Discrete Random Variable

A **discrete random variable** is a random variable which has a countable number of possible outcomes.

When describing a discrete random variable, you should do the following.

1. State the variable.
2. List all of the possible values of the variable.
3. Determine the probabilities of these values.

DEFINITION

Continuous Random Variable

A **continuous random variable** is a random variable that can assume any value on a continuous segment(s) of the real number line.

DEFINITION

Example 7.1.1

Random Phenomenon: Toss a die and observe the outcome of the toss.

X = outcome of the roll

Values → 1, 2, 3, 4, 5, 6

X	1	2	3	4	5	6
P(x)	1/6	1/6	1/6	1/6	1/6	1/6

Example 7.1.3

Random Phenomenon: The head nurse of the pediatric division of the Sisters of Mercy Hospital is trying to determine the capacity requirement for the nursery. She realizes that the number of babies born at the hospital each day is a random variable. She will have to develop a description of the randomness in order to develop her plan.

X = number of Babies

Values → any number from 0 to ⟶

Example 7.1.4

Random Phenomenon: A local restaurant has an express policy that states that lunch will be served within 15 minutes of ordering, or it will be free. Obviously, the restaurant is keenly interested in not giving away its product and in delivering on its promise of a timely lunch. But the length of time to prepare each meal varies because of the difference in preparation times of each dish, the load on the kitchen, and the experience of the chefs and waitresses. Since time is measured on a continuous scale and the variability of meal preparation is not predictable, the time between ordering and receiving a meal is considered to be a continuous random variable.

X = time from ordering

values $\to 0 \to \infty$

Classify the following as either a discrete random variable or a continuous random variable.

a. The number of emergency phone calls received per day by a local fire department.

b. The speed of pitches of major league baseball pitchers.

c. The weight of a lobster caught in Maine.

d. The number of defective circuits on a computer chip.

e. The time it takes for a 5-year battery to die.

Discrete Probability Distribution

A **discrete probability distribution** consists of all possible values of the discrete random variable along with their associated probabilities.

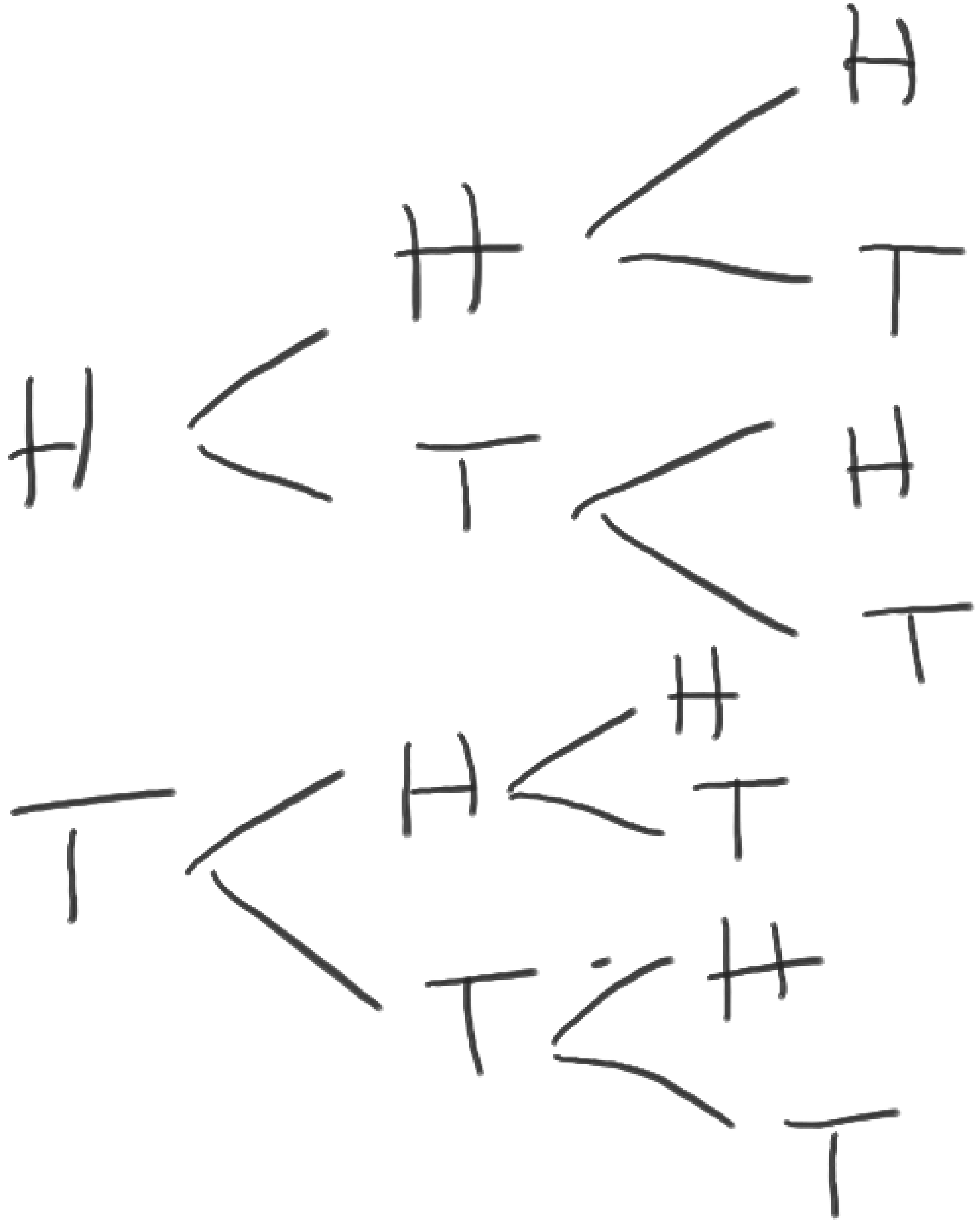
Discrete probability distributions always have two characteristics.

1. The sum of all of the probabilities must equal 1.
2. The probability of any value must be between 0 and 1, inclusively.

DEFINITION

Consider the random phenomenon of tossing a coin three times and counting the number of heads. What is the probability distribution for the number of heads observed in three tosses of a coin?

X = number of Heads	$\boxed{P(X=x)}$
0	$1/8$ $P(X=0)$
1	$3/8$ $P(X=1)$
2	$3/8$ $P(X=2)$
3	$1/8$ $P(X=3)$



H H H	3
H H T	2
H T H	2
H T T	1
T H H	2
T H T	1
T T H	1
T T T	0

K. J. Johnson is a computer salesperson. During the last year he has kept records of his computer sales for the last 200 days.

Frequency Distribution					
Sales	0	1	2	3	4
Frequency	40	20	60	40	40

$X = \text{Sales}$

He recognizes that his daily sales constitute a random process and he wishes to determine the probability distribution for daily sales. From the probability distribution he would like to determine the following.

a. The probability that he will sell at least 2 computers each day.

b. The probability he will sell at most 2 computers each day.

X	$P(X=x)$
0	0.2
1	0.1
2	0.3
3	0.2
4	0.2

a) 0.7

b) 0.6

The following function is a discrete probability distribution function.

$$P(X=x)=\begin{cases} \dfrac{x^2}{30}, & \text{if } x = 1,2,3,4 \\ 0 & \text{elsewhere} \end{cases}$$

Summarize the probability distribution for this function.

x	$P(X=x)$
1	1/30
2	4/30
3	9/30
4	16/30

Expected Value

The **expected value** of the random variable X is the mean of the random variable X. It is denoted $E(X)$ and is given by computing the following expression

$$\mu = E(X) = \sum\left[x \cdot p(x)\right]$$

where $p(x) = P(X = x)$.

DEFINITION

$x(P(x))$

Investment Alternatives			
Option A		**Option B**	
Profit (Dollars)	Probability	Profit (Dollars)	Probability
–2000	0.2	–3000	0.2
0	0.1	–1000	0.1
1000	0.3	2000	0.2
2000	0.3	3000	0.3
4000	0.1	4000	0.2

−400
0
300
600
400

$900

Option B	
Profit (Dollars)	Probability
–3000	0.2
–1000	0.1
2000	0.2
3000	0.3
4000	0.2

$X(P(x))$

$$\begin{array}{r} -600 \\ -100 \\ 400 \\ 900 \\ 800 \\ \hline \$1400 \end{array}$$

$-0{,}0526$

Variance and Standard Deviation of a Discrete Random Variable

The **variance of a discrete random variable** X is given by the following formula.

$$\sigma^2 = V(X) = \sum\left[(x-\mu)^2 p(x)\right]$$

The **standard deviation of a discrete random variable** X is therefore,

$$\sigma = \sqrt{V(x)} = \sqrt{\sum (x-\mu)^2 p(x)}.$$

DEFINITION

Investment Alternatives			
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–2000	0.2	–3000	0.2
0	0.1	–1000	0.1
1000	0.3	2000	0.2
2000	0.3	3000	0.3
4000	0.1	4000	0.2

$\mu = 900$ $\mu = 1400$

$\sigma = 1758$ $\sigma = 2577$