Standardizing a Normal Random Variable

The following formula can transform any normal random variable into a standard normal random variable, z.

$$z = \frac{x - \mu}{\sigma}$$

where x is a normal random variable with mean μ and standard deviation σ.

FORMULA

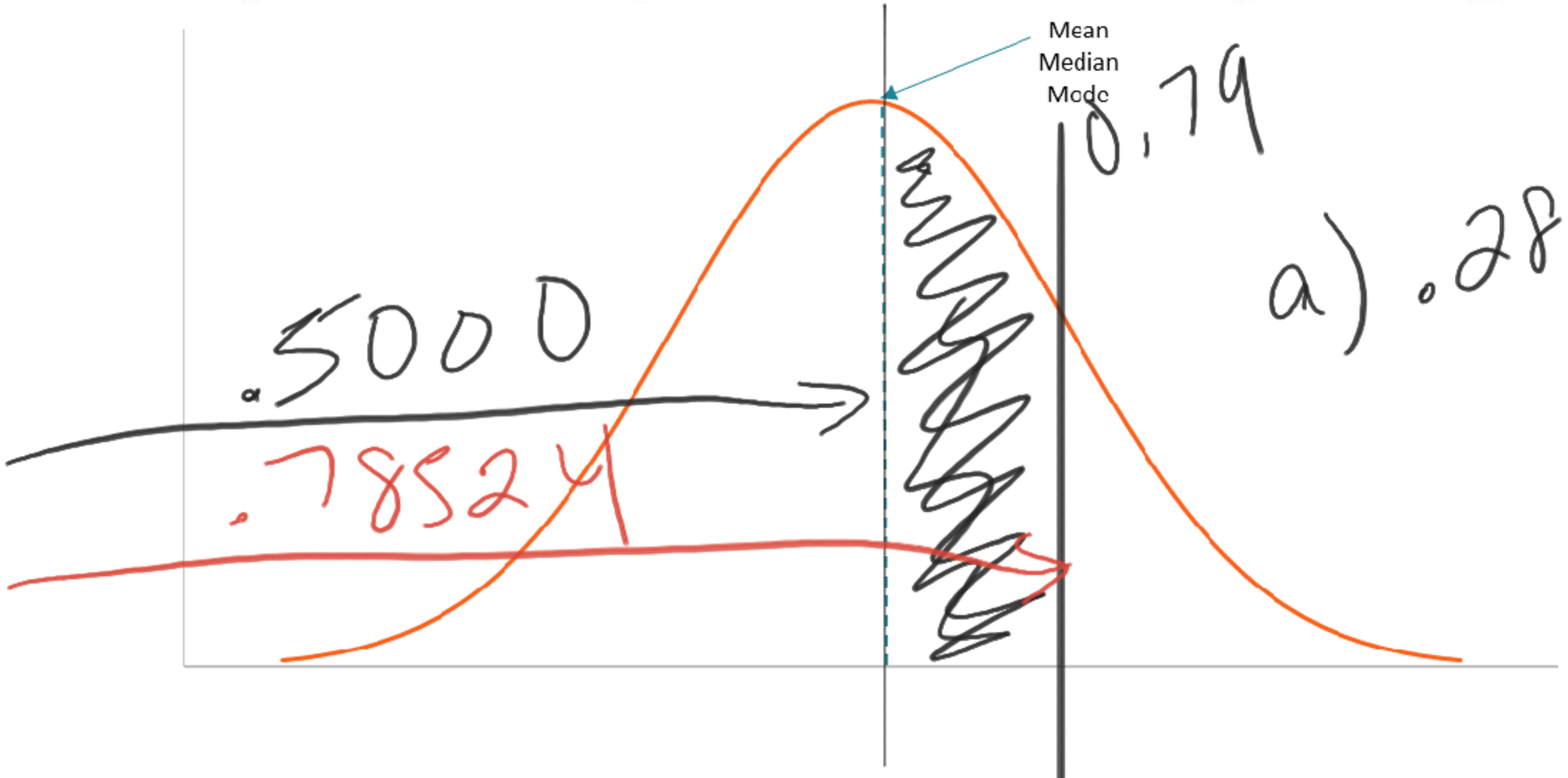
10. Using the standard normal tables in Appendix A, determine the following probabilities. Sketch the associated areas.

a. $P(0 \le z \le 0.79)$

b. $P(-1.57 \le z \le 2.33)$

c. $P(z \ge 1.89)$

d. $P(z \le -2.77)$



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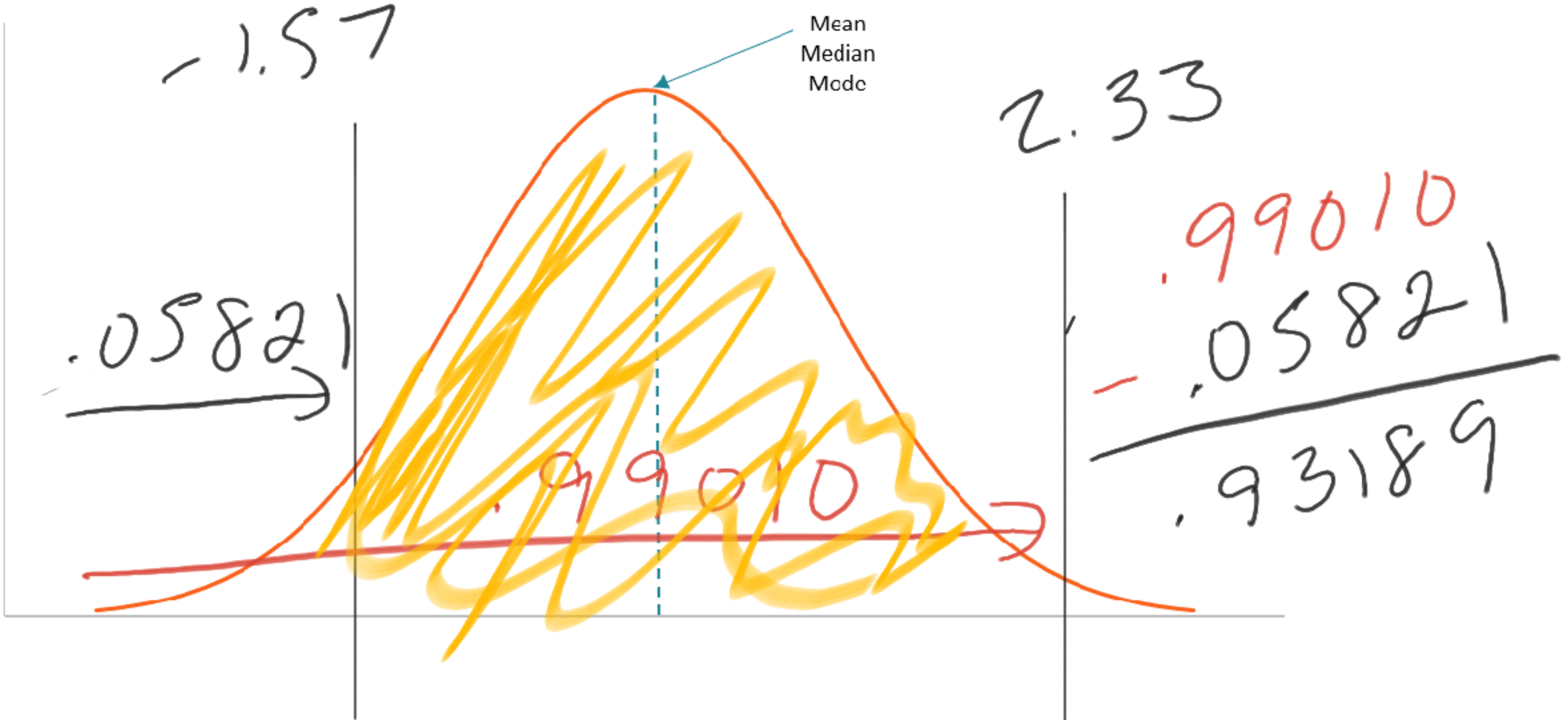

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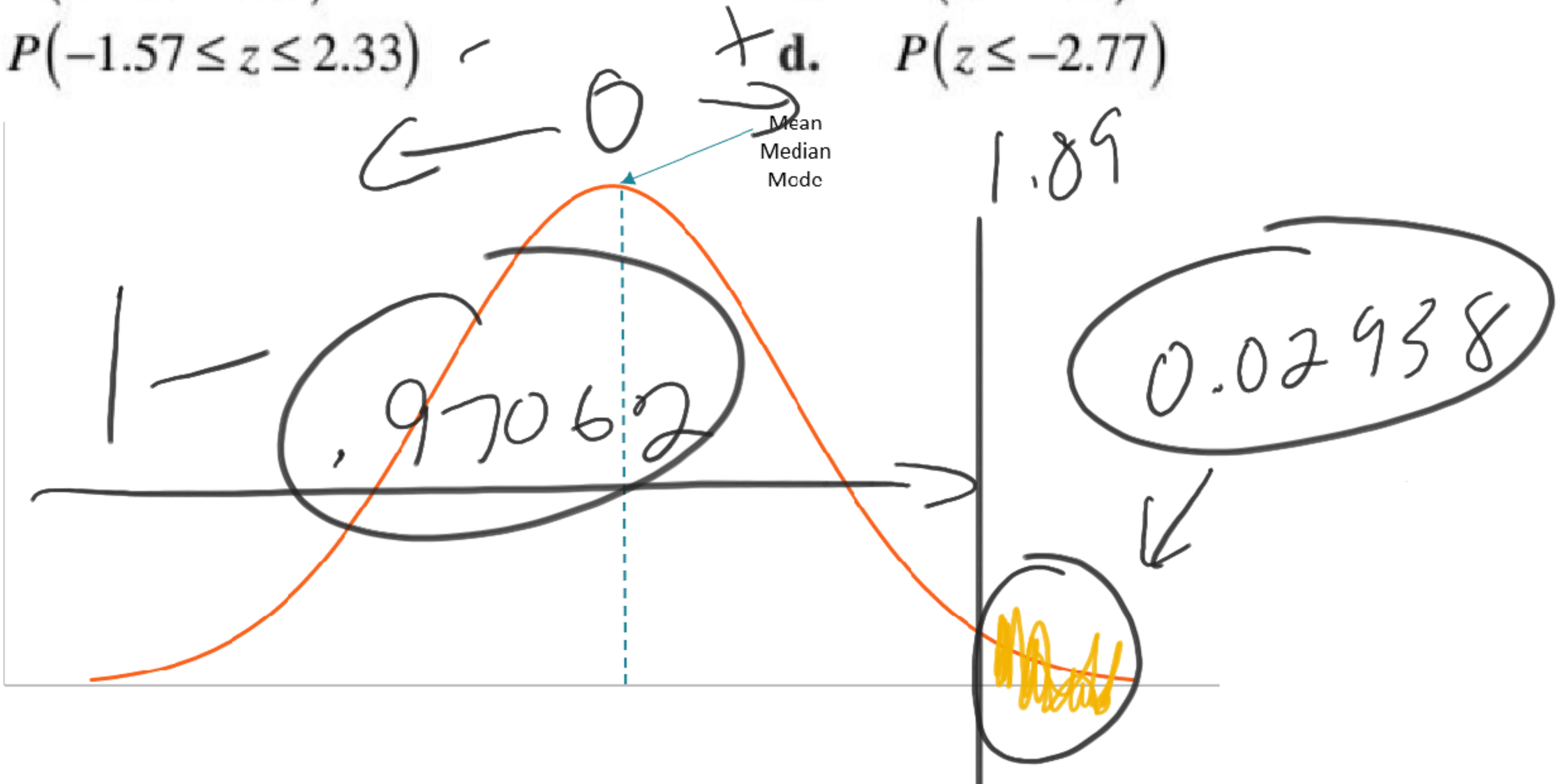

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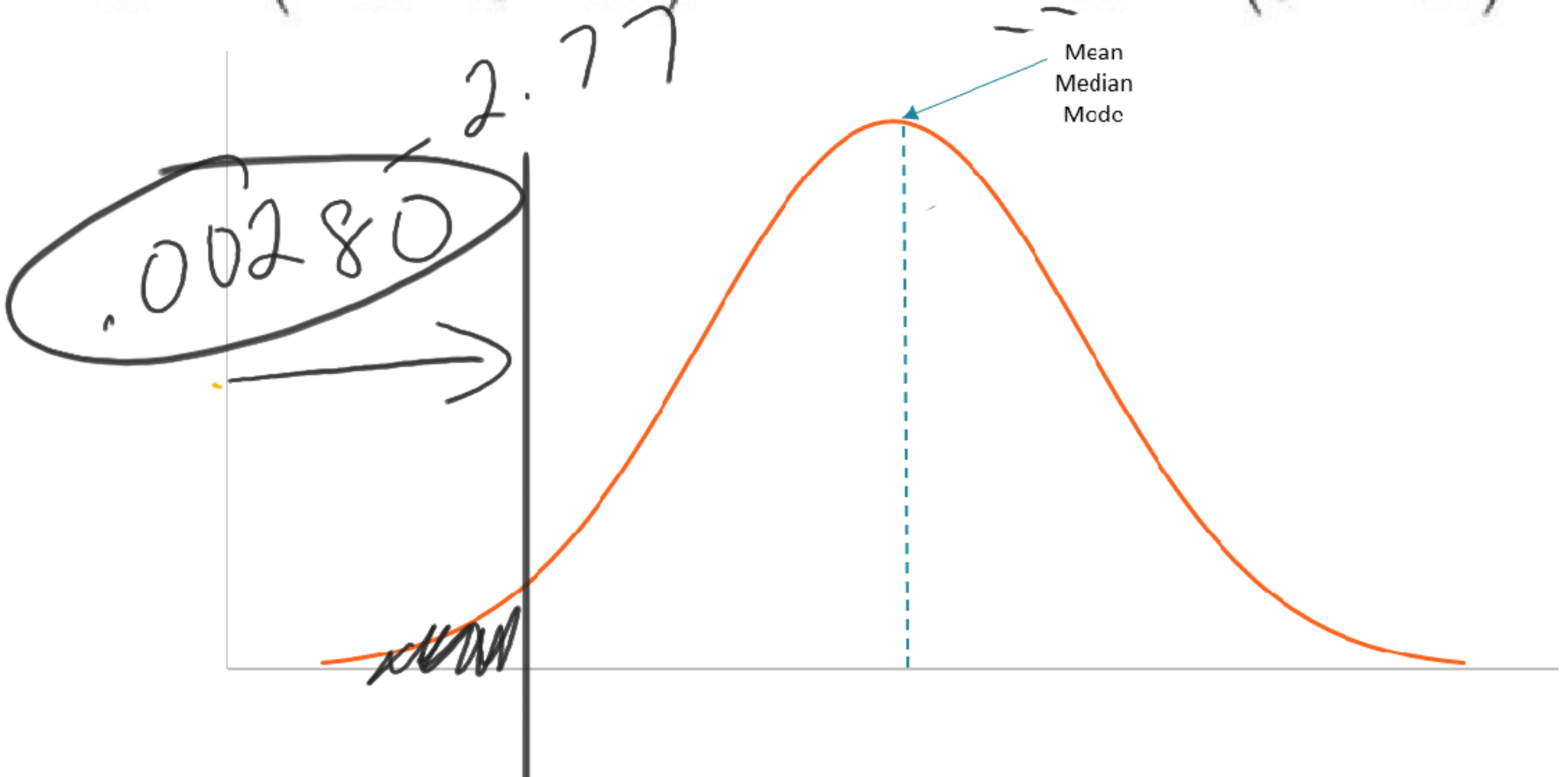
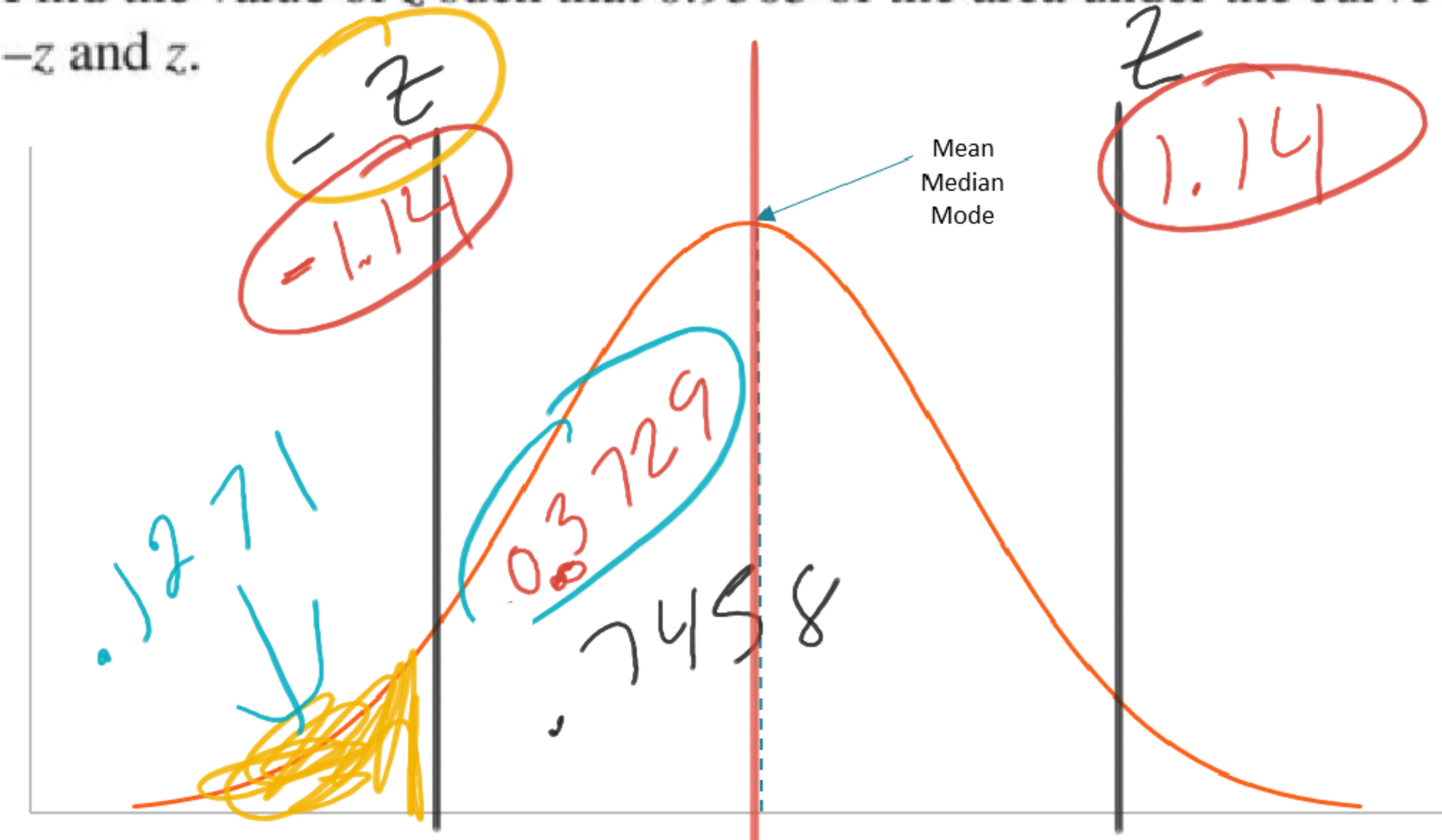


18. Find the value of z such that 0.7458 of the area under the curve lies between $-z$ and z.

19. Find the value of z such that 0.9505 of the area under the curve lies between $-z$ and z.



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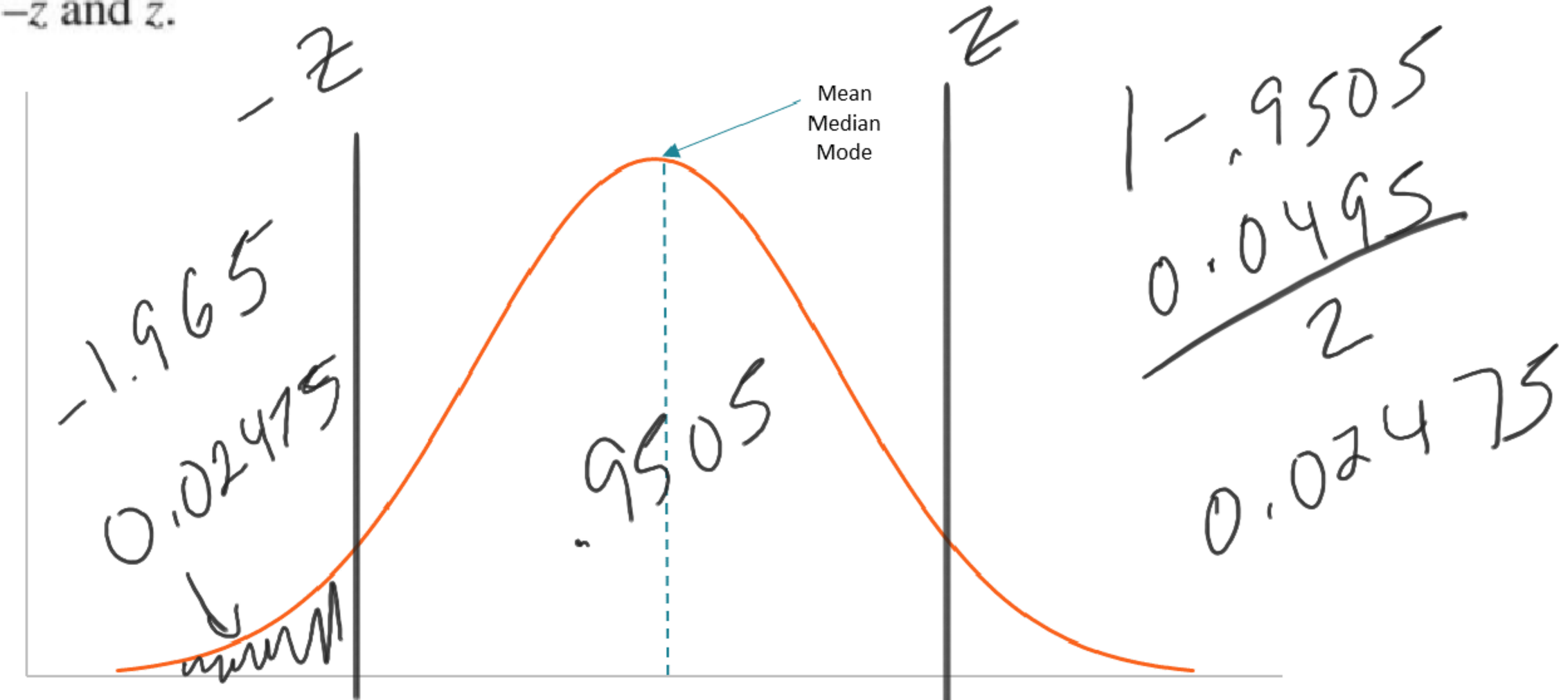
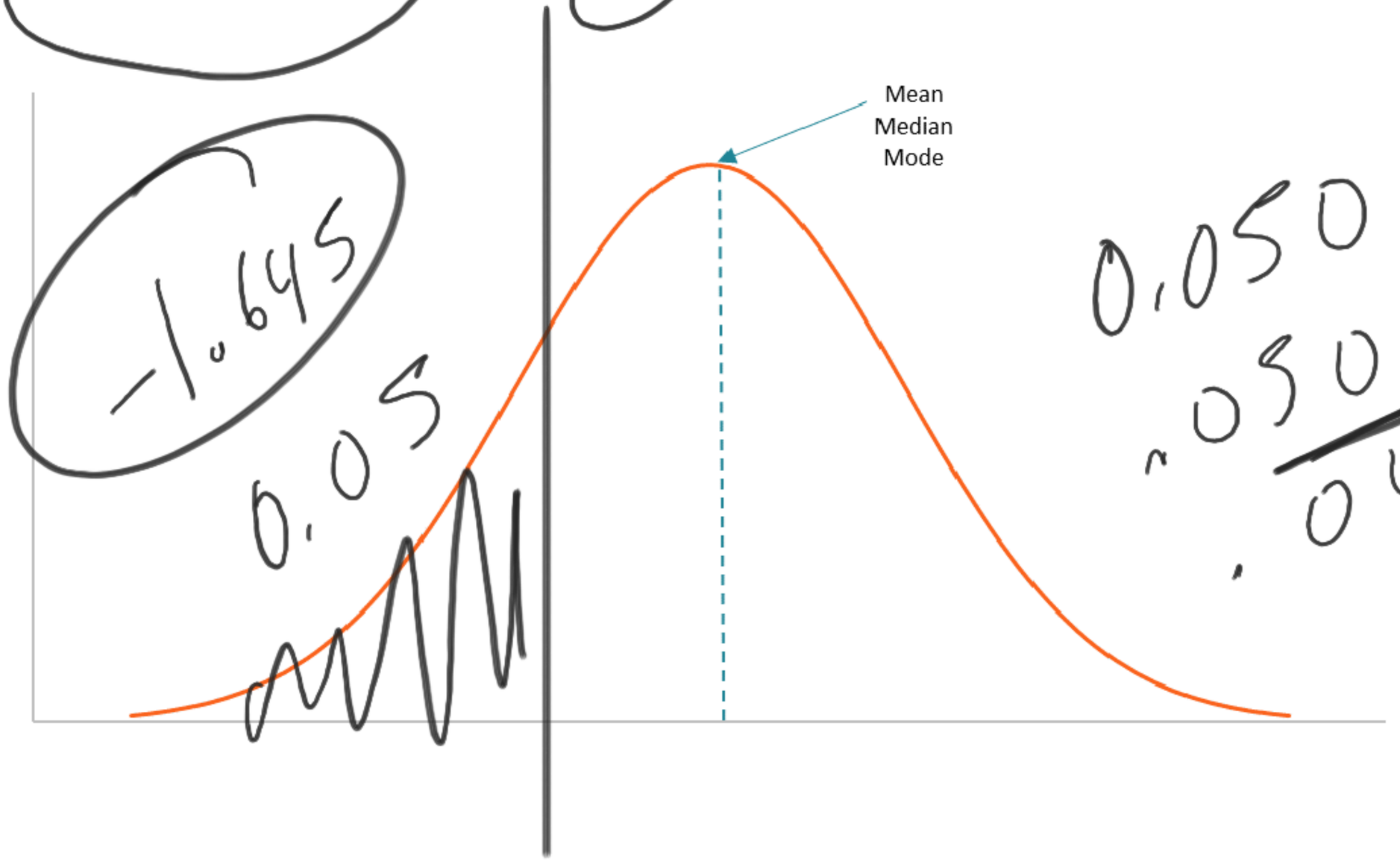


15. Find the value of z such that 0.05 of the area under the curve lies to the left of z.



-1.645

0.05

0.05000

.05050

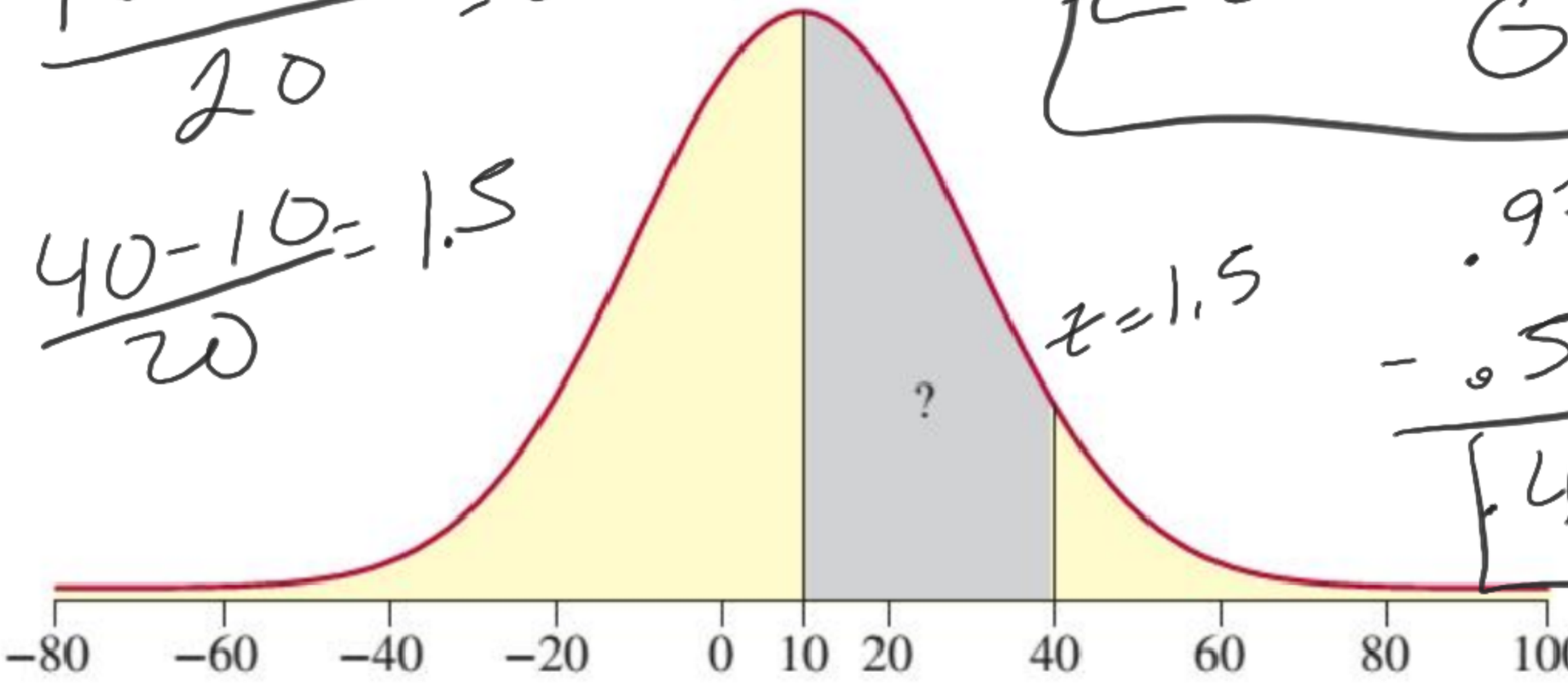
.04947

Find the probability that a normal random variable with a mean of 10 and a standard deviation of 20 will lie between 10 and 40.

$$z = \frac{10 - 10}{20} = 0 \qquad z = 0$$

$$z = \frac{40 - 10}{20} = 1.5 \qquad z = 1.5$$

$$z = \frac{x - \mu}{\sigma}$$



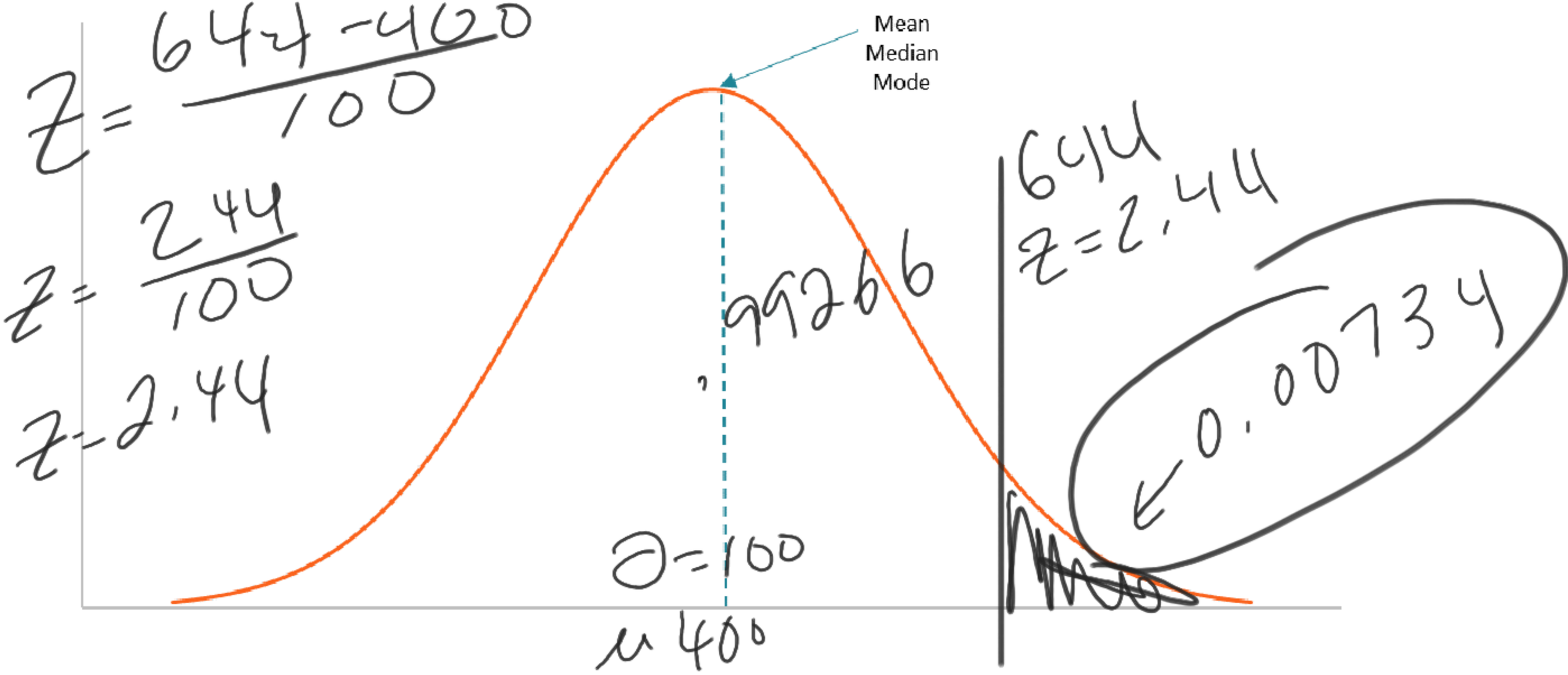
.93319
− .5000
.43319

Suppose that a national testing service gives a test in which the results are normally distributed with a mean of 400 and a standard deviation of 100. If you score a 644 on the test, what fraction of the students taking the test exceeded your score?

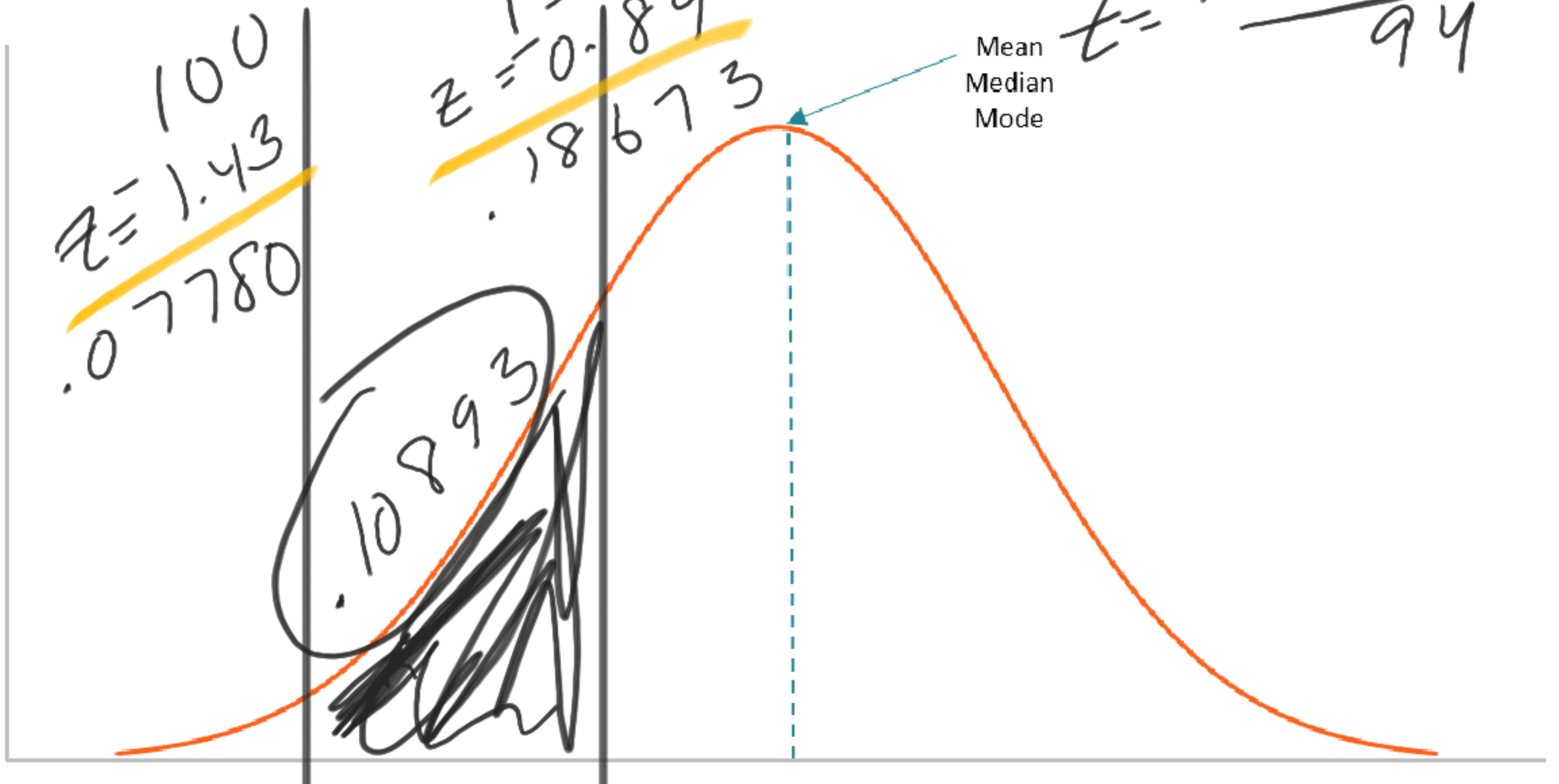
$$z = \frac{644 - 400}{100}$$

$$z = \frac{244}{100}$$

$$z = 2.44$$



Suppose that for 132 shoppers making a purchase at a clothing store, the total each shopper will spend follows a normal distribution with a mean of $234 and a standard deviation of $94. What is the probability that the next purchase total will be between $100 and $150?



The body temperatures of adults are normally distributed with a mean of 98.60 °F and a standard deviation of 0.73 °F. What temperature represents the 90th percentile?

$z = \frac{x - \mu}{\sigma}$

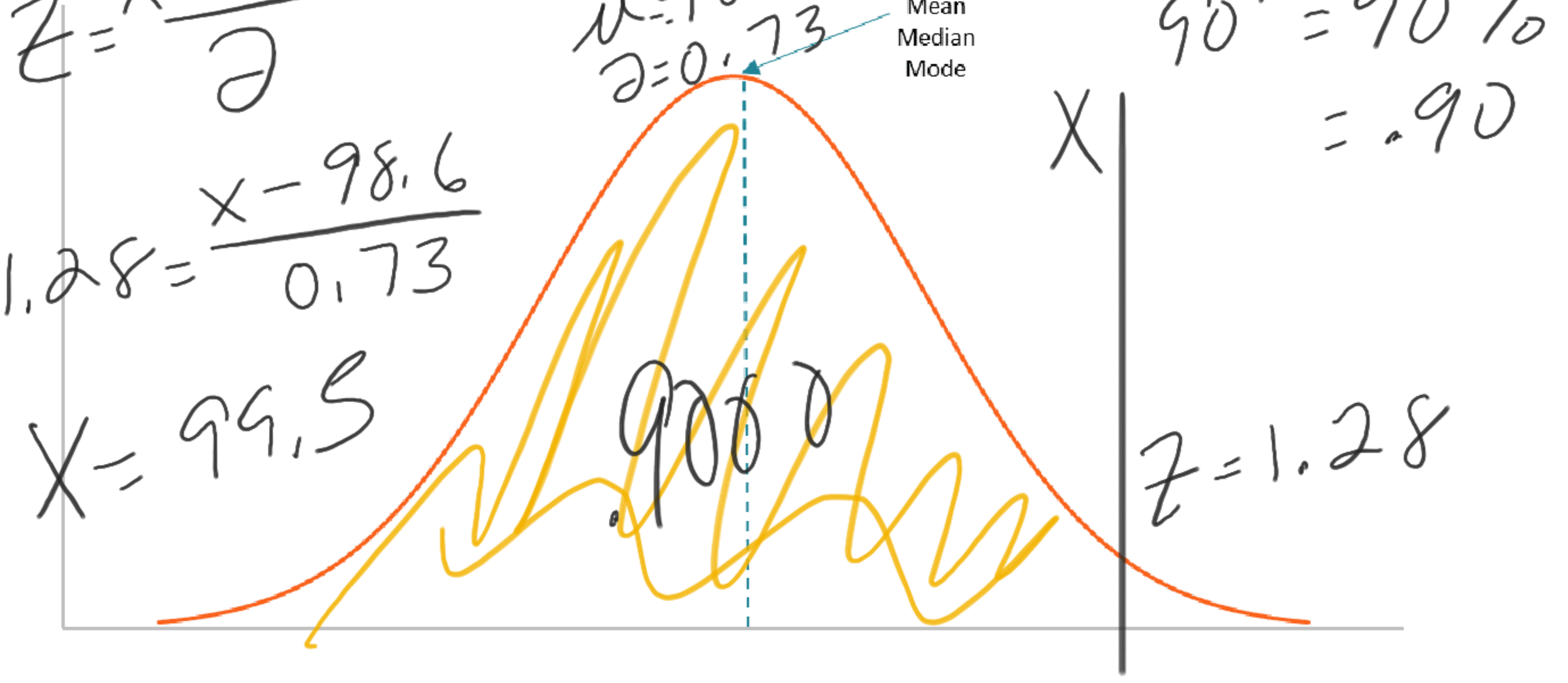
$\mu = 98.6$

$\sigma = 0.73$

$90^{th} = 90\% = .90$

$1.28 = \frac{x - 98.6}{0.73}$

$x = 99.5$



A statistics teacher believes that the final exam grades for her business statistics class have a normal distribution with a mean of 82 and a standard deviation of 8.

a. Find the score which separates the top 10% of the scores from the lowest 90% of the scores.

b. The teacher plans to give all students who score in the top 10% of scores an A. Will a student who scored a 90 on the exam receive an A? Explain.

c. Find the score which separates the lowest 20% of the scores from the highest 80% of the scores.

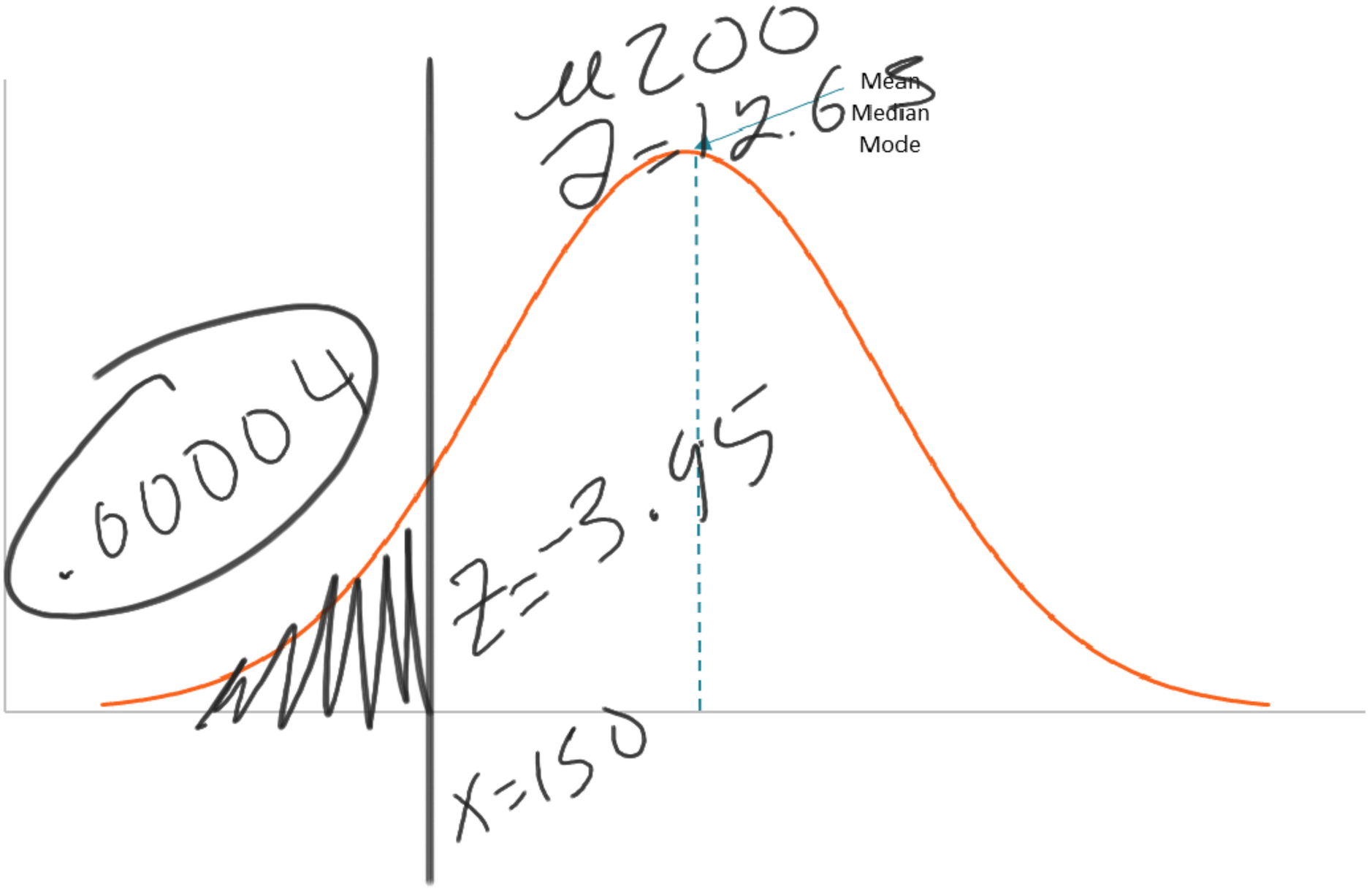
d. The teacher plans to give all students who score in the lowest 10% of scores an F. Will a student who scored a 65 on the exam receive an F? Explain.

An advertising agency hired on behalf of Tech's development office conducted an ad campaign aimed at making alumni aware of their new capital campaign. Upon completion of the new campaign, the agency claimed that 20% of alumni in the state were aware of the new campaign. To validate the claim of the agency, the development office surveyed 1000 alumni in the state and found that 150 were aware of the campaign. Assuming that the ad agency's claim is true, what is the probability that no more than 150 of the alumni in the random sample were aware of the new campaign?

$\mu = np = 1000(.20) = 200$

$\sigma = \sqrt{npq(1-p)} = \sqrt{1000(.2)(.8)} = 12.65$

$X = 150$


μ200
x̄=12.65
Mean
Median
Mode
.00004
z=-3.95
x=150

Suppose a virus is believed to infect two percent of the population. If a sample of 3000 randomly selected subjects are tested, answer the following questions.

a. Find the expected number of subjects sampled that will be infected.

b. Find the standard deviation of the number of subjects sampled that will be infected.

c. What is the probability that fewer than 30 of the subjects in the sample will be infected?

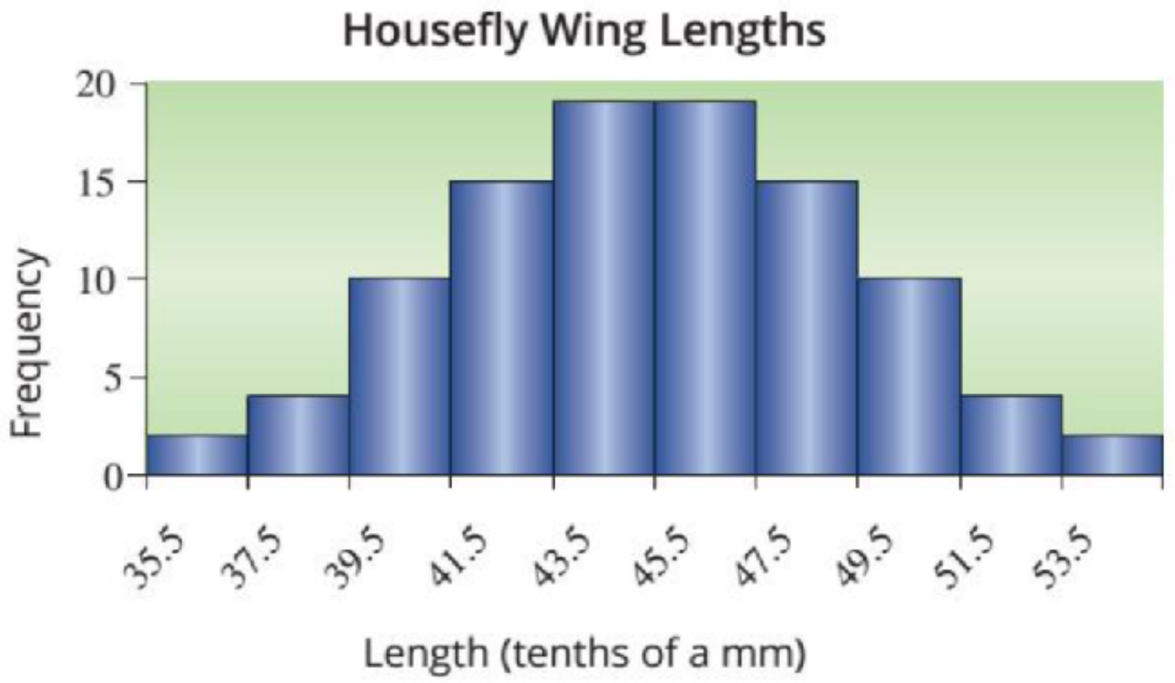
d. What is the probability that between 40 and 80 (inclusive) of the subjects in the sample will be infected?

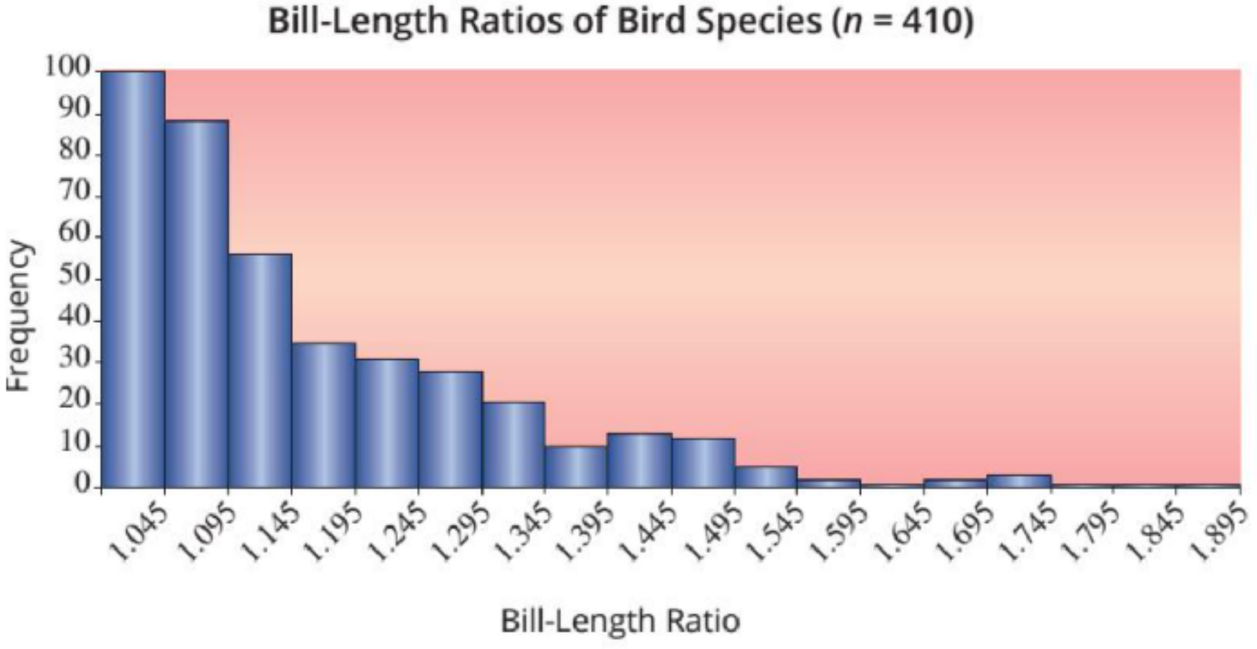
e. Find the probability that at least 70 of the subjects in the sample will be infected.

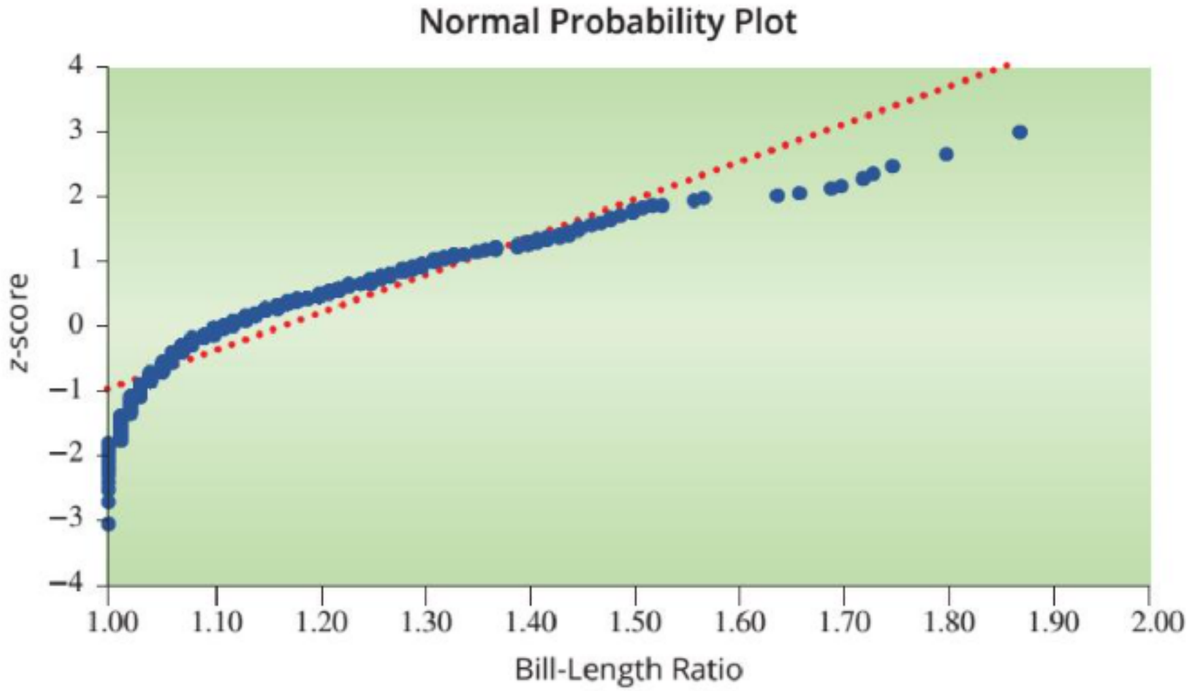
Assessing Normality

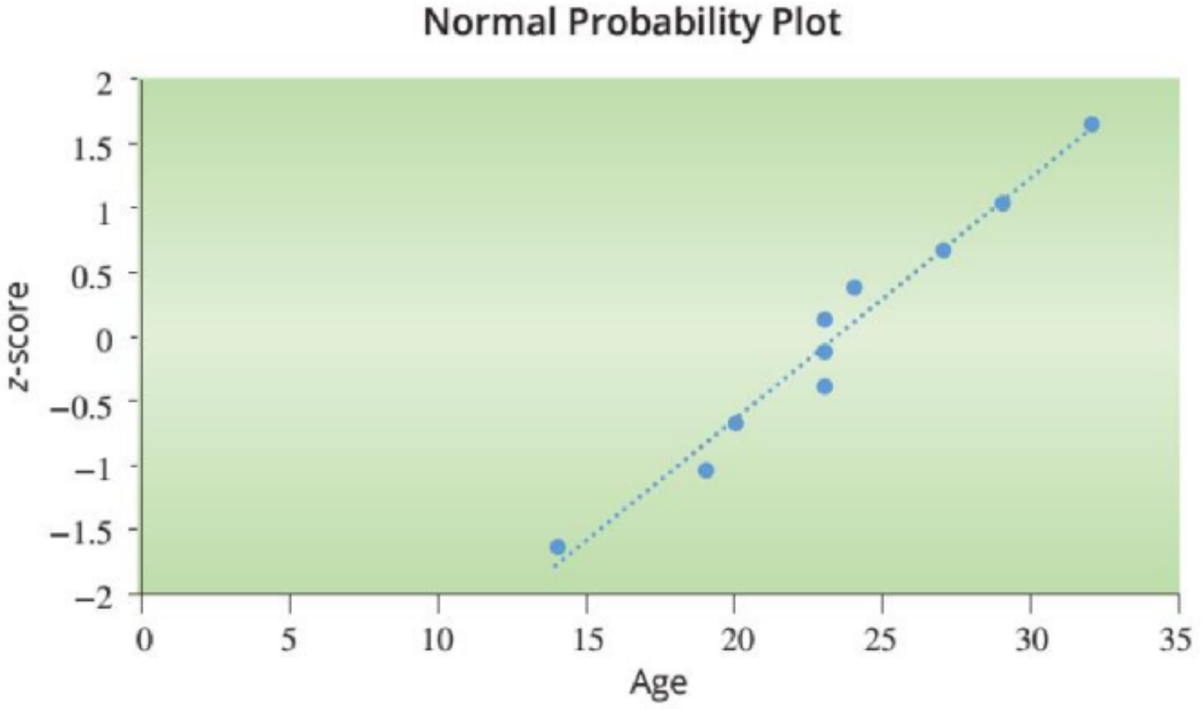
1. Graphically confirm the distribution of the sample data.
   - **a.** If there are enough data values to draw a histogram, then the histogram of the data must follow a bell-shaped curve, or a "normal curve". If the shape of the histogram differs drastically from a bell shape, then we conclude that the data are not drawn from a normal distribution.
   - **b.** For small data sets ($n \leq 30$), construct a normal probability plot. If the points on the normal probability plot do not follow a linear pattern, or if there is a systematic pattern that is not linear, then we conclude that the data are not drawn from a normal distribution.
2. Inspect the histogram or normal probability plot for outliers.
   - **a.** The data must have no more than one outlier present. A single outlier could be the result of an error or some chance variation. However, since even a single outlier may affect results, we must reject normality if there are two or more outliers. A method to detect outliers was described in Section 4.3. Box plots of the data can also be used to detect outliers.

PROCEDURE


Housefly Wing Lengths
Frequency
0
5
10
15
20
35.5
37.5
39.5
41.5
43.5
45.5
47.5
49.5
51.5
53.5
Length (tenths of a mm)


Bill-Length Ratios of Bird Species (n = 410)
Frequency
0
10
20
30
40
50
60
70
80
90
100
1.045
1.095
1.145
1.195
1.245
1.295
1.345
1.395
1.445
1.495
1.545
1.595
1.645
1.695
1.745
1.795
1.845
1.895
Bill-Length Ratio


Normal Probability Plot
z-score
4
3
2
1
0
−1
−2
−3
−4
1.00
1.10
1.20
1.30
1.40
1.50
1.60
1.70
1.80
1.90
2.00
Bill-Length Ratio


Normal Probability Plot
z-score
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
0
5
10
15
20
25
30
35
Age