$$\cos y = 0$$

$$\cos^{-1}(\cos y) = \cos^{-1}(0)$$

$$y = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$y = \frac{\pi}{2} + 2k\pi$$

$$= \frac{3\pi}{2} + 2k\pi$$

$$= \frac{\pi}{2} + k\pi$$

Find all solutions to the equation.

$\cos 2x = 0$

$$\frac{2x}{2} = \frac{\frac{\pi}{2}}{2} + \frac{k\pi}{2}$$

$$x = \frac{\pi}{4} + \frac{1}{2}k\pi$$

$$x = \frac{\pi}{4} + k\pi$$

$$= \frac{3\pi}{4} + k\pi$$

Find all solutions of the equation in the interval $[0, 2\pi)$.

$$2\cos\frac{2x}{3} - 1 = 0$$

$$\cos\frac{2x}{3} = \frac{1}{2}$$

$$\frac{2x}{3} = \cos^{-1}\frac{1}{2}$$

$$\frac{2x}{3} = \frac{\pi}{3} \Rightarrow x = \frac{\pi}{2}$$

$$\frac{2x}{3} = \frac{5\pi}{3} \Rightarrow x = \frac{5\pi}{2}$$

$$\cos 2x = \frac{1}{2}$$

$$2x = \frac{\pi}{3} \quad 2x = \frac{5\pi}{3} \quad 2x = \frac{7\pi}{3} \quad 2x = \frac{11\pi}{3}$$

$$x = \frac{\pi}{6} \quad x = \frac{5\pi}{6} \quad x = \frac{7\pi}{6} \quad x = \frac{11\pi}{6}$$

Find all solutions of the equation in the interval $[0, 2\pi)$.

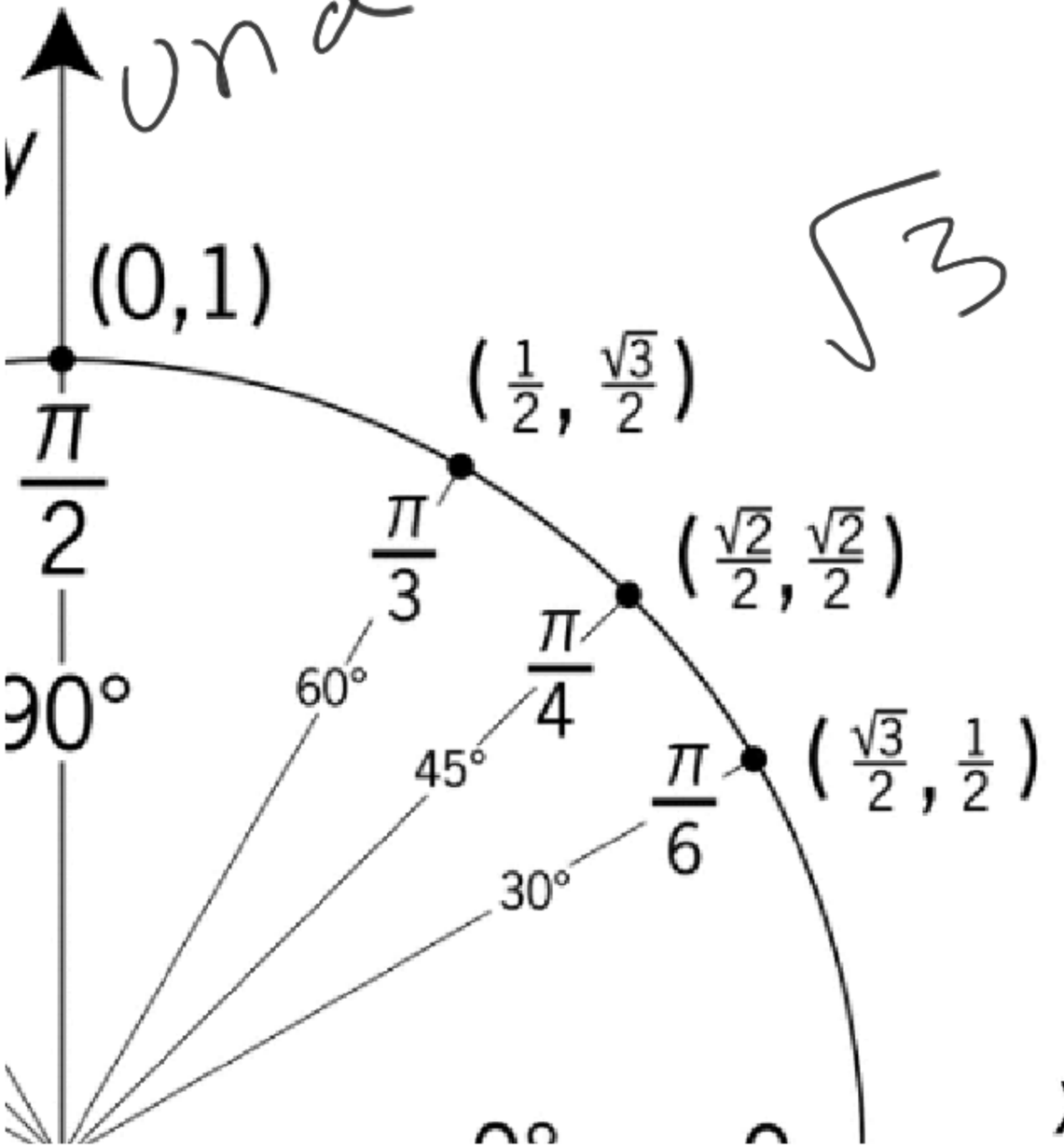
$$\sqrt{3}\cot\frac{2x}{3}+1=0$$

$$\cot\frac{2x}{3}=-\frac{1}{\sqrt{3}}$$

$$\tan\frac{2x}{3}=-\sqrt{3}$$

$$\frac{2x}{3}=\frac{2\pi}{3} \rightarrow x=\pi$$

$$\frac{2x}{3}=\frac{5\pi}{3} \rightarrow x=\frac{5\pi}{2}$$



und

$\sqrt{3}$

1

$\frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

0

Find all solutions of the equation in the interval $[0, 2\pi)$.

$$\sec(2x) + 1 = 0$$

$$\sec(2x) = -1$$

$$\cos 2x = -1$$

$$\frac{2x}{2} = \frac{\pi}{2}$$

$$x = \frac{\pi}{2}$$

$$\frac{2x}{2} = \frac{3\pi}{2}$$

$$x = \frac{3\pi}{2}$$

Find all solutions of the equation in the interval $[0, 2\pi)$.

$$\cos x = -\sin^2 x - 1$$

$\cos^2 x + \sin^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$\cos x = -(1 - \cos^2 x) - 1$

$\cos x = -2 + \cos^2 x$

$\cos^2 x - \cos x - 2 = 0$

$(\cos x - 2)(\cos x + 1) = 0$

~~$\cos x = 2$~~

$\cos x = -1$

$x = \pi$

Find all solutions of the equation in the interval $[0, 2\pi)$.

$$2\cos^2\theta - 3\sin\theta = 3$$

$\cos^2\theta = 1 - \sin^2\theta$

$2(1-\sin^2\theta) - 3\sin\theta = 3$

$2 - 2\sin^2\theta - 3\sin\theta = 3$

$2\sin^2\theta + 3\sin\theta + 1 = 0$

$(2\sin\theta + 1)(\sin\theta + 1) = 0$

$\sin\theta = -\frac{1}{2}$ $\quad \sin\theta = -1$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\theta = \frac{3\pi}{2}$

Find all solutions of the equation in the interval $[0, 2\pi)$.

$$\left(\sin x = \sqrt{1-\cos x}\right)^2$$

$$\sin^2 x = 1 - \cos x$$

$$1 - \cos^2 x = 1 - \cos x$$

$$\cos^2 x - \cos x = 0$$

$$\cos x(\cos x - 1) = 0$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos x = 0 \rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\cos x = 1 \rightarrow x = 0$$

$x = 0 \rightarrow 0 = \sqrt{1-1}$ ✓

$x = \frac{\pi}{2} \rightarrow 1 = \sqrt{1-0}$ ✓

$x = \frac{3\pi}{2} \rightarrow -1 = \sqrt{1-0}$ ✗

Solving a trigonometric equation modeling a real-world situation

Suppose that the height above ground of a person sitting on a Ferris wheel is described by the following.

$$h(t) = 18.5 - 16.4\cos\left(\frac{2\pi}{5}t\right)$$

In this equation, $h(t)$ is the height above ground (in meters) and t is the time (in minutes). The ride begins at $t = 0$ minutes.

During the first 5 minutes of the ride, when will the person be 28 meters above the ground?

$28 = 18.5 - 16.4\cos\left(\frac{2\pi}{5}t\right)$

$9.5 = -16.4\cos\left(\frac{2\pi}{5}t\right)$

$\frac{-9.5}{16.4} = \cos\left(\frac{2\pi}{5}t\right)$

$t = 0 \to 5$

$[0, 5)$

↑ one period

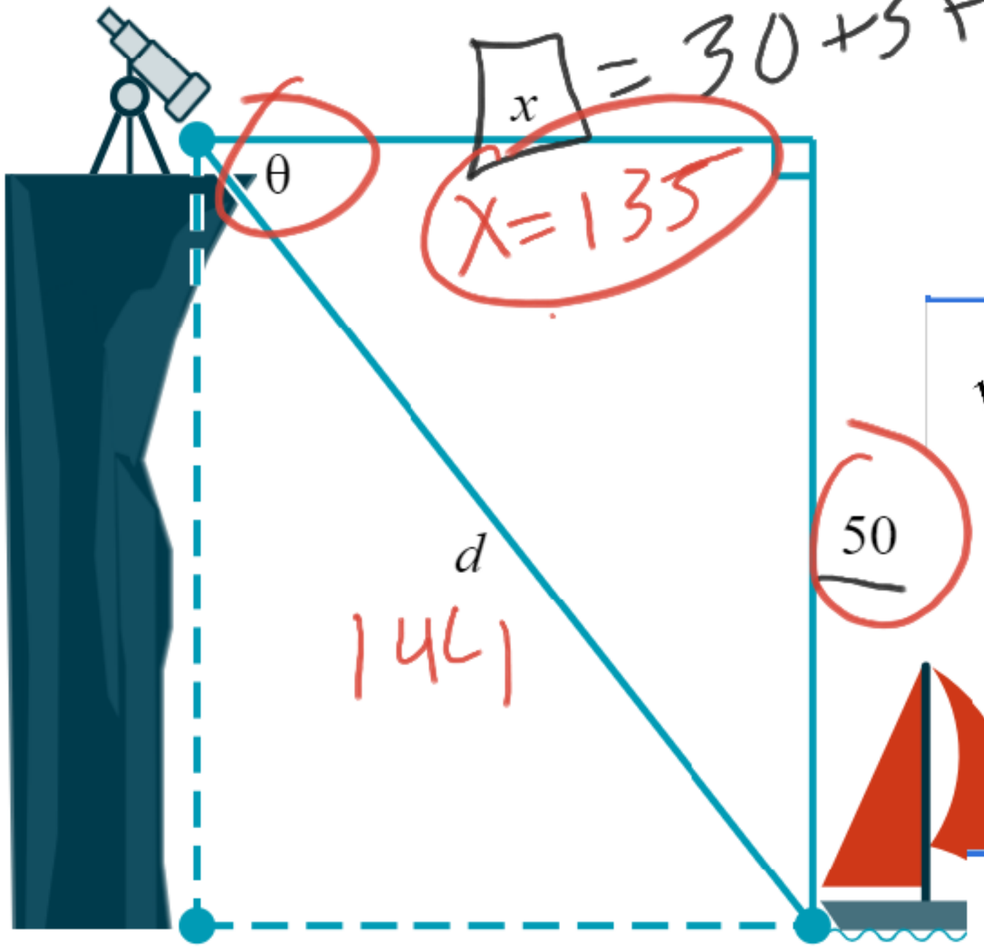
$$\frac{-9.5}{16.4} = \cos\left(\frac{2\pi}{5}t\right)$$

$$\cos^{-1}\left(\frac{-9.5}{16.4}\right) = \frac{2\pi}{5}t \rightarrow \pi + \cos^{-1}\left(\frac{-9.5}{16.4}\right) = \frac{2\pi}{5}t$$

$$\frac{5}{2\pi}\cos^{-1}\left(\frac{-9.5}{16.4}\right) = t \rightarrow \frac{5}{2\pi}\left(\pi + \cos^{-1}\left(\frac{-9.5}{16.4}\right)\right) = t$$

$$t = 1.74 \qquad t = 4.24$$

A child watches a sailboat through a coin-operated telescope that is 50 meters above the bay, on a cliff. The angle of depression from the telescope to the boat is θ. See the figure below. At time $t = 0$, the boat is 30 meters from the base of the cliff and sailing directly away from the cliff at a constant rate of 3 meters per second. The distance x (in meters) between the base of the cliff and the boat is given by $x = 30 + 3t$.



$x = 30 + 3t$

$X = 135$

$x^2 + 50^2 = d^2$

$(30+3t)^2 + 50^2 = d(t)$

$\sqrt{(30+3(35))^2 + 50^2}$ at 35 sec $= 143.9618005$

141

$\tan\theta = \frac{50}{135}$

$\theta = \tan^{-1}\left(\frac{50}{135}\right) = 20.3°$

$\tan^{-1}\left(\frac{50}{135}\right) = 20.32313683$