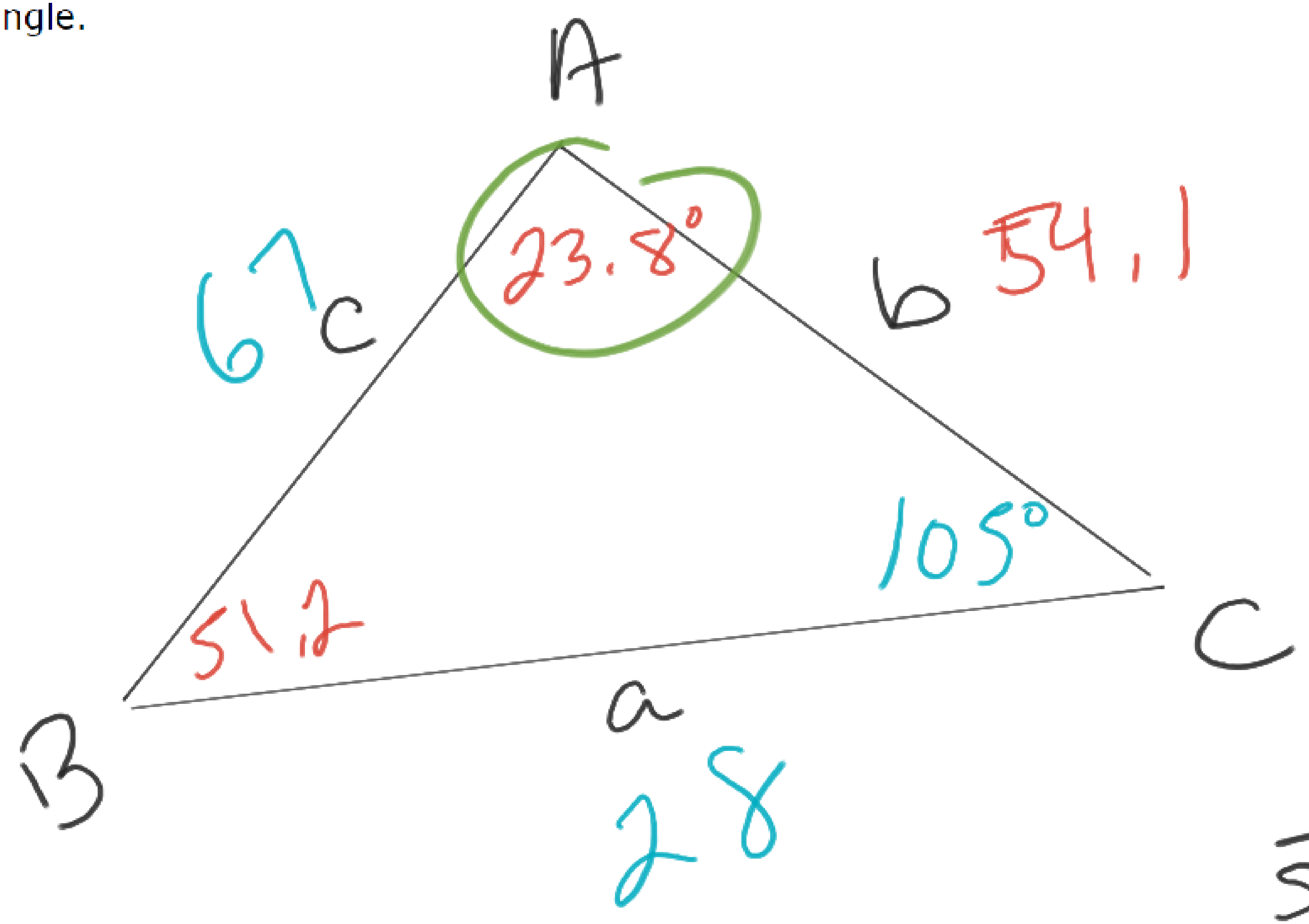
Consider a triangle ABC like the one below. Suppose that $c = 67$, $a = 28$, and $C = 105°$. (The figure is not drawn to scale.) Solve the triangle.



$$180 - 105 - 23.8 = 51.2$$

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin 105}{67} = \frac{\sin A}{28}$$

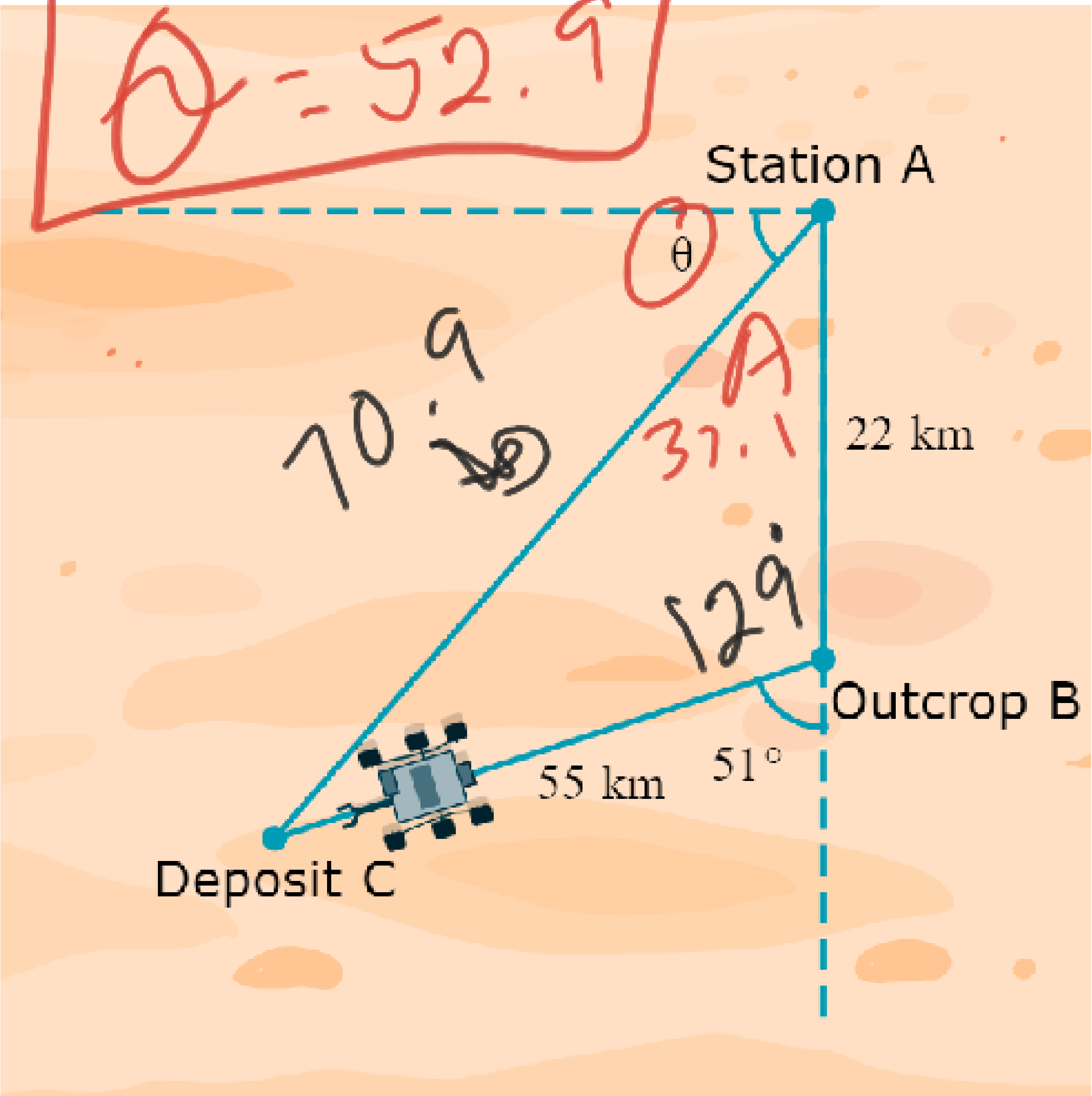
$$\frac{28 \sin 105}{67} = \sin A$$

$$\sin^{-1}\left(\frac{28 \sin 105}{67}\right) = A$$

$$\frac{c}{\sin C} = \frac{b}{\sin B} \qquad b = \frac{67 \sin 51.2}{\sin 105}$$

$$\frac{67}{\sin 105} = \frac{b}{\sin 51.2}$$

A rover on Mars leaves Station A and travels 22 km due south to sample rocks at Outcrop B. It then adjusts its course 51° westward. It travels 55 km in that direction until it reaches Deposit C to sample soil. What angle θ with respect to due west could the rover have used to travel directly from Station A to Deposit C? See the figure below. (The figure is not drawn to scale.)



$A = 37.1^\circ$

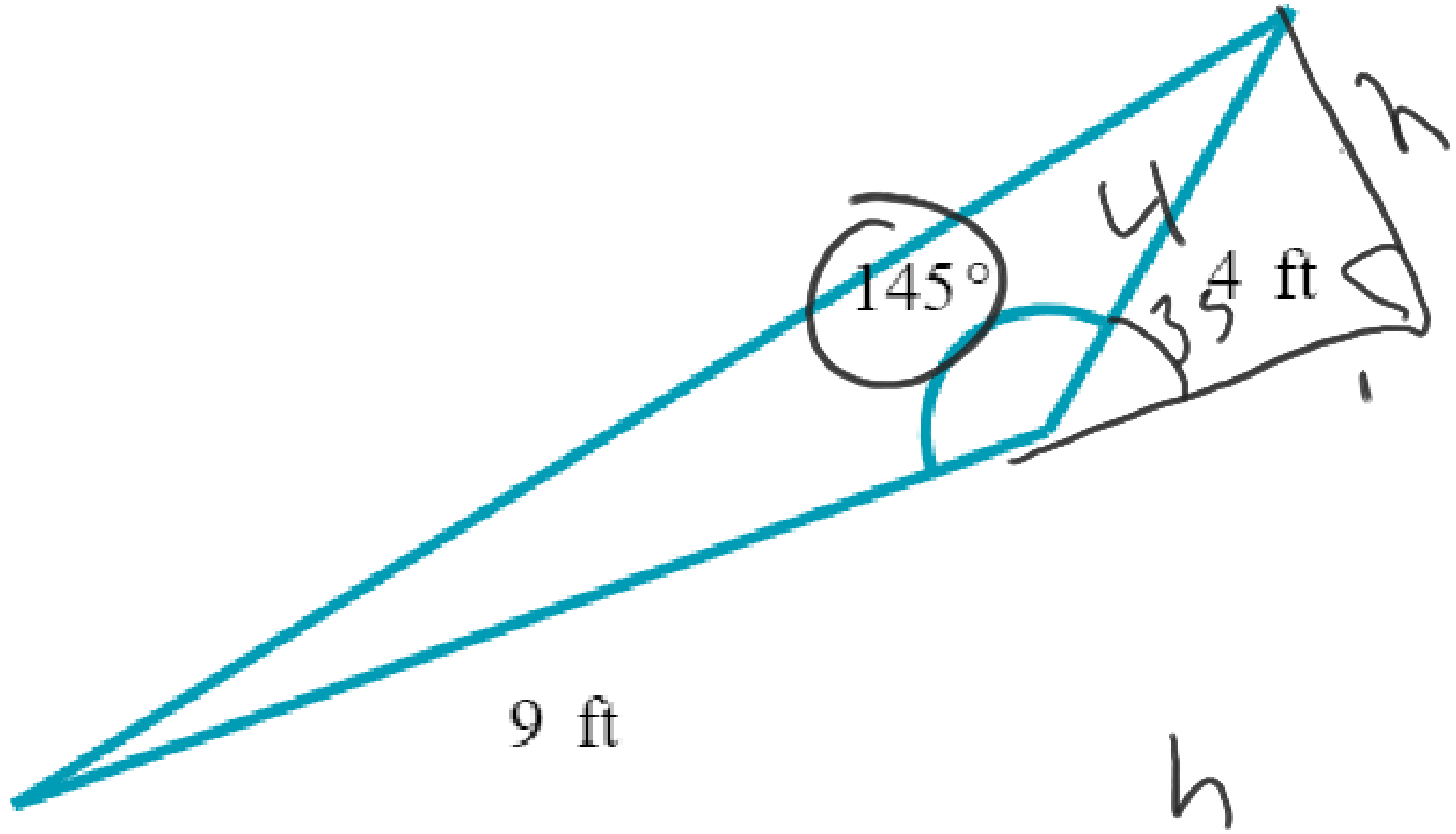
$b^2 = a^2 + c^2 - 2ac\cos B$

$\sqrt{b^2} = \sqrt{55^2 + 22^2 - 2(55)(22)(\cos 129)}$

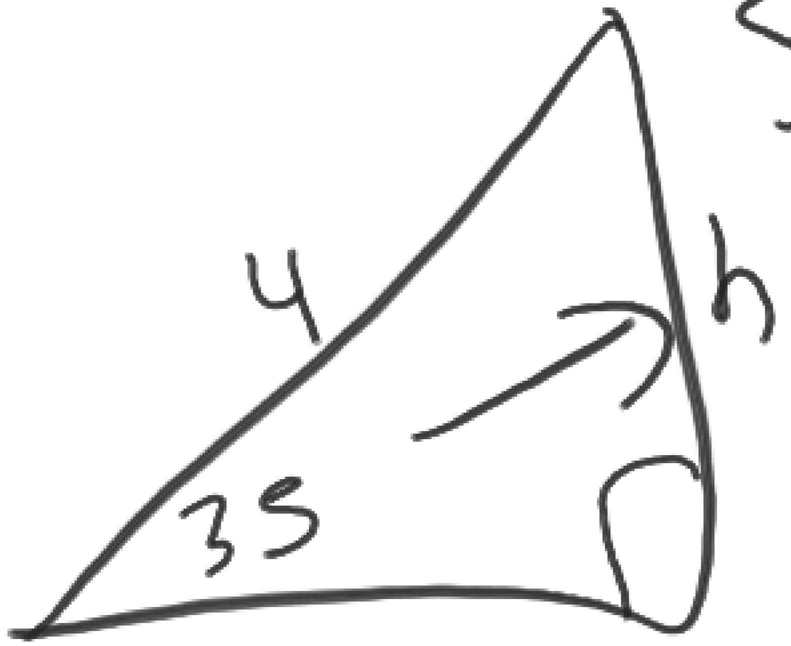
$b = 70.9$

$\frac{\sin B}{b} = \frac{\sin A}{a}$

$\frac{\sin 129}{70.9} = \frac{\sin A}{55}$ $\quad \sin^{-1}\left(\frac{55\sin 129}{70.9}\right)$



$$\sin 35 = \frac{h}{4}$$



$$h = 4\sin 35$$

$$A = \frac{1}{2}bh$$

$$b = 9$$

$$h = 4\sin 35$$

$$A = \frac{1}{2}(9)(4\sin 35)$$

$$\boxed{A = 10.3\ ft^2}$$

Find all solutions of the equation in the interval $[0, 2\pi)$.

$$\left(\cot x - 1 = \csc x\right)^2$$

$$\boxed{\csc^2 x = 1 + \cot^2 x}$$

$$\cot^2 x - 2\cot x + 1 = \csc^2 x$$

$$\cot^2 x - 2\cot x + 1 = 1 + \cot^2 x$$

$$-2\cot x = 0$$

$$\cot x = 0$$

$$x = \frac{\pi}{2}, \boxed{\frac{3\pi}{2}}$$

$$\cot\frac{\pi}{2} - 1 = \csc\frac{\pi}{2}$$

$$-1 = 1 \quad \times$$

$$\cot\frac{3\pi}{2} - 1 = \csc\frac{3\pi}{2}$$

$$-1 = -1$$

Find all solutions to the equation. $\sin 2x = 2\sin x \cos x$.

$$\sqrt{3}\sin x - \sin 2x = 0$$

$$\sqrt{3}\sin x - 2\sin x\cos x = 0$$

$$\sin x(\sqrt{3} - 2\cos x) = 0$$

$$\sin x = 0$$

$$\boxed{0, \pi}$$

$0 + 2k\pi$

$\pi + 2k\pi$

$$\sqrt{3} - 2\cos x = 0$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$\frac{\pi}{6}, \frac{11\pi}{6}$$

$$\boxed{k\pi, \quad \pi/6 + 2k\pi, \quad \frac{11\pi}{6} + 2k\pi}$$

Let θ be an angle in quadrant IV such that $\cot\theta = -\frac{4}{5}$.

Find the exact values of $\sin\theta$ and $\sec\theta$.

$x = 4 \quad y = -5 \quad r = \sqrt{41}$

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{16 + 25} = \sqrt{41}$

$\boxed{\sin\theta = \frac{y}{r}} = \frac{-5}{\sqrt{41}}$

$\sec\theta = \frac{r}{x} = \frac{\sqrt{41}}{4}$

Prove the identity.

$$\sec^2 x - 1 = \tan^2 x$$

$$\frac{\tan x}{\tan x}\left(\frac{\tan x}{\sec x+1}\right) = \frac{\sec x - 1}{\tan x}$$

$$\frac{\tan^2 x}{\tan x(\sec x+1)} \quad A$$

$$\frac{\sec^2 x - 1}{\tan x(\sec x+1)} \quad P$$

$$\frac{(\sec x-1)\cancel{(\sec x+1)}}{\tan x\cancel{(\sec x+1)}} \quad A$$

$$\boxed{\frac{\sec x - 1}{\tan x} \quad A}$$

$$\frac{\cos x}{1+\sin x} = \sec x - \tan x \rightarrow \frac{1}{\cos x} - \frac{\sin x}{\cos x} = \frac{1-\sin x}{\cos x}$$

$$\frac{\cos x(1-\sin x)}{(1+\sin x)(1-\sin x)} \quad A$$

$$\frac{\cos x(1-\sin x)}{1-\sin^2 x}$$

$$\frac{\cos x(1-\sin x)}{\cos^2 x} \quad P$$

$$\frac{1-\sin x}{\cos x} \quad A$$

$$\frac{1}{\cos x} - \frac{\sin x}{\cos x} \quad A$$

$$\sec x - \tan x \quad R, Q$$

$$\frac{1+\tan^2 x}{1-\tan^2 x} = \sec 2x$$

$$\frac{\sec^2 x}{1-\tan^2 x} \quad P$$

$$\frac{1}{\cos^2 x} \cdot \frac{1}{1-\frac{\sin^2 x}{\cos^2 x}} \quad R, Q$$

$$\frac{1}{\cos^2 x - \sin^2 x} \quad A$$

$$\frac{1}{\cos 2x} \quad DA$$

$$\sec 2x \quad R$$

Use the information given below to find $\boxed{\tan(\alpha+\beta)}$. $= 84/13$

$\rightarrow$ $\cos\alpha = \dfrac{15}{17}$, with α in quadrant I

$\tan\beta = \dfrac{4}{3}$, with β in quadrant III

$$\tan(\theta \pm \phi) = \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta \tan\phi}$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$x = 15 \quad y = 8 \quad r = 17$

$y = \sqrt{17^2 - 15^2} = \quad y = 8$

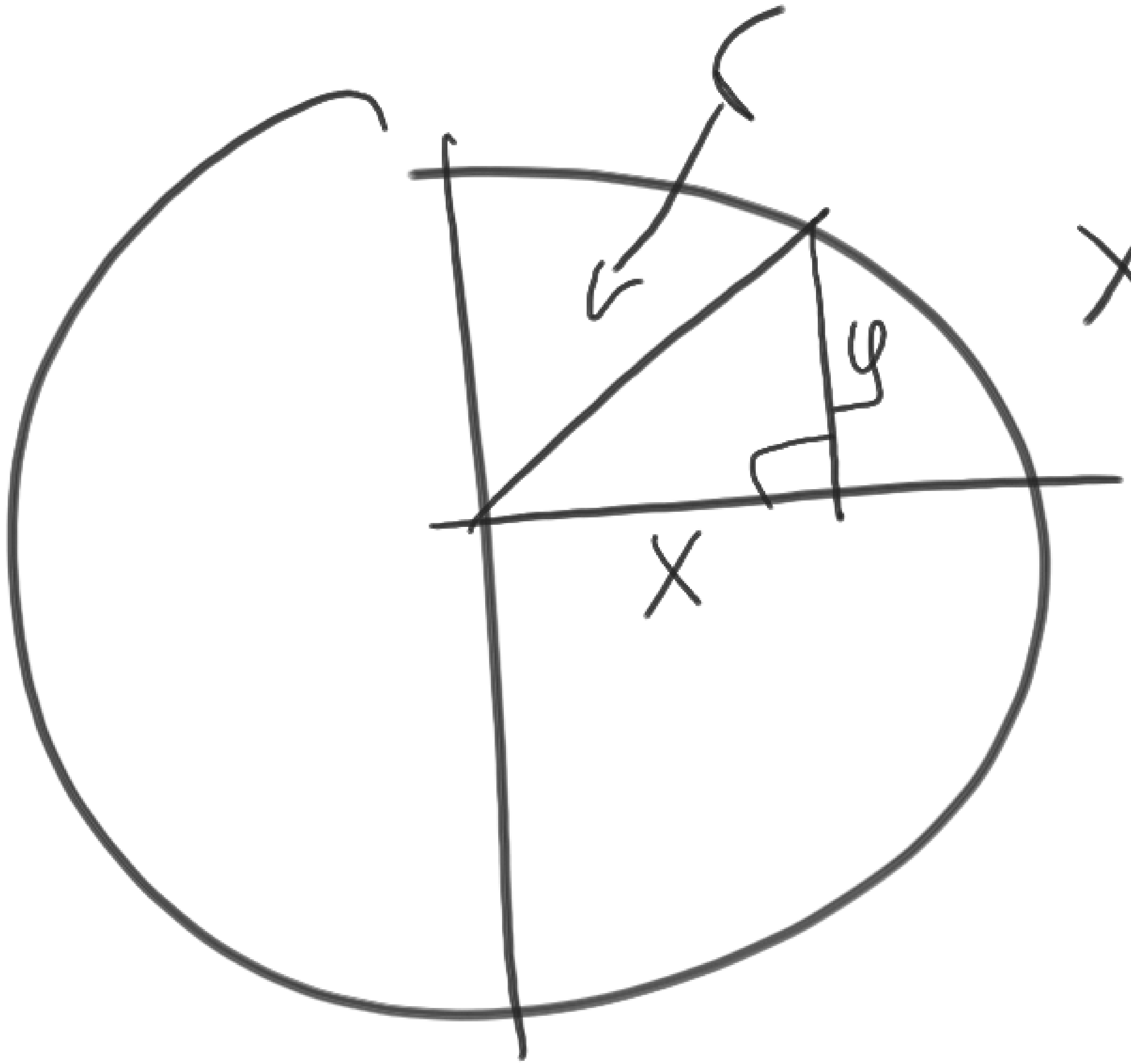
$$\boxed{\tan a = \frac{8}{15}}$$

$$= \frac{\left(\frac{8}{15}\right) + \left(\frac{4}{3}\right)}{1 - \left(\frac{8}{15}\right)\left(\frac{4}{3}\right)}$$

Use a product-to-sum formula to rewrite $\cos 5d \cos 3d$ as a sum or difference.

$$\cos\theta \cos\phi = \frac{[\cos(\theta - \phi) + \cos(\theta + \phi)]}{2}$$

$$\boxed{\frac{\cos(2d) + \cos(8d)}{2}}$$



$$x^2+y^2=r^2$$

$$y^2=r^2-x^2$$

$$\cot x\,(1-\cos 2x) = \sin 2x$$

$\cot x(2\sin^2 x)$ DA

$\dfrac{\cos x}{\cancel{\sin x}}(2\sin^{\cancel{2}} x)$ Q

$2\cos x\sin x$ A $2\sin x\cos x = \sin 2x$

$\sin 2x$ DA

Find all solutions to the equation.

$$\cos 2x = 2\cos^2 x - 1.$$

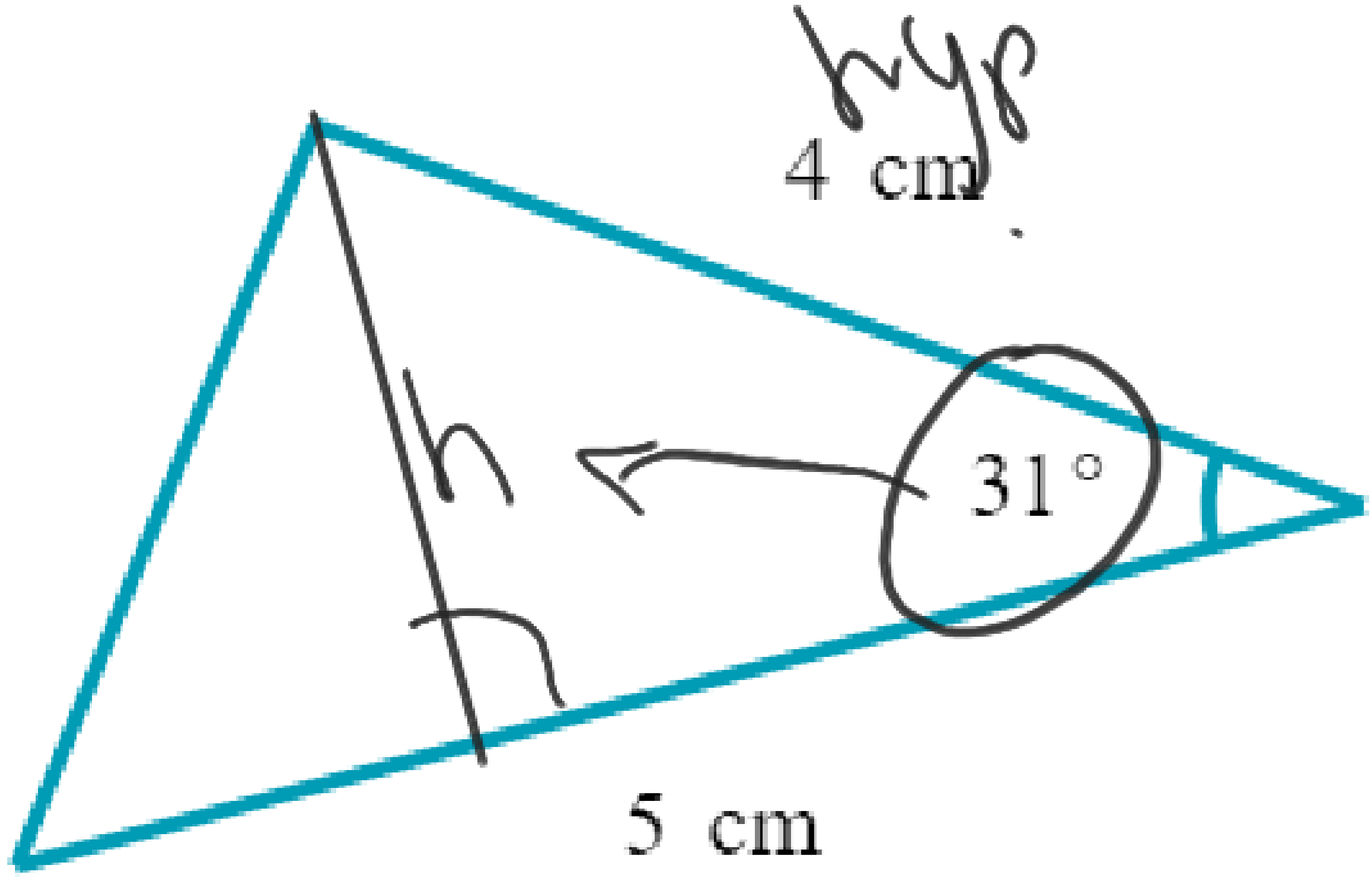
$$\cos 2x - \sqrt{2}\cos x = -1$$

$$(2\cos^2 x - 1) - \sqrt{2}\cos x = -1$$

$$2\cos^2 x - \sqrt{2}\cos x = 0 \qquad \cos x = 0 \qquad \cos x = \frac{\sqrt{2}}{2}$$

$$\cos x\,(2\cos x - \sqrt{2}) = 0 \qquad x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad \frac{\pi}{4}, \frac{7\pi}{4}$$

$$+2k\pi$$



$$\sin 31 = \frac{h}{4}$$

$$A = \frac{1}{2} b h$$

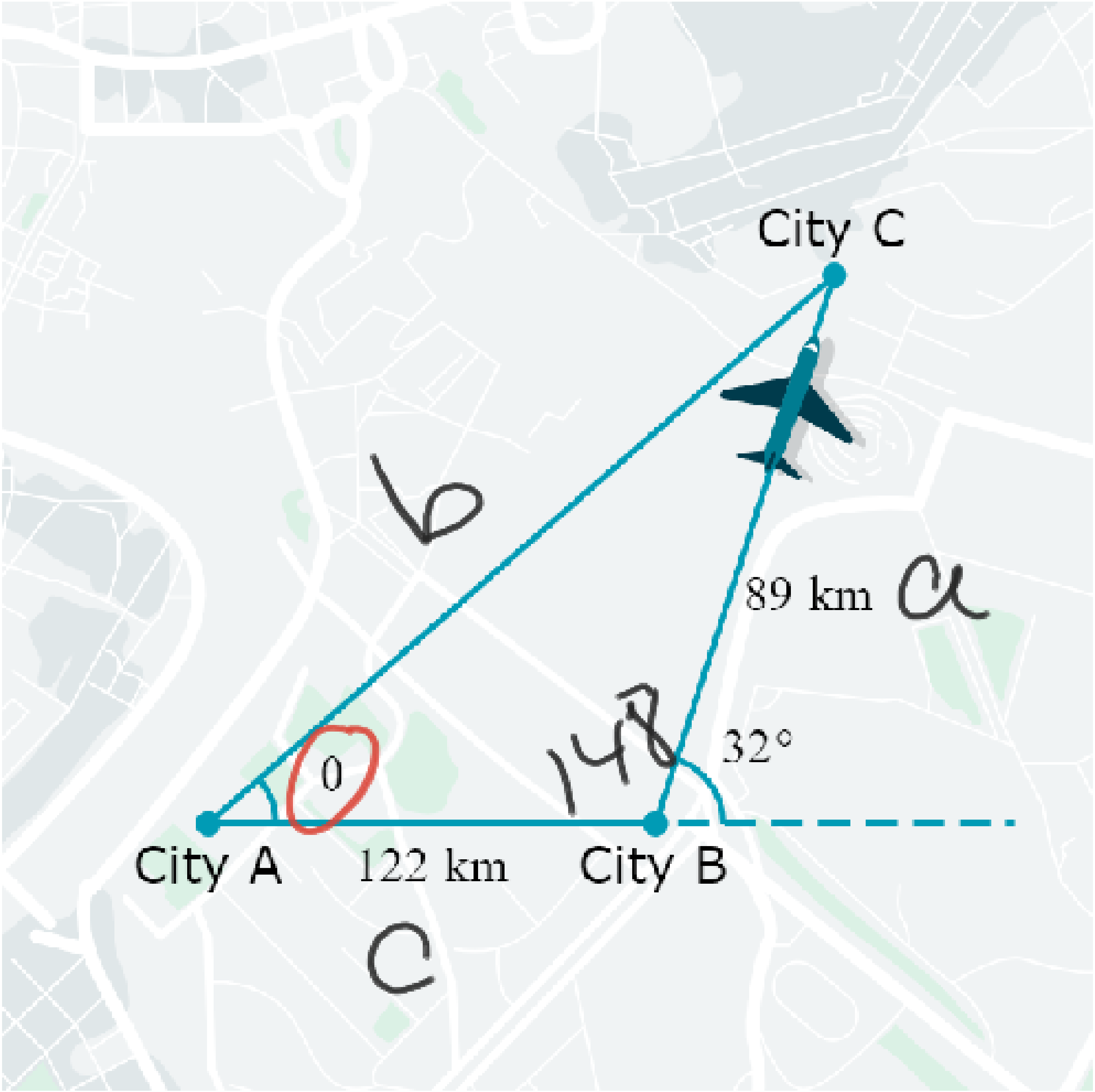
$$b = 5$$

$$h = \boxed{4 \sin 31}$$

$$A = \frac{1}{2}(5)(4 \sin 31)$$

$$A =$$

An airplane leaves City A and flies 122 km due east to City B. It then turns through an angle of 32° northward and flies 89 km to City C. What angle θ with respect to due east could the pilot have used to fly directly from City A to City C? See the figure below. (The figure is not drawn to scale.)



A = 13.4

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$b = \sqrt{89^2 + 122^2 - 2(89)(122)\cos 148}$$

$$b = 203.0$$

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin 148}{203} = \frac{\sin A}{89}$$

$$\sin A = \frac{89\sin 148}{203}$$

$$A = \sin^{-1}\left(\frac{89\sin 148}{203}\right)$$

$\sin = \frac{y}{r}$ $\cos = \frac{x}{r}$ $\tan = \frac{y}{x}$

$\csc = \frac{r}{y}$ $\sec = \frac{r}{x}$ $\cot = \frac{x}{y}$

Let θ be an angle in quadrant I such that $\cot\theta = \frac{3}{4}$.

Find the exact values of $\sin\theta$ and $\sec\theta$. $x = 3$ $y = 4$ $r = 5$

$\sin\theta = \frac{4}{5}$ $\sec\theta = \frac{5}{3}$