## Odd/Even function identities:

$$\sin(-u) = -\sin(u) \qquad \cos(-u) = \cos(u) \qquad \tan(-u) = -\tan(u)$$

$$\csc(-u) = -\csc(u) \qquad \sec(-u) = \sec(u) \qquad \cot(-u) = -\cot(u)$$

### Double-angle Identities

1) $\sin 2u = 2\sin u \cos u$

2) $\cos 2u = \cos^2 u - \sin^2 u$

3) $\cos 2u = 2\cos^2 u - 1$

4) $\cos 2u = 1 - 2\sin^2 u$

5) $\tan 2u = \dfrac{2\tan u}{1 - \tan^2 u}$

### Sum and Difference Identities

1) $\sin(u+v) = \sin u \cos v + \cos u \sin v$

2) $\sin(u-v) = \sin u \cos v - \cos u \sin v$

3) $\cos(u+v) = \cos u \cos v - \sin u \sin v$

4) $\cos(u-v) = \cos u \cos v + \sin u \sin v$

5) $\tan(u+v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v}$

6) $\tan(u-v) = \dfrac{\tan u - \tan v}{1 + \tan u \tan v}$

**Reciprocal identities:**

$$\sin u = \frac{1}{\csc u} \qquad \cos u = \frac{1}{\sec u} \qquad \tan u = \frac{1}{\cot u}$$

$$\csc u = \frac{1}{\sin u} \qquad \sec u = \frac{1}{\cos u} \qquad \cot u = \frac{1}{\tan u}$$

**Quotient identities:**

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u}$$

**Pythagorean identities:**

$$\sin^2 u + \cos^2 u = 1$$

$$\tan^2 u + 1 = \sec^2 u$$

$$\cot^2 u + 1 = \csc^2 u$$

$$\frac{\csc x}{\sec x} = \frac{\frac{1}{\sin x}}{\frac{1}{\cos x}} = \frac{1}{\sin x} \cdot \frac{\cos x}{1} = \frac{\cos x}{\sin x}$$

$$\frac{\cos x}{\sin x} = \boxed{\cot(x)}$$

$\sin x \cot x$

$$\cancel{\sin x}\left(\frac{\cos x}{\cancel{\sin x}}\right) = \boxed{\cos x}$$

$$\sin(3x)\sec(3x) = \sin(3x)\cdot\frac{1}{\cos(3x)}$$

$$= \boxed{\tan(3x)}$$

$$\left(1+\cot^2 x\right)\cos^2 x$$

Pyth

$$\csc^2(x)\cos^2(x)$$

Reciprocal

$$\frac{\cos^2(x)}{\sin^2(x)}$$

Quotient

$$\cot^2(x)$$

**Pythagorean identities:**

$$\sin^2 u+\cos^2 u=1$$

$$\tan^2 u+1=\sec^2 u$$

$$\cot^2 u+1=\csc^2 u$$

$$\sin^2(x) + \cos^2(x) = 1$$

$$\cos^2(x) = 1 - \sin^2(x)$$

$\cot x \sec^2 x - \cot x$

$\big\downarrow$ Alg

$\cot(x)\left(\sec^2 x - 1\right)$

$\big\downarrow$ pyth

$\cot(x)\left(\tan^2(x)\right)$

$\big\downarrow$ Reciprical

$\dfrac{1}{\tan(x)} \quad \tan^2(x)$

$\big\downarrow$ Alg

$\tan(x)$

**Pythagorean identities:**

$$\sin^2 u + \cos^2 u = 1$$

$$\boxed{\tan^2 u + 1 = \sec^2 u}$$

$$\cot^2 u + 1 = \csc^2 u$$

$$(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$$

To see a detailed description of a Rule, select the |

| Statement | Rule |
|---|---|
| $(\sin x + \cos x)^2$ | |
| $= \sin^2 x + 2\sin x\cos x + \cos^2 x$ | Alg **Rule ?** |
| $= 1 + 2\sin x\cos x$ | Pyth **Rule ?** |

$(\sin x + \cos x)(\sin x \cos x$

$\sin^2 x + \sin x\cos x$
$+ \sin x\cos x + \cos^2 x$

$$\left(\sec^2 x - 1\right)\cos^2 x = \sin^2 x$$

To see a detailed description of a Rule, select the M

| Statement | Rule |
| --- | --- |
| $\left(\sec^2 x - 1\right)\cos^2 x$ | |
| $= \left(\tan^2 x\right)\cos^2 x$ | **Rule ?** |
| $= \left(\dfrac{\sin^2 x}{\cos^2 x}\right)\cos^2 x$ | **Rule ?** |
| $= \sin^2 x$ | **Rule ?** |

$$\left(1+\tan^2 x\right)\cot x = \sec x \csc x$$

| Statement | Rule |
|---|---|
| $\left(1+\tan^2 x\right)\cot x$ | |
| $= \sec^2 x \cot x$ | **Rule ?** |
| $= \left(\dfrac{1}{\cos^2 x}\right)\cot x$ | **Rule ?** |
| $= \left(\dfrac{1}{\cos^2 x}\right)\left(\dfrac{\cos x}{\sin x}\right)$ | **Rule ?** |
| $= \left(\dfrac{1}{\cos x}\right)\left(\dfrac{1}{\sin x}\right)$ | **Rule ?** |
| $= \sec x \csc x$ | **Rule ?** |

Prove the identity.

3) $1+\cot^2 u=\csc^2 u$

$\cot^2 u=\csc^2 u-1$

$$\csc^4 x-\csc^2 x=\cot^2 x+\cot^4 x$$

$\csc^2(x)\left(\csc^2(x)-1\right)$ alg

$(1+\cot^2 x)(\cot^2 x)$ pyth

$\cot^2 x+\cot^4 X$ alg

$(1-\cos x)(1+\cos x) = \sin^2 x$

$$x^2 - y^2 = (x-y)(x+y)$$

$\downarrow$

$$1-\cos^2 x \rightarrow 1 + \cancel{\boxed{\cos x\,\cos x}} - \cos^2 x$$

$\downarrow$

$\sin^2 x$

$$\sin^2 x + \cos^2 x = 1$$
$$-\cos^2 x$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\csc^2 x - \cot^2 x = 1$$

$$(\cot^2 x + 1) - \cot^2 x$$

$$\csc^2 x = \cot^2 x + 1$$

$$(\sec^2 x - 1)\cos^2 x = \sin^2 x$$

$$\tan^2 x \cos^2 x$$

P

$$\frac{\sin^2 x}{\cos^2 x}\cos^2 x$$

Q

$$\sin^2 x$$

A

$$\sec^2 x = \tan^2 x + 1$$

$-1 \quad -1$

$$\sec^2 x - 1 = \tan^2 x$$

$$\tan^2 x\left(\csc^2 x - 1\right) = 1$$

$$\cot^2 x + 1 = \csc^2 x$$

$$\left(\frac{1}{\cot^2 x}\right)(\csc^2 x - 1)$$

$$\tan^2 x \cot^2 x$$

$$\frac{\csc^2 x - 1}{\csc^2 x - 1}$$

$$\tan^2 x\left(\frac{1}{\tan^2 x}\right)$$

$$1$$

$$1$$

$$\tan x\left(1+\underline{\cot^2 x}\right)=\frac{1}{\cos x\,\sin x} = \sec x \csc x$$

$$\tan x \csc^2 x$$

$$\frac{\sin x}{\cos x}\csc^2 x$$

$$\frac{\sin x}{\cos x}\,\frac{1}{\sin^2 x} = \frac{1}{\cos x \sin x}$$

$(1 - \sin^2 x)\csc x = \cos x \cot x$

$\cos^2 x \csc x$ Pyth

$\cos^2 x \dfrac{1}{\sin x}$ R

$\cos x \dfrac{\cos x}{\sin x}$ A

$\cos x \cot x$ Q

## Using cofunction identities

Use a cofunction to write an expression equal to $\cot \frac{\pi}{11}$.

$$\tan\left(\frac{\pi}{2} - \frac{\pi}{11}\right) = \tan \frac{9}{22}\pi$$

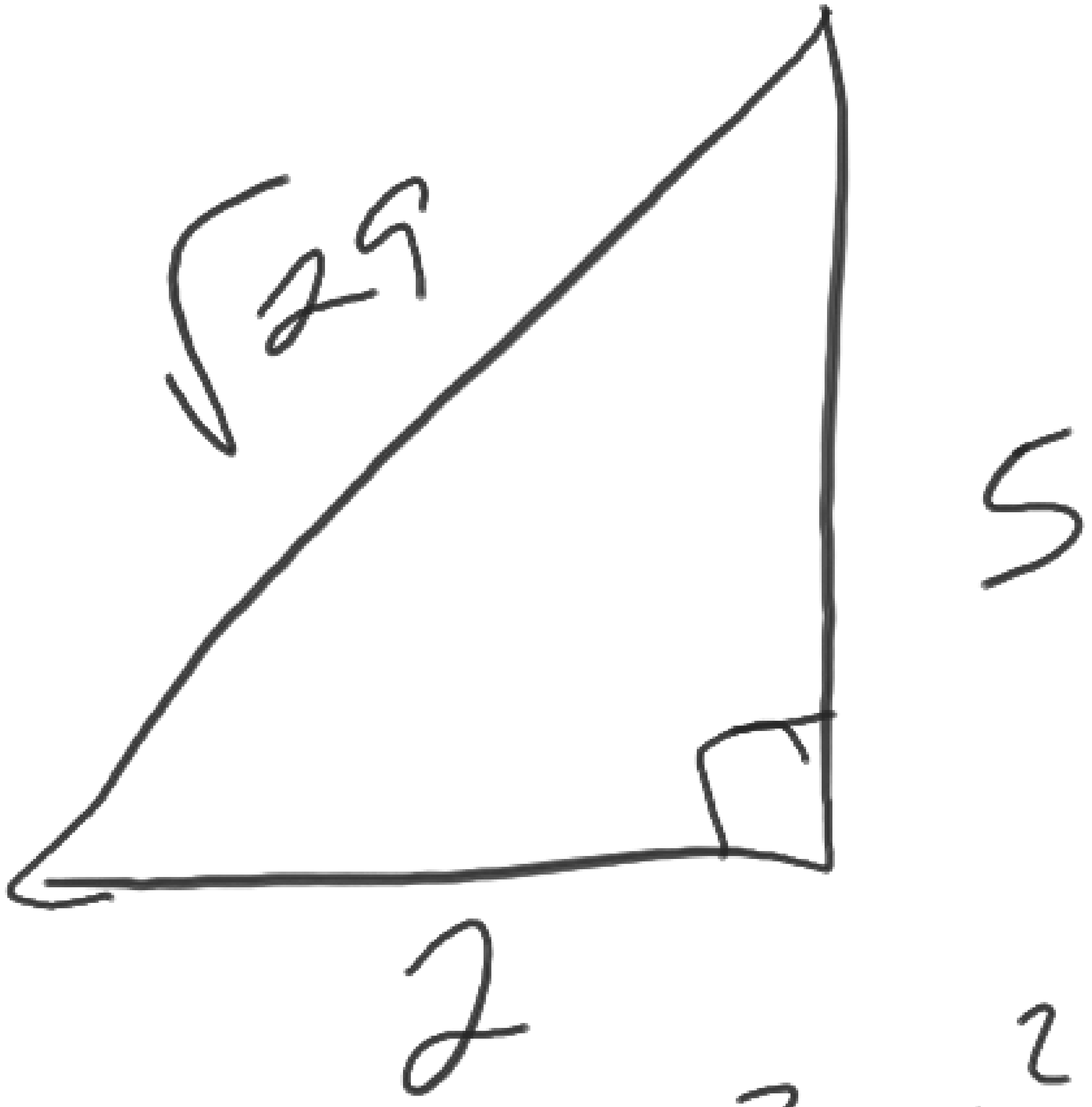
| Sine and Cosine | Tangent and Cotangent | Secant and Cosecant |
|---|---|---|
| $\sin x = \cos\left(\frac{\pi}{2} - x\right)$ | $\tan x = \cot\left(\frac{\pi}{2} - x\right)$ | $\sec x = \csc\left(\frac{\pi}{2} - x\right)$ |
| $\cos x = \sin\left(\frac{\pi}{2} - x\right)$ | $\cot x = \tan\left(\frac{\pi}{2} - x\right)$ | $\csc x = \sec\left(\frac{\pi}{2} - x\right)$ |

Let $\theta$ be an angle in quadrant I such that $\cot\theta = \frac{2}{5}$.

Find the exact values of $\sin\theta$ and $\sec\theta$.

$$\sin \frac{y}{r} = \frac{5}{\sqrt{29}}$$

$$\cos = \frac{x}{r} \rightarrow \sec \frac{r}{x} = \frac{\sqrt{29}}{2}$$




$$\tan = \frac{y}{x} \qquad \cot \frac{2}{5} \; \frac{x}{y}$$

$$2^2 + 5^2 = r^2$$

$$\sqrt{29} = r^2$$