Properties of H_0 and H_a

The **null hypothesis**, denoted H_0, is a statement about the value of a population parameter. This statement is assumed to be true unless we find sample evidence that indicates it is not. If the sample evidence is strong enough to indicate it is not true, then we reject the null hypothesis. Otherwise, we fail to reject the null hypothesis.

The **alternative hypothesis**, denoted H_a or H_1, is also a statement about the value of a population parameter. It is the statement or claim that we are trying to find evidence to support.

Table 11.2.1 - Hypotheses Concerning a Test about a Single Mean

	Is the population mean different from μ_0?	Is the population mean greater than μ_0?	Is the population mean less than μ_0?
Null Hypothesis, H_0	$\mu = \mu_0$	$\mu = \mu_0$	$\mu = \mu_0$
Alternative Hypothesis, H_a	$\mu \neq \mu_0$	$\mu > \mu_0$	$\mu < \mu_0$
Type of Hypothesis Test	Two-tailed	Right-tailed	Left-tailed

Suppose the average national reading level for high school sophomores is 150 words per minute with a standard deviation of 15. A local school board member wants to know if sophomore students at Lincoln High School read at a level different from the national average for tenth graders. The level of the test is to be set at 0.05. A random sample size of 100 tenth graders from Lincoln High School has been drawn, and the resulting average is 154 words per minute.

$H_0 \quad \mu = 150$ Null

$H_a \quad \mu \neq 150$ Alternate

Steps in the Hypothesis Test

Step 1: Determine the null and alternative hypotheses. ✓

Step 2: Specify the significance level α. $\alpha = 0.05$

Step 3: Validate the assumptions of the hypothesis test, identify the appropriate test statistic, and compute its value.

Before proceeding with the test, we must check our assumptions.

- ☑ Quantitative data
- ☑ Random sample used to obtain data
- ☑ $n > 30$ or population is normally distributed
- ☑ σ known or ☐ σ unknown

Step 4: Determine the critical value(s) or *P*-value.

Critical Value Method

Find the critical value(s).

(It may help to draw a graph displaying the critical value(s), the rejection region, and the test statistic.)

***P*-Value Method**

Find the *P*-value based on the value of the test statistic and the alternative hypothesis.

(It may help to draw a graph displaying the test statistic and *P*-value.)

Test statistic:

The test statistic is given by

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

where,

n = the sample size,

$\bar{x}$ = the sample mean,

μ_0 = the population mean (from the null hypothesis), and

σ = the known population standard deviation.

PROCEDURE

Suppose the average national reading level for high school sophomores is 150 words per minute with a standard deviation of 15. A local school board member wants to know if sophomore students at Lincoln High School read at a level different from the national average for tenth graders. The level of the test is to be set at 0.05. A random sample size of 100 tenth graders from Lincoln High School has been drawn, and the resulting average is 154 words per minute.

$$z = \frac{154-150}{15/\sqrt{100}} = 2.67$$

P-score $\rightarrow$ 0.9962

$1-0.9962 = .0038$

Step 5: Make the decision to reject or fail to reject H_0.

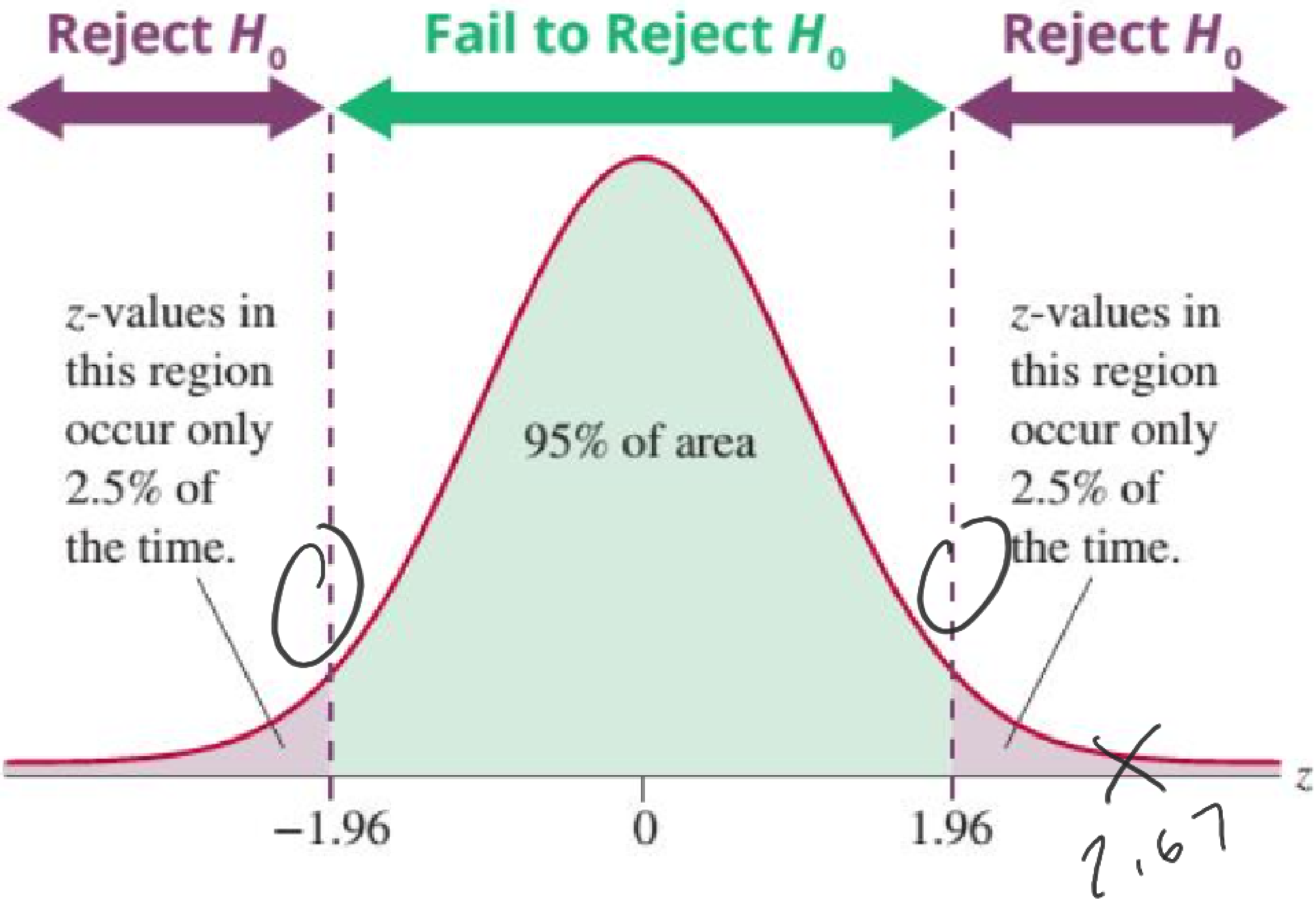
- **Reject** H_0 if the test statistic is in the rejection region. ✓
- **Fail to reject** H_0 if the test statistic is not in the rejection region.

- **Reject** H_0 if P-value is $< \alpha$. ✓
- **Fail to reject** H_0 if P-value is $\geq \alpha$.

$.0038 < 0.05$

Step 6: State the conclusion in terms of the original question.

Standard Normal Random Variable



P-Value

A ***P*-value** is the probability of observing a value of the test statistic as extreme or more extreme than the observed one, assuming the null hypothesis is true and all the assumptions related to the hypothesis test are satisfied.

DEFINITION

A microprocessor designer has developed a new fabrication process which he believes will increase the usable life of a chip. Currently the usable life is 16,000 hours with a standard deviation of 2500 hours. Test the hypothesis that the process increases the usable life of a chip, at the 0.01 level. A random sample of 1000 microprocessors will be tested. Assume the standard deviation of the life of the new chips will be equal to the standard deviation of the current chips. $\bar{x} = 16200$

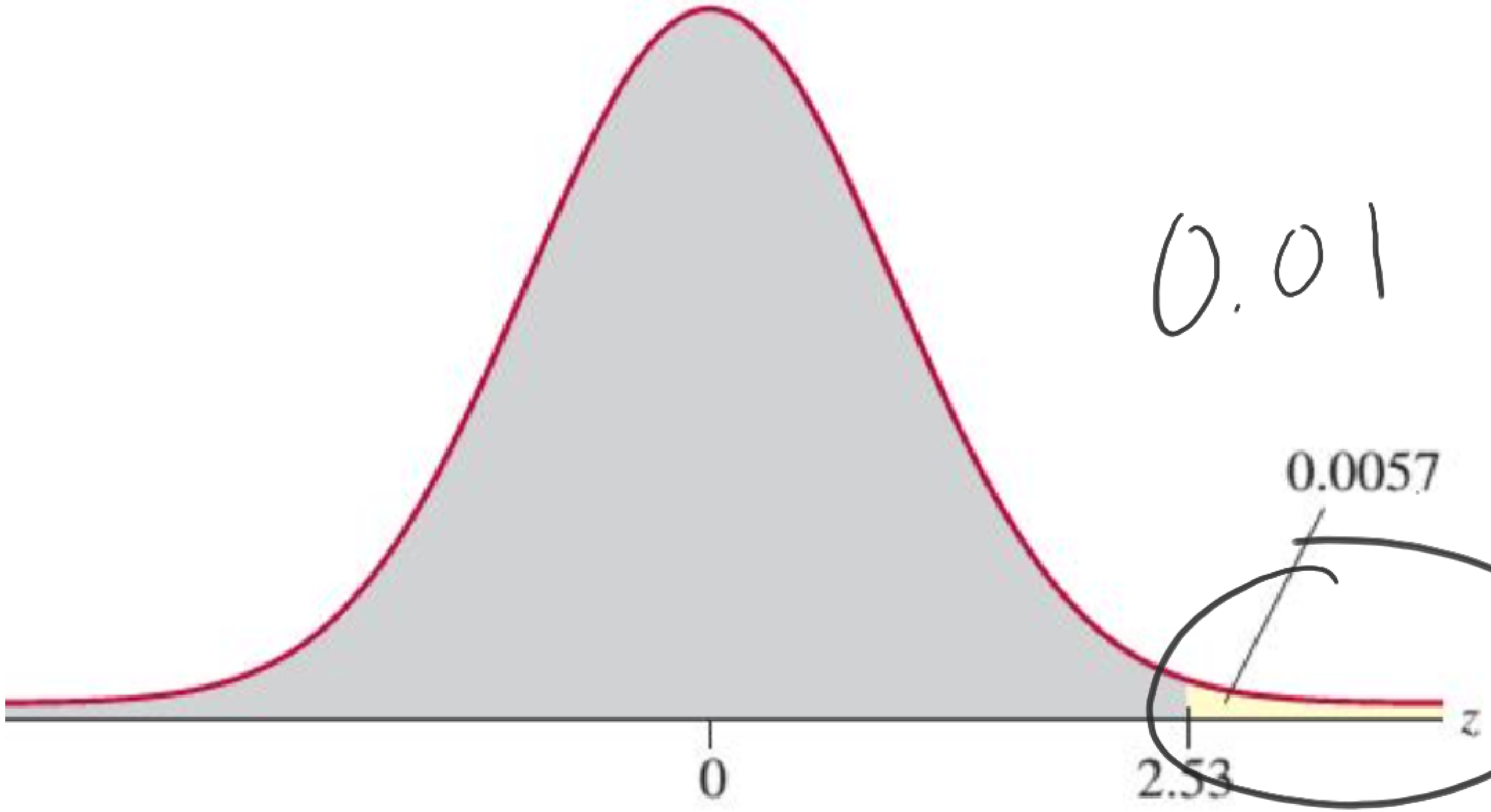
$H_0 \quad \mu = 16000$

$\alpha = 0.01$

$H_a \quad \mu > 16000$

$$z = \frac{16200 - 16000}{2500/\sqrt{1000}}$$

$z = 2.53$ p-value = .0057



0.01

Reject H_0

Testing a Hypothesis About a Mean, σ Unknown

Assumptions:

1. The data is quantitative.
2. The data is obtained via a random sample of size n.
3. The population is normally distributed or the sample size n is large, $n > 30$.
4. The population standard deviation σ is unknown.

Test Statistic:

The test statistic is given by $$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$
where,

n = the sample size,

$\bar{x}$ = the sample mean,

μ_0 = the population mean (from the null hypothesis), and

s = the sample standard deviation.

The test statistic has a t-distribution with $n-1$ degrees of freedom.

PROCEDURE

A personnel researcher has designed a questionnaire she believes will take an average time of 35 minutes to complete. Suppose she randomly samples 20 employees and finds ~~that the~~ mean time to take the test is 29 minutes with a standard deviation of $s = 8$ minutes. Determine if there is sufficient evidence to conclude that the completion time of the newly designed test differs from its intended duration. Conduct the test at the 0.05 level. Assume the sample comes from a normally distributed population.

$H_0 \quad \mu = 35$

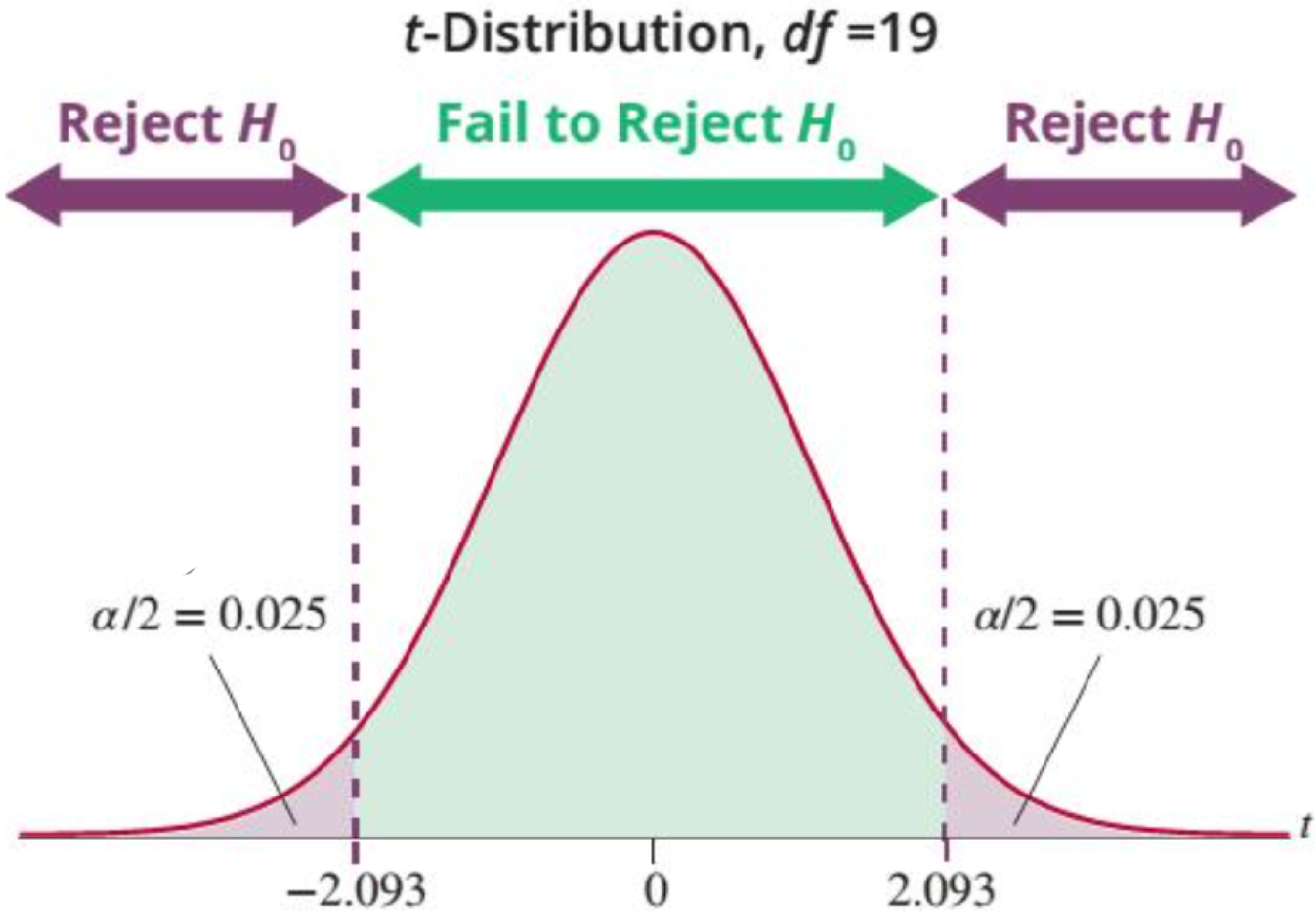
$H_a \quad \mu \boxed{\neq} 35$

two tail

$\boxed{\alpha = 0.05}$

$t = \dfrac{29 - 35}{8/\sqrt{20}} = \boxed{-3.35}$

CV = 2.093


t-Distribution, df =19
Reject H_0
Fail to Reject H_0
Reject H_0
$\alpha/2 = 0.025$
$\alpha/2 = 0.025$
t
−2.093
0
2.093

Hurricane Andrew swept through southern Florida causing billions of dollars of damage. Because of the severity of the storm and the type of residential construction used in this semitropical area, there was some concern that the average claim size would be greater than the historical average hurricane claim. Historically, the average claim size was \$24,000 with standard deviation \$2400. Several insurance companies collaborated in a data gathering experiment. They randomly selected 84 homes and sent adjusters to settle the claims. In the sample of 84 homes, the average claim was \$27,500.

$H_0 \ \mu = 24000$

$H_a \ \mu > 24000$

$CV = 2.326$

$\alpha = 0.01$

$$z = \frac{27500 - 24000}{2400/\sqrt{84}}$$

$z = 13.37$