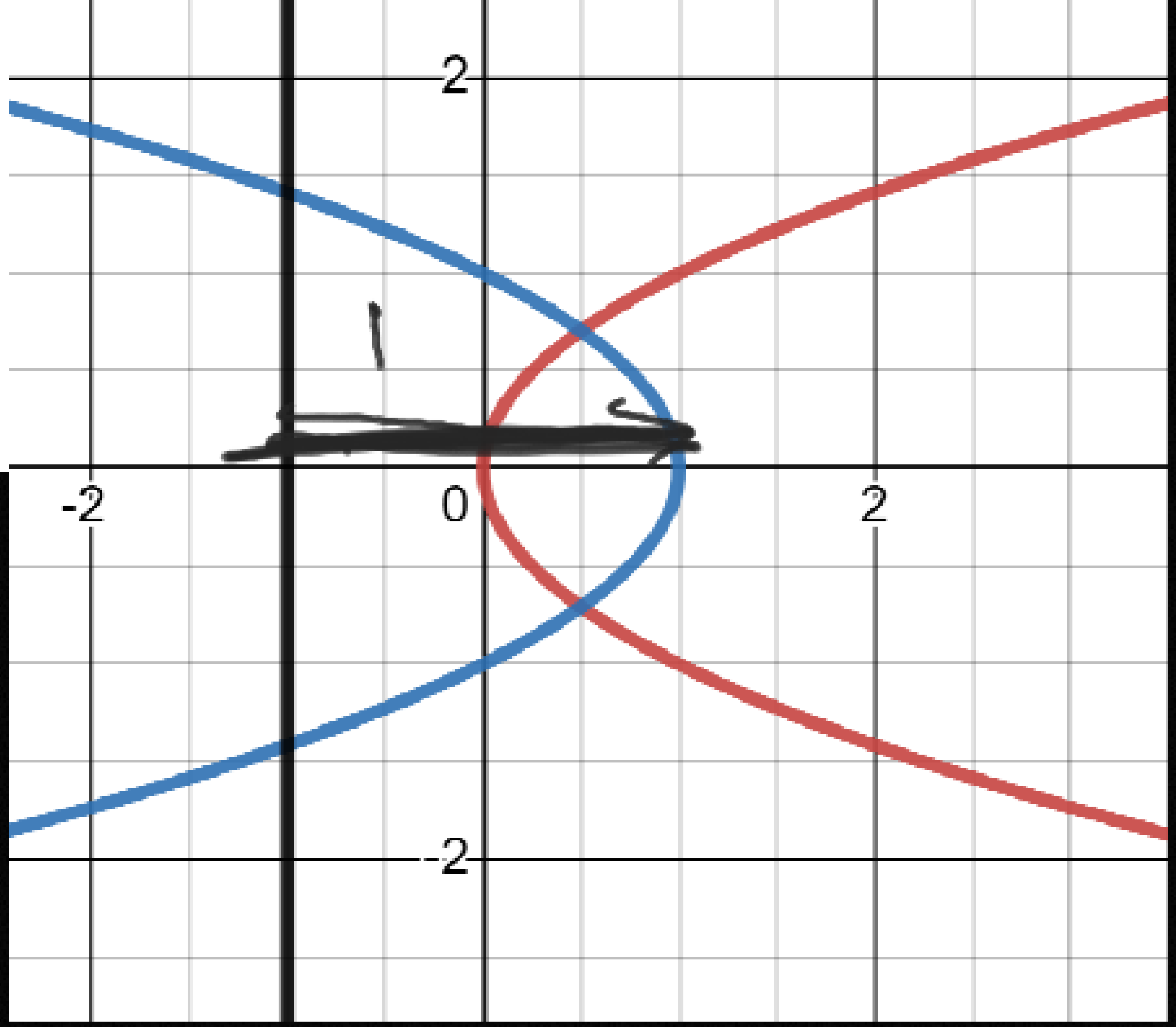
1) Find the volume of the rotated region bounded by the curves

$$x = y^2, x = 1 - y^2; \text{about } x = -1$$



$R \to 1 + (1 - y^2) = 2 - y^2$

$L \to 1 + y^2$

$y^2 = 1 - y^2$

$2y^2 = 1$

$y^2 = 1/2$

$y = \sqrt{\frac{1}{2}} \pm \frac{\sqrt{2}}{2}$

$.707$

$-.707$

$\frac{\sqrt{2}}{2}$

$$2\int_0^{\frac{\sqrt{2}}{2}} \pi\left[(2-y^2)^2-(1+y^2)^2\right]$$

$$(4-4y^2+y^4)-(1+2y^2+y^4)$$

$$3-6y^2$$

$$\frac{2\sqrt{2}}{4}=\frac{\sqrt{2}}{2}$$

$$2\pi\int_0^{\frac{\sqrt{2}}{2}} 3-6y^2\,dy$$

$$2\pi\left[3y-2y^3\right]_0^{\sqrt{2}/2}=2\pi\left[\frac{3\sqrt{2}}{2}-2\left(\frac{\sqrt{2}}{2}\right)^3\right]=2\pi\sqrt{2}$$

$$2\pi\left[\frac{3\sqrt{2}}{2} - 2\left(\frac{\sqrt{2}}{2}\right)^3\right]$$

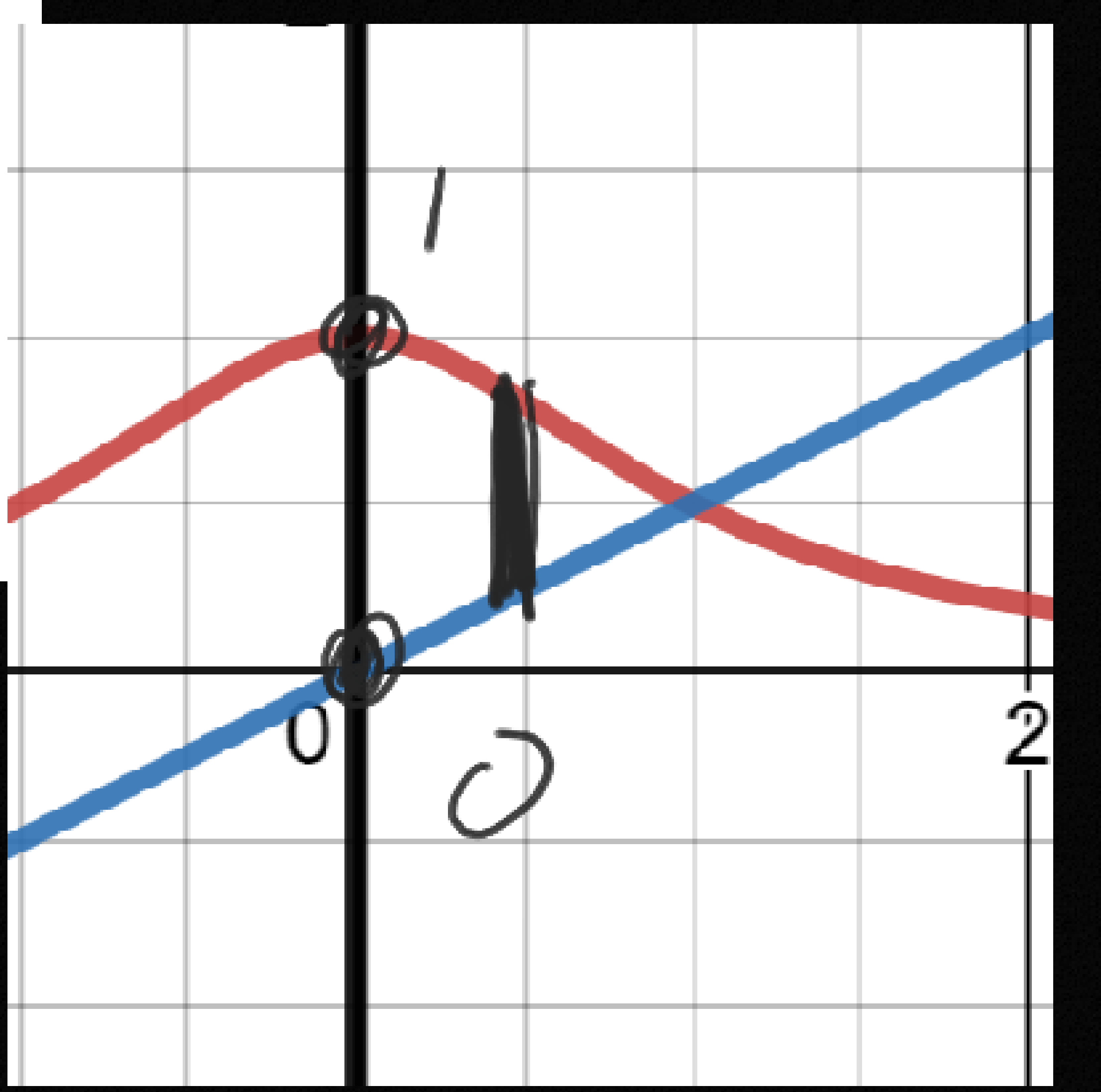
$$\frac{2\sqrt{2}^{\,3}}{2^3} = \frac{4\sqrt{2}}{8} = \frac{\sqrt{2}}{2}$$

$$2\pi\left[\frac{3\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right] = 2\pi\left[\frac{2\sqrt{2}}{2}\right]$$

$$= 2\pi\sqrt{2}$$

2) Find the volume of the rotated region bounded by the curves

$$y = \frac{1}{1+x^2}, y = \frac{x}{2}; \text{around the } y - \text{axis}$$



$$\int 2\pi x(h)$$

$$2\pi \int_0^1 x\left(\frac{1}{1+x^2} - \frac{x}{2}\right) dx$$

$$2\pi\int_0^1 x\left(\frac{1}{1+x^2}-\frac{x}{2}\right)dx$$

$$\frac{x}{1+x^2}$$

$u = 1+x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$2\pi\int_0^1 \frac{x}{1+x^2}-\frac{x^2}{2}\,dx$$

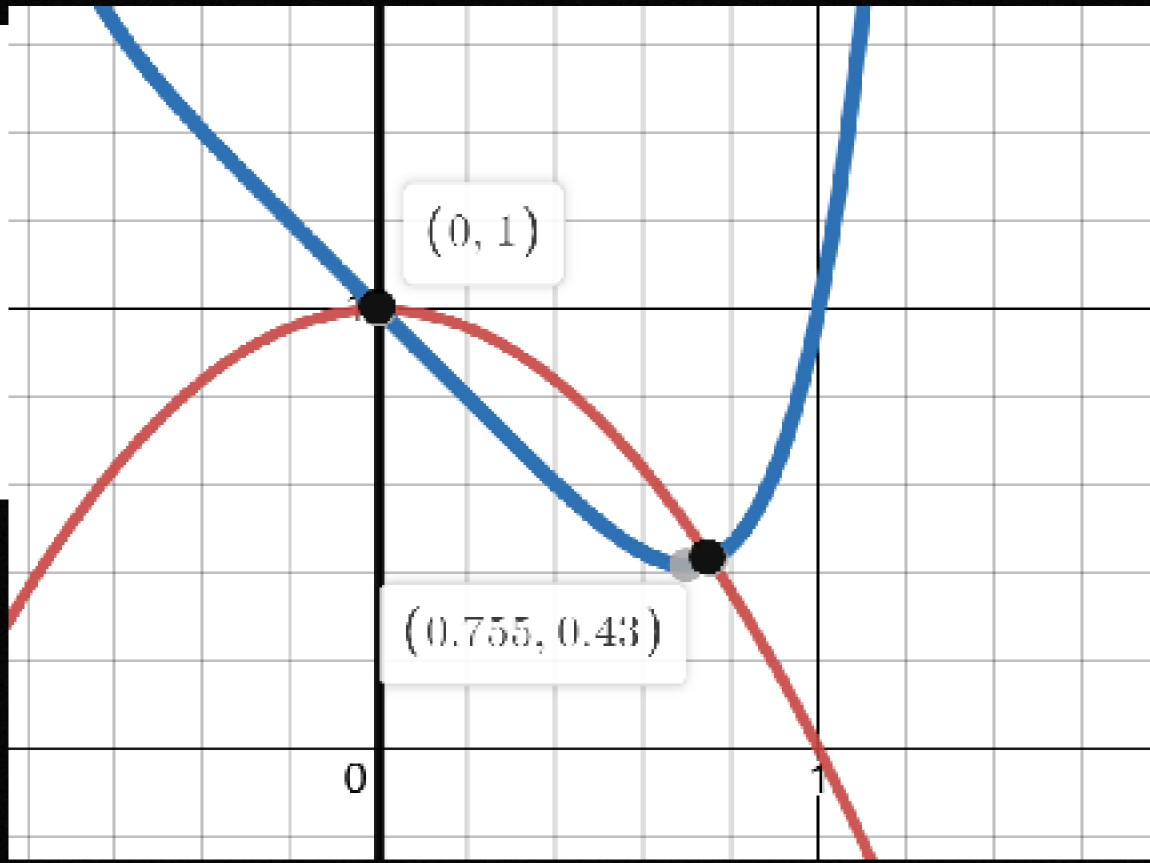
$$\frac{1}{2}\,\frac{1}{u}$$

$$2\pi\left[\frac{1}{2}\ln|1+x^2|-\frac{x^3}{6}\right]_0^1$$

$$2\pi\left[\frac{1}{2}\ln(2)-\frac{1}{6}\right] = \boxed{\pi\ln(2)-\frac{\pi}{3}}$$

3) Find the volume of the rotated region bounded by the curves

$$y = 1 - x^2, y = x^6 - x + 1; \text{around the } y - \text{axis}$$



1. **SIMPLIFY THE INTEGRAND IF POSSIBLE** Sometimes the use of algebraic manipulation or trigonometric identities will simplify the integrand and make the method of integration obvious. Here are some examples:

2. **LOOK FOR AN OBVIOUS SUBSTITUTION** Try to find some function $u = g(x)$ in the integrand whose differential $du = g'(x)\ dx$ also occurs, apart from a constant factor. For instance, in the integral

$$\int \frac{x}{x^2 - 1} dx$$

we notice that if $u = x^2 - 1$, then $du = 2x\ dx$. Therefore we use the substitution $u = x^2 - 1$ instead of the method of partial fractions.

(a) *Trigonometric functions.* If $f(x)$ is a product of powers of $\sin x$ and $\cos x$, of $\tan x$ and $\sec x$, or of $\cot x$ and $\csc x$, then we use the substitutions recommended in Section 7.2.

(b) *Rational functions.* If f is a rational function, we use the procedure of Section 7.4 involving partial fractions.

(c) *Integration by parts.* If $f(x)$ is a product of a power of x (or a polynomial) and a transcendental function (such as a trigonometric, exponential, or logarithmic function), then we try integration by parts, choosing u and dv according to the advice given in Section 7.1. If you look at the functions in Exercises 7.1, you will see that most of them are the type just described.

(d) *Radicals.* Particular kinds of substitutions are recommended when certain radicals appear.

(i) If $\sqrt{x^2 + a^2}$, $\sqrt{x^2 - a^2}$, or $\sqrt{a^2 - x^2}$ occurs, we use a trigonometric substitution according to the table in Section 7.3.

(ii) If $\sqrt[n]{ax+b}$ occurs, we use the rationalizing substitution $u = \sqrt[n]{ax+b}$. More generally, this sometimes works for $\sqrt[n]{g(x)}$.

4) $\int x^4 \ln x \, dx$

$u = \ln x \qquad v = \frac{1}{5}x^5$

$du = \frac{1}{x}dx \qquad dv = x^4 dx$

$uv - \int v \, du$

$\frac{x^4}{5}$

$(\ln x)\left(\frac{1}{5}x^5\right) - \int \frac{1}{5}x^5\left(\frac{1}{x}\right)dx$

$\boxed{\frac{1}{5}x^5 \ln x - \frac{1}{25}x^5 + C}$

5) $\int_0^{\frac{\pi}{3}} e^{3x}\cos x\,dx$

$u = \cos x \quad v = \frac{1}{3}e^{3x}$

$du = -\sin x\,dx \quad dv = e^{3x}dx$

$(\cos x)(\frac{1}{3}e^{3x}) - \boxed{\int(\frac{1}{3}e^{3x})(-\sin x)dx}$

$u = -\sin x \quad v = \frac{1}{9}e^{3x}$

$du = -\cos x\,dx \quad dv = \frac{1}{3}e^{3x}dx$

$\frac{1}{3}e^{3x}\cos x - \left[(-\sin x)\frac{1}{9}e^{3x} - \int \frac{1}{9}e^{3x}(-\cos x)\,dx\right]$

$$\frac{1}{3}e^{3x}\cos x - \left[(-\sin x)\frac{1}{9}e^{3x} - \int \frac{1}{9}e^{3x}(-\cos x)\,dx\right]$$

$$\left(\frac{1}{3}e^{3x}\cos x + \frac{1}{9}e^{3x}\sin x\right) - \frac{1}{9}\int e^{3x}\cos x\,dx$$

$$\frac{9}{10}$$

$$\int e^{3x}\cos x\,dx =$$

$$+\frac{1}{9}$$

$$\frac{10}{9}\int e^{3x}\cos x\,dx = \boxed{\frac{3}{10}e^{3x}\cos x + \frac{1}{10}e^{3x}\sin x}$$

$$\left[\frac{3}{10}e^{3x}\cos x + \frac{1}{10}e^{3x}\sin x\right]_0^{\pi/3}$$

$$\left[\frac{3}{10}e^{\pi}\left(\frac{1}{2}\right) + \frac{1}{10}e^{\pi}\left(\frac{\sqrt{3}}{2}\right)\right] - \left[\frac{3}{10}\right]$$

$$\boxed{\frac{3}{20}e^{\pi} + \frac{\sqrt{3}}{20}e^{\pi} - \frac{3}{10}}$$

6) $\int \tan^4 x \sec^2 x \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$$\int u^4 \, du$$

$$\frac{u^5}{5} + C \rightarrow \boxed{\frac{\tan^5 x}{5} + C}$$

7) $\int \sin^5 x \cos^2 x \, dx$

$$\int \sin^4 x \cos^2 x \boxed{\sin x \, dx}$$

$$u = \cos x$$

$$-du = \sin x \, dx$$

$$\sin^4 x = (1-\cos^2 x)^2$$

$$-\int (1-u^2)^2 u^2 \, du$$

$$(1-2u^2+u^4)u^2$$

$$-\int u^2 - 2u^4 + u^6 \, du$$

$$-\left[\frac{u^3}{3} - \frac{2u^5}{5} + \frac{u^7}{7}\right] + C$$

$$= \boxed{-\frac{1}{3}\cos^3 x + \frac{2}{5}\cos^5 x - \frac{1}{7}\cos^7 x + C}$$

8) $\int \frac{1}{x^2\sqrt{x^2-16}}\,dx$

$x = 4\sec\theta$

$dx = 4\sec\theta\tan\theta\,d\theta$

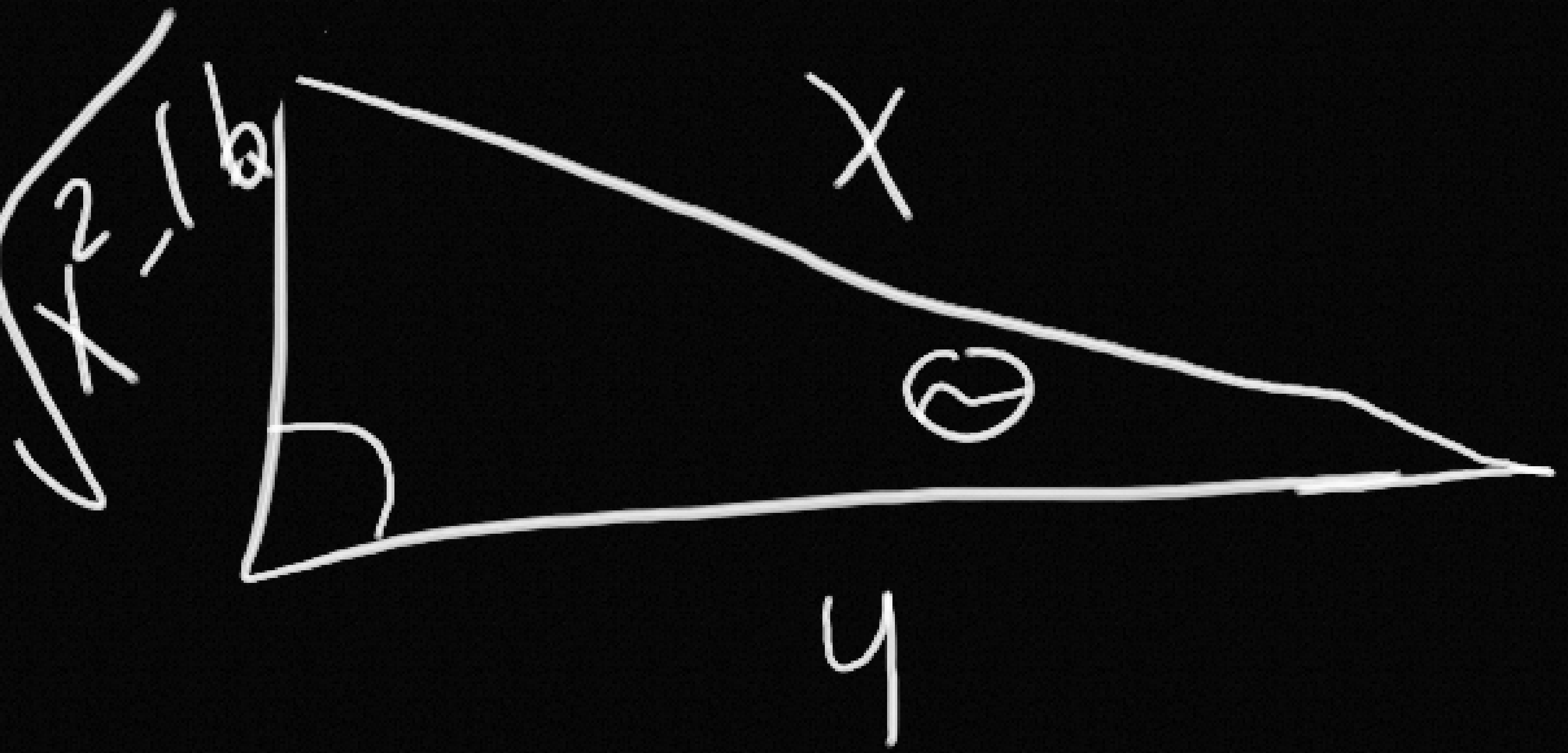
$\sqrt{x^2-16} \to 4\tan\theta$

$$\int \frac{4\sec\theta\tan\theta}{16\sec^2\theta\;4\tan\theta}\,d\theta$$

$$\int \frac{1}{16\sec\theta}\,d\theta = \frac{1}{16}\int \cos\theta\,d\theta$$

$$\frac{1}{16}\int \cos\theta \, d\theta = \frac{1}{16}\sin\theta + C$$

$$x = 4\sec\theta$$



$$\boxed{\frac{\sqrt{x^2-16}}{16x} + C}$$

9) $\int_1^2 \frac{3x^2+6x+2}{x^2+3x+2}\,dx$

$$\begin{array}{r} 3 \\ x^2+3x+2 \overline{\big) 3x^2+6x+2} \\ -\ 3x^2 \mp 9x \mp 6 \\ \hline -3x-4 \end{array}$$

$$\int_1^2 3 - \frac{3x+4}{\underbrace{x^2+3x+2}_{(x+1)(x+2)}}\,dx$$

$$3x+4 = A(x+2) + B(x+1)$$

$$\frac{3x+4}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$3x+4 = A(x+2) + B(x+1)$$

$x=-2$ $\quad 2 = B$

$x=-1$ $\quad 1 = A$

$$\int_1^2 3 - \frac{1}{x+1} - \frac{2}{x+2}\,dx$$

$$3x - \ln|x+1| - 2\ln|x+2| \Big]_1^2$$

$$(6 - \ln 3 - 2\ln 4) - (3 - \ln 2 - 2\ln 3)$$

$$\boxed{3 + \ln 2 + \ln 3 - 2\ln 4}$$

$$3 + \ln\frac{3}{8}$$

$$\int \frac{10}{(x-1)(x^2+9)}dx$$

$$\rightarrow \frac{A}{x-1} + \frac{Bx+C}{x^2+9}$$

$$10 = A(x^2+9) + (Bx+C)(x-1)$$

$x=1 \quad 10 = 10A \quad A = 1$

$$10 = x^2+9+Bx^2-Bx+Cx-C$$

$$1 = (B+1)x^2 + \boxed{(C-B)}x - C$$

$B=-1 \quad C=-1$

$$\int \frac{1}{x-1} + \frac{-x-1}{x^2+9}\,dx$$

$U = x^2 + 9$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\int \frac{1}{x-1} - \boxed{\frac{x}{x^2+9}} - \frac{1}{x^2+9}\,dx$$

$\frac{1}{2}\frac{1}{U} = \frac{1}{2}\ln|U|$

$$\boxed{\ln|x-1| - \frac{1}{2}\ln|x^2+9| - \frac{1}{3}\tan^{-1}\frac{x}{3} + C}$$

$\frac{1}{a}\tan^{-1}\frac{x}{a}$