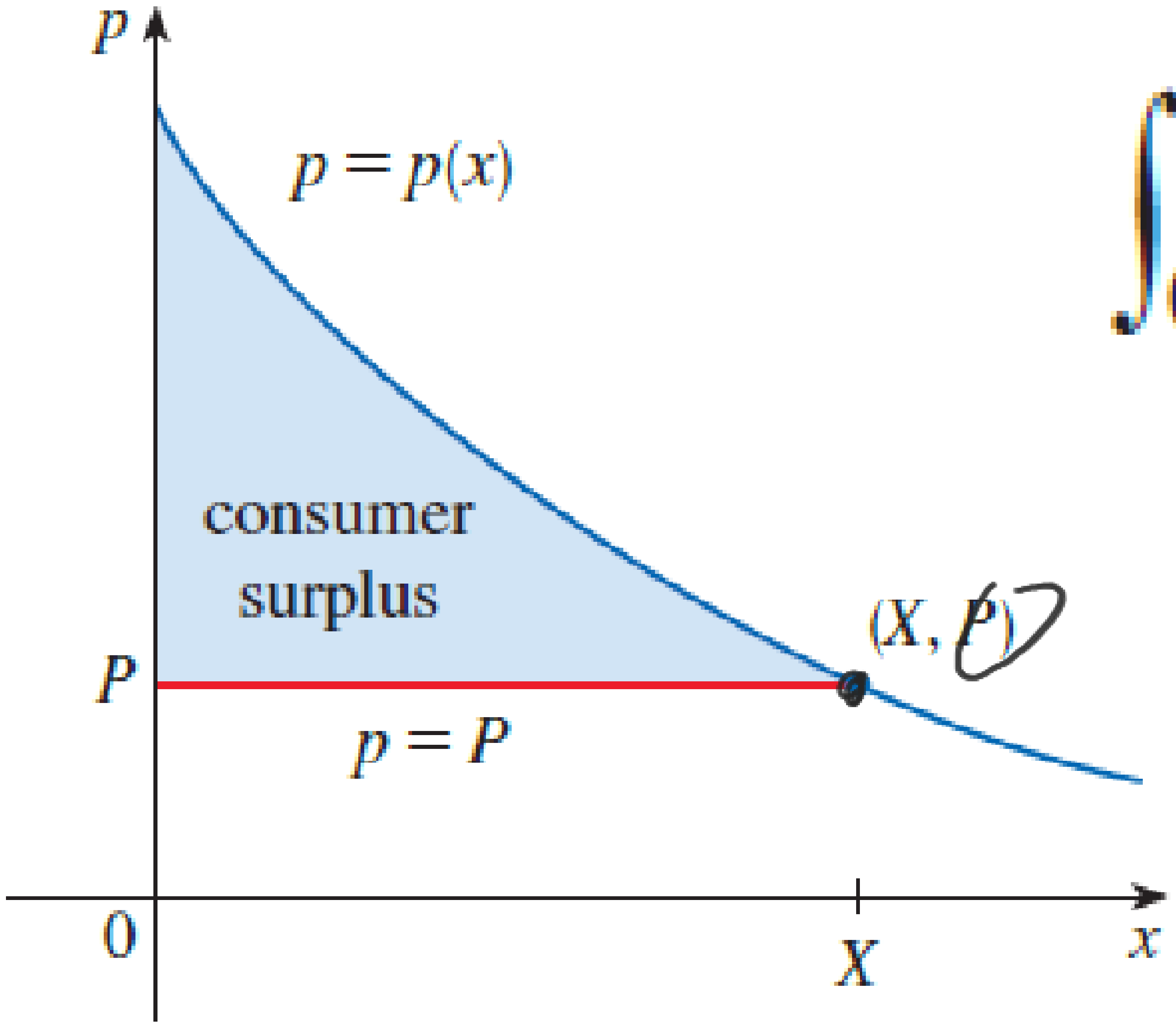
Consumer Surplus



$$\int_0^X [p(x) - P]\, dx$$

(500,)

EXAMPLE 1 The demand for a product, in dollars, is

$$p = 1200 - 0.2x - 0.0001x^2$$

Find the consumer surplus when the sales level is 500.

$X = 500$

$P = 1075$

$$\int_0^X [p(x) - P]\,dx$$

$$\int_0^{500} [(1200 - 0.2x - 0.0001x^2) - 1075)]\,dx$$

$$\int_0^{500} 125 - 0.2x - 0.0001x^2 dx$$

$$\int_0^{500} 125 - 0.2x - 0.0001x^2 \, dx$$

$$\left[125x - 0.1x^2 - \frac{0.0001x^3}{3}\right]_0^{500}$$

$33,333

The demand function for a manufacturer's microwave oven is $p(x) = 870e^{-0.03x}$, where x is measured in thousands. Calculate the consumer surplus when the sales level for the ovens is 45,000.

$x = 45$ $P = 225.54$

$$\text{Consumer surplus} = \int_0^{45} [p(x) - p(45)]\, dx$$

$$= \int_0^{45} (870e^{-0.03x} - 870e^{-0.03(45)})\, dx$$

$$= 870\left[-\frac{1}{0.03}e^{-0.03x} - e^{-1.35}x\right]_0^{45}$$

$$= 870\left(-\frac{1}{0.03}e^{-1.35} - 45e^{-1.35} + \frac{1}{0.03}\right) \approx \$11{,}332.78$$

9–11 Producer Surplus The *supply function* $p_S(x)$ for a commodity gives the relation between the selling price and the number of units that manufacturers will produce at that price. For a higher price, manufacturers will produce more units, so p_S is an increasing function of x. Let X be the amount of the commodity currently produced and let $P = p_S(X)$ be the current price. Some producers would be willing to make and sell the commodity for a lower selling price and are therefore receiving more than their minimal price. The excess is called the *producer surplus*. An argument similar to that for consumer surplus shows that the surplus is given by the integral

$$\int_0^X [P - p_S(x)]\, dx$$

10. If a supply curve is modeled by the equation $p = 125 + 0.002x^2$, find the producer surplus when the selling price is \$625.

$$\int_0^X [P - p_S(x)]\,dx$$

$625 = 125 + 0.002x^2$

$500 = 0.002x^2$

$\sqrt{250000} = \sqrt{x^2} \rightarrow x = 500$

$$\int_0^{500} \left[625 - (125 + 0.002x^2)\right] dx$$

$$P = p_S(x) \quad\Rightarrow\quad 625 = 125 + 0.002x^2 \quad\Rightarrow\quad 500 = \tfrac{1}{500}x^2 \quad\Rightarrow\quad x^2 = 500^2 \quad\Rightarrow\quad x = 500.$$

$$\text{Producer surplus} = \int_0^{500}[P - p_S(x)]\,dx = \int_0^{500}[625 - (125 + 0.002x^2)]\,dx = \int_0^{500}\left(500 - \tfrac{1}{500}x^2\right)dx$$

$$= \left[500x - \tfrac{1}{1500}x^3\right]_0^{500} = 500^2 - \tfrac{1}{1500}(500^3) \approx \$166{,}666.67$$

A hot, wet summer is causing a mosquito population explosion in a lake resort area. The number of mosquitoes is increasing at an estimated rate of $2200 + 10e^{0.8t}$ per week (where t is measured in weeks). By how much does the mosquito population increase between the fifth and ninth weeks of summer?

$$\int_5^9 2200 + 10e^{0.8t}\, dt$$

$$n(9) - n(5) = \int_5^9 (2200 + 10e^{0.8t})\, dt = \left[2200t + \frac{10e^{0.8t}}{0.8}\right]_5^9 = [2200t]_5^9 + \tfrac{25}{2}\left[e^{0.8t}\right]_5^9$$

$$= 2200(9 - 5) + 12.5(e^{7.2} - e^4) \approx 24{,}860$$

EXAMPLE 5 Intelligence Quotient (IQ) scores are distributed normally with mean 100 and standard deviation 15. (Figure 6 shows the corresponding probability density function.)

(a) What percentage of the population has an IQ score between 85 and 115?

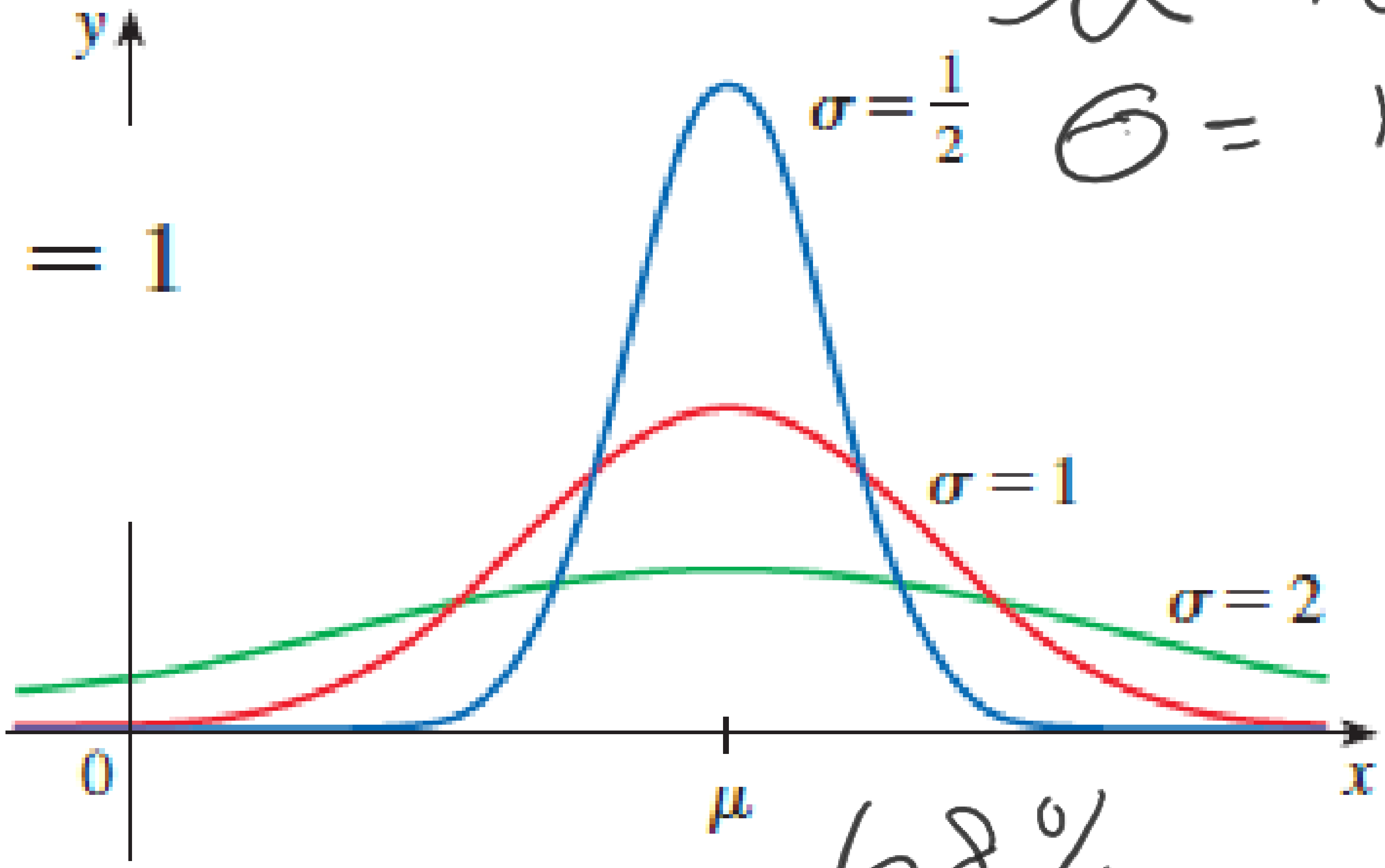
(b) What percentage of the population has an IQ above 140?

$\mu = 100$

$\sigma = 15$

$$\int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}\,dx = 1$$

$$\int_{85}^{115} \frac{1}{15\sqrt{2\pi}} e^{-\frac{(x-100)^2}{2(15)^2}}$$



68%

$$\int_{85}^{115} \frac{1}{15\sqrt{2\pi}} e^{-\frac{(x-100)^2}{2(15)^2}}$$

$$\frac{1}{15\sqrt{2\pi}} \int_{85}^{115} e^{-\frac{x^2-200x+10000}{450}}$$

Lengths of human pregnancies are normally distributed with mean 268 days and standard deviation 15 days. What percentage of pregnancies last between 250 days and 280 days?

$\mu = 268 \qquad \sigma = 15$

$$\int_{250}^{280} \frac{1}{15\sqrt{2\pi}} e^{-\frac{(x-268)^2}{2(15)^2}} \quad =$$

The heights of adult men in the United States are approximately normally distributed with a mean of 70 inches and a standard deviation of 3 inches. Heights of adult women are approximately normally distributed with a mean of 64.5 inches and a standard deviation of 2.5 inches.

$$\frac{1}{3\sqrt{2\pi}} e^{-\frac{(x-70)^2}{2(3)^2}}$$

Centers of Mass

$$\bar{x} = \frac{1}{A}\int_a^b x f(x)\, dx \qquad \bar{y} = \frac{1}{A}\int_a^b \tfrac{1}{2}[f(x)]^2\, dx$$

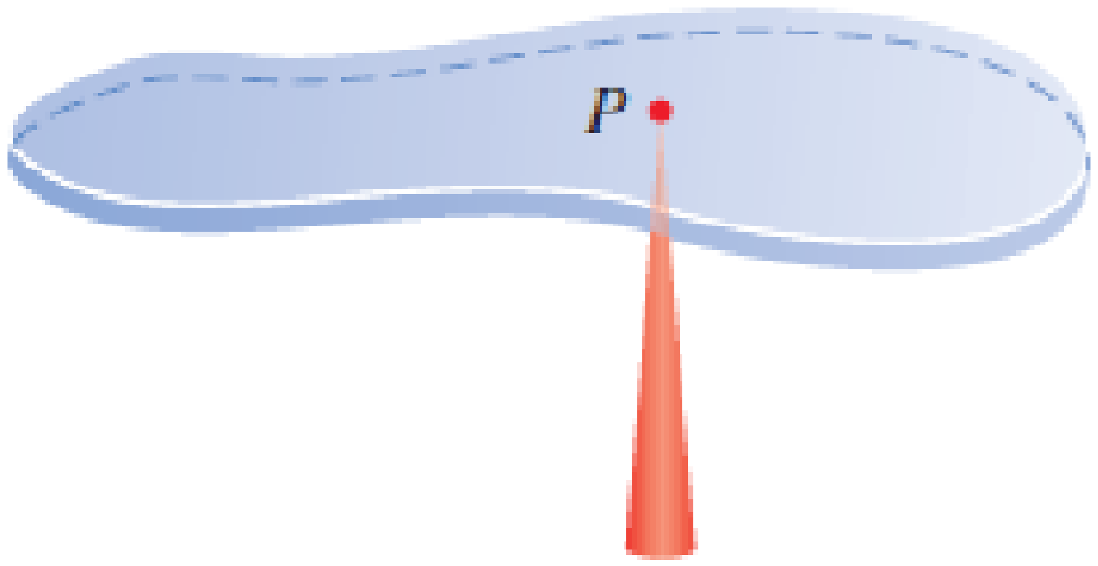


FIGURE 5

EXAMPLE 5 Find the centroid of the region in the first quadrant bounded by the curves $y = \cos x$, $y = 0$, and $x = 0$.

EXAMPLE 6 Find the centroid of the region bounded by the line $y = x$ and the parabola $y = x^2$.