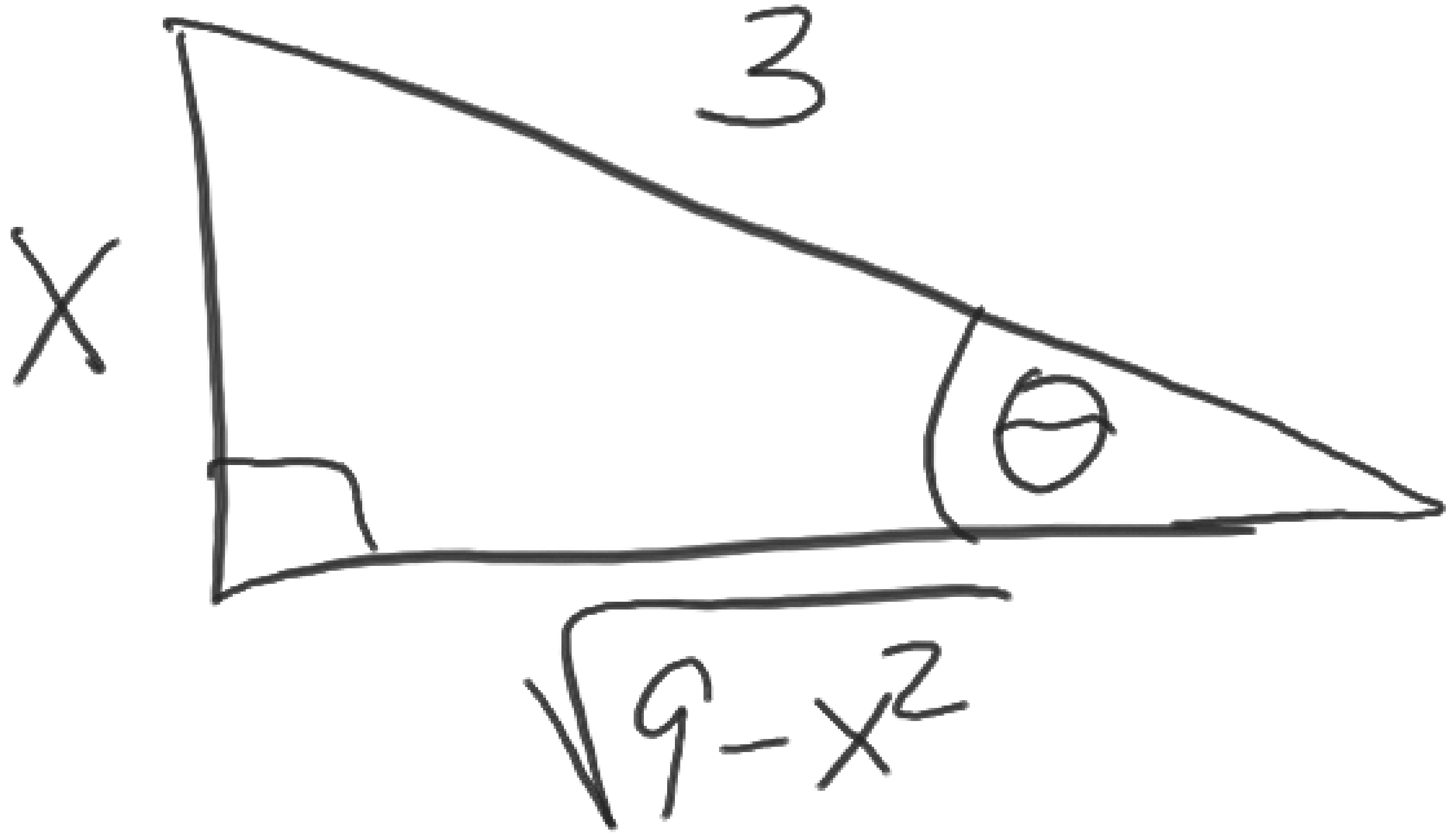
$$= \cot\theta - \theta + C$$

$\cot\theta - \theta + C$

$$\boxed{\frac{\sqrt{9-x^2}}{x} - \sin^{-1}\left(\frac{x}{3}\right) + C}$$

$x = 3\sin\theta$

$\frac{x}{3} = \sin\theta$



$\sin^{-1}\left(\frac{x}{3}\right) = \theta$

$$\int \frac{1}{x^2\boxed{\sqrt{x^2+4}}}\,dx.$$

$$x = 2\tan\theta$$

$$dx = 2\sec^2\theta \; d\theta$$

$$\sqrt{(2\tan\theta)^2+4}$$

$$\sqrt{4\tan^2\theta+4}$$

$$\sqrt{4(\tan^2\theta+1)}$$

$$\sqrt{4\sec^2\theta}$$

$$2\sec\theta$$

$$\int \frac{1}{(2\tan\theta)^2\cancel{(2\sec\theta)}} \cdot \cancel{2}\sec^{\cancel{2}}\theta \, d\theta$$

$$\frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\, d\theta$$

$$\frac{1}{4}\int \frac{\boxed{\sec\theta}}{\boxed{\tan^2\theta}}\,d\theta = \frac{1}{4}\int \boxed{\frac{1}{\cos\theta}} \cdot \boxed{\frac{\cos^2\theta}{\sin^2\theta}}\,d\theta$$

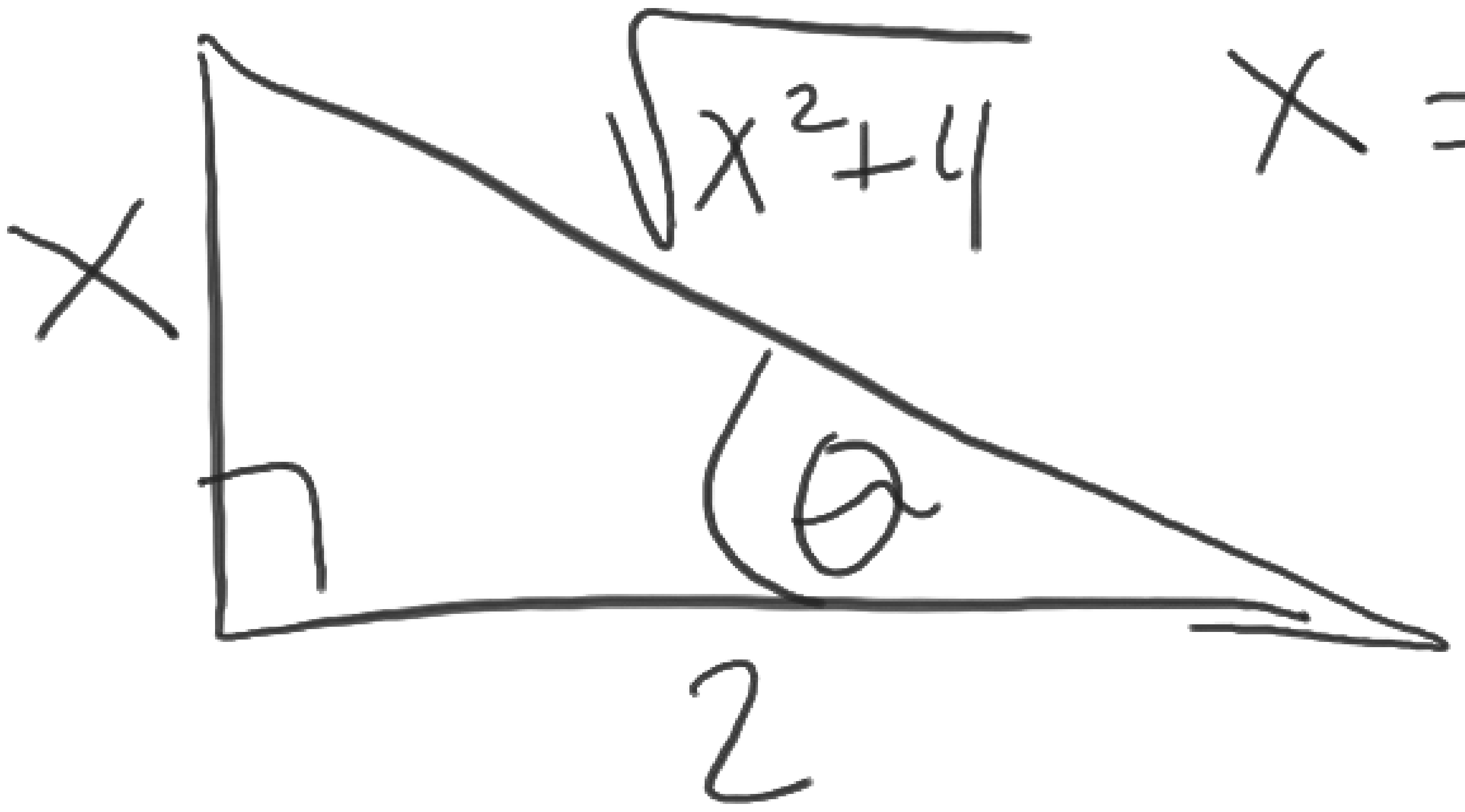
$$\frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

$$u = \sin\theta$$

$$du = \cos\theta\,d\theta$$

$$\frac{1}{4}\int \frac{1}{u^2}\,du = \frac{1}{4}\left(-\frac{1}{u}\right) = \frac{-1}{4u} + C$$

$$\frac{-1}{4u}+C \rightarrow \frac{-1}{4\sin\theta}+C = -\frac{1}{4}\csc\theta + C$$



$$x = 2\tan\theta \rightarrow \frac{x}{2} = \tan\theta$$

$$-\frac{1}{4}\left(\frac{\sqrt{x^2+4}}{x}\right)+C$$

$$\boxed{\frac{-\sqrt{x^2+4}}{4x}+C}$$

Find $\int \frac{x}{\sqrt{x^2+4}}\,dx.$

$$u = x^2 + 4$$

$$du = 2x \; dx$$

$$\int \frac{\cancel{x}}{\sqrt{u}} \frac{du}{2\cancel{x}} = \frac{1}{2}\int u^{-1/2}\,du$$

$$u^{1/2} + C = \boxed{\sqrt{x^2+4} + C}$$

$$\int_0^{3\sqrt{3}/2} \frac{x^3}{(4x^2+9)^{3/2}}\,dx.$$

$\sqrt{x^2-a^2}$

$\sqrt{a^2-x^2}$

$\sqrt{x^2+a^2}$

$$\left(\sqrt{4x^2+9}\right)^3$$

$$\left(\sqrt{(2x)^2+(3)^2}\right)^3$$

$X = 2x$

$a = 3$

$2x = 3\tan\theta$

$x = \frac{3}{2}\tan\theta$

$$\left(\sqrt{4x^2+9}\right)^3 \qquad x = \frac{3}{2}\tan\theta$$

$$\left(\sqrt{4\left(\frac{3}{2}\tan\theta\right)^2+9}\right)^3$$

$$\left(\sqrt{9\tan^2\theta+9}\right)^3$$

$$\left(\sqrt{9\sec^2\theta}\right)^3$$

$$(3\sec\theta)^3 = 27\sec^3\theta$$

$$\int_0^{3\sqrt{3}/2} \frac{x^3}{(4x^2+9)^{3/2}}\,dx.$$

$x = \frac{3}{2}\tan\theta$

$dx = \frac{3}{2}\sec^2\theta\,d\theta$

$\left(\sqrt{4x^2+9}\right)^3 \to 27\sec^3\theta$

$$\int_0^{\frac{\pi}{3}} \frac{\left(\frac{3}{2}\tan\theta\right)^3}{27\sec^3\theta} \cdot \frac{3}{2}\sec^2\theta\,d\theta$$

$0 = \frac{3}{2}\tan\theta$

$0 = \tan\theta \to \boxed{\theta = 0}$

$3\sqrt{3}/2 = 3/2\,\tan\theta$

$\sqrt{3} = \tan\theta \to \theta = \frac{\pi}{3}$

$$\int_0^{\frac{\pi}{3}} \frac{\left(\frac{3}{2}\tan\theta\right)^3}{(27\sec^3\theta)} \cdot \left(\frac{3}{2}\sec^2\theta \; d\theta\right)$$

$$\frac{\left(\frac{3}{2}\right)^3 \cdot \left(\frac{3}{2}\right)}{27} = \frac{\frac{81}{16}}{27} = \boxed{\frac{3}{16}}$$

$$\frac{3}{16}\int_0^{\pi/3} \frac{\tan^3\theta}{\sec\theta} \, d\theta$$

$$\frac{3}{16}\int_0^{\pi/3} \frac{\tan^3\theta}{\sec\theta}\, d\theta = \frac{3}{16}\int_0^{\pi/3} \frac{\sin^3\theta}{\cos^2\theta}\, d\theta$$

$$\frac{3}{16}\int_0^{\pi/3} \frac{(1-\cos^2\theta)}{\cos^2\theta} \sin\theta \, d\theta$$

$u = \cos\theta$

$du = -\sin\theta \, d\theta$

$$\frac{3}{16}\int_1^{1/2} \frac{1-u^2}{u^2}(-du)$$

$u(0) = 1$

$u(\pi/3) = \frac{1}{2}$

$$\frac{3}{16}\int_{1/2}^{1} u^{-2} - 1 \, du$$

$$\frac{3}{16}\int_{1/2}^{1} u^{-2}-1\,du$$

$$\frac{3}{16}\left[-\frac{1}{u}-u\right]_{1/2}^{1} = \frac{3}{16}\left[\left(-1-1\right)-\left(-2-\frac{1}{2}\right)\right]$$

$$3/16\left[-2+\frac{5}{2}\right] = \frac{3}{16}\left(\frac{1}{2}\right)$$

$$= \boxed{\frac{3}{32}}$$

Evaluate $\displaystyle\int \frac{x}{\sqrt{3 - 2x - x^2}}\,dx.$