Multiplication of probability

Independent

Two events, A and B, are **independent** if and only if

$$P(A|B) = P(A) \text{ and } P(B|A) = P(B).$$

DEFINITION

Probability Law 10: Multiplication Rule for Independent Events

If two events, A and B, are independent, then

$$P(A \cap B) = P(A) \cdot P(B).$$

If n events, $A_1, A_2, \ldots, A_n$, are independent, then

$$P(A_1 \cap A_2 \cap \ldots \cap A_n) = P(A_1) \cdot P(A_2) \cdot \ldots \cdot P(A_n).$$

DEFINITION

A coin is flipped, a die is rolled, and a card is drawn from a standard deck of 52 cards. Find the probability of getting a tail on the coin, a five on the die, and a jack of clubs from the deck of cards.

$$P(\text{Tails}) = \frac{1}{2}$$

$$P(5) = \frac{1}{6}$$

$$P(\text{J clubs}) = \frac{1}{52}$$

$$P(T \cap 5, \text{Jclubs}) = \left(\frac{1}{2}\right)\left(\frac{1}{6}\right)\left(\frac{1}{52}\right) = \frac{1}{624}$$

$$= .0016$$

Choose two cards from a standard deck, replacing the first card and shuffling before choosing the second one. What is the probability of choosing a king and then a queen?

$$P(K) = \frac{4}{52}$$

$$P(Q) = \frac{4}{52}$$

$$\left(\frac{4}{52}\right)\left(\frac{4}{52}\right) = \frac{1}{169}$$

In a production process, a product is assembled by using four different independent parts (A, B, C, and D). In order for the product to operate properly, each part must be free of defects. The probabilities of the parts being nondefective are given by $P(A) = 0.9$, $P(B) = 0.7$, $P(C) = 0.8$, and $P(D) = 0.9$.

a. What is the probability that all four parts are defective? .0006

b. What is the probability that the product does not work?

$P(A) = 0.9$ $\quad P(A^c) = 0.1$

$P(B) = 0.7$ $\quad P(B^c) = 0.3$ $\quad .0006$

$P(C) = 0.8$ $\quad P(C^c) = 0.2$

$P(D) = 0.9$ $\quad P(D^c) = 0.1$

In a production process, a product is assembled by using four different independent parts (A, B, C, and D). In order for the product to operate properly, each part must be free of defects. The probabilities of the parts being nondefective are given by $P(A) = 0.9$, $P(B) = 0.7$, $P(C) = 0.8$, and $P(D) = 0.9$.

a. What is the probability that all four parts are defective?

b. What is the probability that the product does not work?

$P(A) = 0.9$

$P(B) = 0.7$

$P(C) = 0.8$

$P(D) = 0.9$

$(0.9)(0.7)(0.8)(0.9)$

$1 - \boxed{0.4536}$ works

$\boxed{0.5464}$ not works

Probability Law 9: Conditional Probability

The **conditional probability** of event A occurring, given that event B has already occurred is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

The notation $P(A|B)$ is read as *the probability of A given the occurrence of B*. The vertical bar within a probability statement will always mean *given*.

DEFINITION

Suppose a marketing research firm has surveyed a panel of consumers to test a new product and produced the following **cross tabulation** indicating the number of panelists that liked the product, the number that did not like the product, and the number that were undecided.

$\frac{213}{1403}$

Market Research Survey				
Age	**Like**	**Not Like**	**Undecided**	**Total**
18–34	213	197	103	513
35–50	193	184	67	444
Over 50	144	219	83	446
Total	**550**	**600**	**253**	**1403**

If an individual is between 35 and 50 years old, what is the probability that he or she will like the product?

$\frac{193}{444}$

Probability Law 11: Multiplication Rule for Dependent Events

If two events, A and B, are **dependent**, then

$$P(A \cap B) = P(A) \cdot P(B \mid A) = P(B) \cdot P(A \mid B)$$

DEFINITION

What is the probability of drawing a king and then a queen from a standard deck if the cards are drawn *without replacement*?

$$P(K) = \frac{4}{52} \qquad P(Q \mid K) = \frac{4}{51}$$

$$P(K) \cdot P(Q \mid K) = \left(\frac{4}{52}\right)\left(\frac{4}{51}\right) = \frac{4}{663}$$

Assume that there are 17 men and 24 women in the Lions Club. Two members are chosen at random each year to serve on the hospitality committee. What is the probability of choosing two members at random and the first being a man and the second being a woman?

$$\left(\frac{17}{41}\right)\left(\frac{24}{40}\right) = \frac{51}{205} = 24.8\%$$

The Fundamental Counting Principle

E_1 is an event with n_1 possible outcomes and E_2 is an event with n_2 possible outcomes. The number of ways the events can occur in sequence is $n_1 \cdot n_2$. This principle can be applied for any number of events occurring in sequence.

PROCEDURE

A local office supply store offers ballpoint pens from three different manufacturers. Each manufacturer's pens come in either red, blue, black, or green and either fine or medium tip is available for each color. How many different pens does the store carry?

$$(3)(4)(2) = 24$$

$$(M)(C)(T)$$

Licence plates in CT have 2 letters followed by 5 numbers.

$(26)(26)(10)(10)(10)(10)(10)$

$67,600,000$

Permutation

A **permutation** is a specific order or arrangement of objects in a set. There are $n!$ permutations of n unique objects.

DEFINITION

Permutation

The number of permutations of n unique objects in which k are selected at a time and repetition is not allowed is given by

$$_nP_k = \frac{n!}{(n-k)!}$$

Note that some alternate notations for permutations that you may see are P_k^n and $P(n,k)$.

FORMULA

order maters → nPr

order doesnt mater → nCr

At a local fast food restaurant, the door to the kitchen is secured by a five button lock, labeled 1, 2, 3, 4, 5. To open the door, the correct three digit code must be pushed, but each button can only be pushed once. How many different codes are possible?

$${}_nP_r(5,3) \leftrightarrow {}_5P_3 = 60$$

Seven sprinters have advanced to the final heat at a track meet. How many ways can they finish in first, second, and third place?

7 6 5

$${}_nP_r(7,3) = 210$$

Combinations

A **combination** is a collection or grouping of objects where the order is ***not*** important.

DEFINITION

Combination

The number of combinations of n unique objects selecting k at a time and repetition is not allowed is given by

$$_nC_k = \frac{n!}{(n-k)!k!}$$

Note that some alternate notations for combinations that you may see are C_k^n and $C(n,k)$.

FORMULA

Seven sprinters have advanced to the final heat at a track meet. How many ways can they finish in ~~first, second, and third place~~? 3 people

$$nCr(7, 3) = 35$$

In South Carolina's *Palmetto Cash 5* lottery, a player selects five different numbers from 1 to 38 (inclusive). If the numbers selected match the player's numbers in any order, the player wins.

a. What is the total number of winning combinations?

b. What is the probability of winning?

a) $_{38}C_5 = 501{,}942$

b) $\dfrac{1}{501942}$

Select five numbers between 1 and 69 for the white balls, then select one number between 1 and 26 for the red Powerball.

$${}_{69}C_5 = (11{,}238{,}513)(26)$$

$$= 292{,}201{,}338$$

$$1/292{,}201{,}338$$