7.3 The Discrete Uniform Distribution

Example 7.3.1

What is the probability distribution for the outcome of the throw of a single six-sided die?

Solution

If the die is *fair*, then each of the outcomes is equally likely and thus we have a uniform distribution in which all probabilities equal $\frac{1}{6}$. The probability distribution is given below.

Throwing a Die						
x	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Sharlene has just put a down payment on a lot in a small subdivision. There are 10 lots in the subdivision and all are approximately 0.25 acres in size. Five builders have been contracted by the subdivision manager to each build two homes in order to finish the subdivision in 6 months. Sharlene's uncle is one of the builders contracted by the subdivision manager. What is the probability that Sharlene's uncle will be the builder that builds her house?

x	1	2	3	4	5
	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

7.4 The Binomial Distribution

Binomial Experiment

A **binomial experiment** is a random experiment which satisfies all of the following conditions.

1. There are only two outcomes on each trial of the experiment. (One of the outcomes is usually referred to as a *success*, and the other as a *failure*.)
2. The experiment consists of n identical trials as described in condition 1.
3. The probability of success on any one trial is denoted by p and does not change from trial to trial. (**Note:** The probability of a failure is $1 - p$ and also does not change from trial to trial.)
4. The trials are independent.
5. The binomial random variable is the count of the number of successes in n trials.

DEFINITION

Binomial Probability Distribution Function

The **binomial probability distribution function** is

$$P(X=x) = {}_nC_x\, p^x (1-p)^{n-x}$$

where ${}_nC_x$ represents the number of possible combinations of n objects taken x at a time (without replacement) and is given by

$$\cancel{{}_nC_x = \frac{n!}{x!(n-x)!} \text{ where } n! = n(n-1)(n-2)\cdots 2\cdot 1} \text{ and } 0! = 1;$$

$n =$ the number of trials,

$p =$ the probability of a success, and

$x =$ the number of successes in n trials.

FORMULA

Example 7.4.1

Toss a coin 4 times and record the number of heads. Is the number of heads in 4 tosses a binomial random variable?

$n = 4$

$x = 3$ heads

$\boxed{0.25}$

$p = \frac{1}{2}$

How many ways can a person toss a coin 8 times so that the number of heads is between 4 and 7 inclusive?

Answer

How to enter your answer (opens in new window)

$n = 8$

$x = 4, 5, 6, 7$

$p = 1/2$

$x = 4 \quad 0.273$

$x = 5 \quad 0.219$

$x = 6 \quad 0.109$

$x = 7 \quad 0.031$

$\boxed{0.632}$

Example 7.4.4

According to a recent national poll, about 40% of Americans believe in ghosts. Assuming this percentage is accurate, if 20 people were randomly selected and asked if they believed in ghosts, what is the probability that 12 ~~or more would say~~ they do?

$n = 20$

$x = 12$

$p = .40$

0.035

Example 7.4.4

According to a recent national poll, about 40% of Americans believe in ghosts. Assuming this percentage is accurate, if 20 people were randomly selected and asked if they believed in ghosts, what is the probability that 12 ~~or more would say~~ they do?

$n = 20$

3 or more

20	0	0.1216	0.0115	0.0008	0.0000
	1	0.2702	0.0576	0.0068	0.0005
	2	0.2852	0.1369	0.0278	0.0031
	3	0.1901	0.2054	0.0716	0.0123
	4	0.0898	0.2182	0.1304	0.0350

$X = 0, 1, 2$

$P = .40$

$1 - 0.0036$

$.9964$

Expected Value of a Binomial Random Variable

The **expected value** of a binomial random variable can be computed using the expression

$$\mu = E(X) = np,$$

where n and p are the parameters of the binomial distribution.

FORMULA

$n = 20 \quad p = .4$

$\mu = 20(.4)$
$= 8$

It is important to remember that this formula is only valid for binomial random variables. Also, there is a simple method for obtaining the variance for binomial random variables.

Variance and Standard Deviation of a Binomial Random Variable

To find the **variance** of a binomial random variable, use the expression

$$\sigma^2 = V(X) = np(1-p).$$

Therefore, the **standard deviation** of a binomial random variable is given by

$$\sigma = \sqrt{V(X)} = \sqrt{np(1-p)}.$$

FORMULA

Variance
$20(.4)\underbrace{(.6)}$
4.8

St Dev $\sqrt{4.8}$
2.19

9. The random variable X is a binomial random variable with $n = 9$ and $p = 0.1$.

a. Find the expected value of X. $0.9 = 9(0.1)$

b. Find the standard deviation of X. $\sqrt{9(.1)(.9)} = 0.9$

c. Find the probability that X equals 2. (Use the formula for $P(X = x)$.) 0.172

d. Find the probability that X is at most 3.

[redacted]

$X = 0, 1, 2, 3$

$0.387 + 0.387 + 0.172 + 0.045$

$\boxed{0.991}$

12. A small commuter airline is concerned about reservation no-shows and, correspondingly, how much they should overbook flights to compensate. Assume their commuter planes will hold 15 people. Industry research indicates that 20% of the people making a reservation will not show up for a flight. Whether or not one person takes the flight is considered to be independent of other persons holding reservations.

$n = 17$

$x = 15$

$p = .80$

0.191

A company that makes traffic signal lights buys switches from a supplier. Out of each shipment of 1000 switches, the company will take a random sample of 10 switches. Let X equal the number of defective switches in the sample.

a. The company has a policy of rejecting a lot if they find any defective switches in the sample. What is the probability that the shipment will be accepted if, in fact, 2% of the switches are actually defective?

$n = 10$ $\quad x = 0$ $\quad p = .02$

0.817 $\quad 81.7\%$

A company that makes traffic signal lights buys switches from a supplier. Out of each shipment of 1000 switches, the company will take a random sample of 10 switches. Let X equal the number of defective switches in the sample.

a. The company has a policy of rejecting a lot if they find any defective switches in the sample. What is the probability that the shipment will be accepted if, in fact, 2% of the switches are actually defective? 81.7%

b. What is the probability that the shipment will be accepted if the percent of defective switches is actually 5%? 59.9%

c. The company decides to change their policy and will accept the lot if they find no more than one defective switch. Repeat parts a. and b. for this new policy.