Introduction to Probability

Random Experiment

A **random experiment** is defined as any activity or phenomenon that meets the following conditions.

1. There is one distinct outcome for each trial of the experiment.
2. The outcome of the experiment is uncertain.
3. The set of all distinct outcomes of the experiment can be specified and is called the **sample space**, denoted by S.

DEFINITION

Outcome

An **outcome** is any member of the sample space.

DEFINITION

Event

An **event** is a set of outcomes.

DEFINITION

Experiment 1: Toss a coin and observe the outcome. Have we met the three conditions of a random experiment?

Sample Space $\{H, T\}$

Experiment 2: Toss a coin three times and observe the number of heads. Have we met the three conditions of a random experiment?

Sample Space {0, 1, 2, 3}

Relative Frequency

If an experiment is performed n times, under identical conditions, and the event A happens k times, the **relative frequency** of A is given by the following expression.

$$\text{Relative Frequency of } A = \frac{k}{n}$$

If the relative frequency converges as n increases, then the relative frequency is said to be the **probability** of A.

DEFINITION

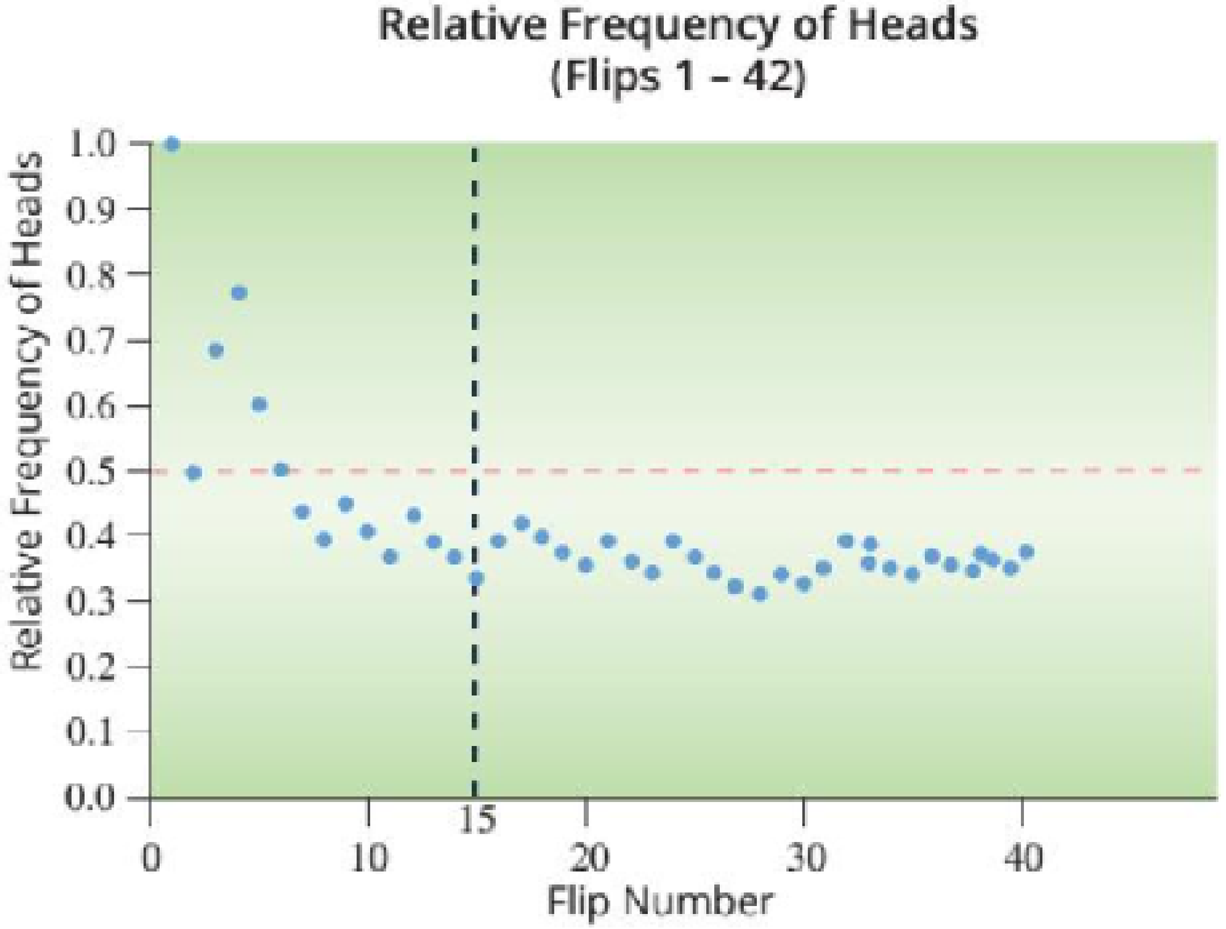
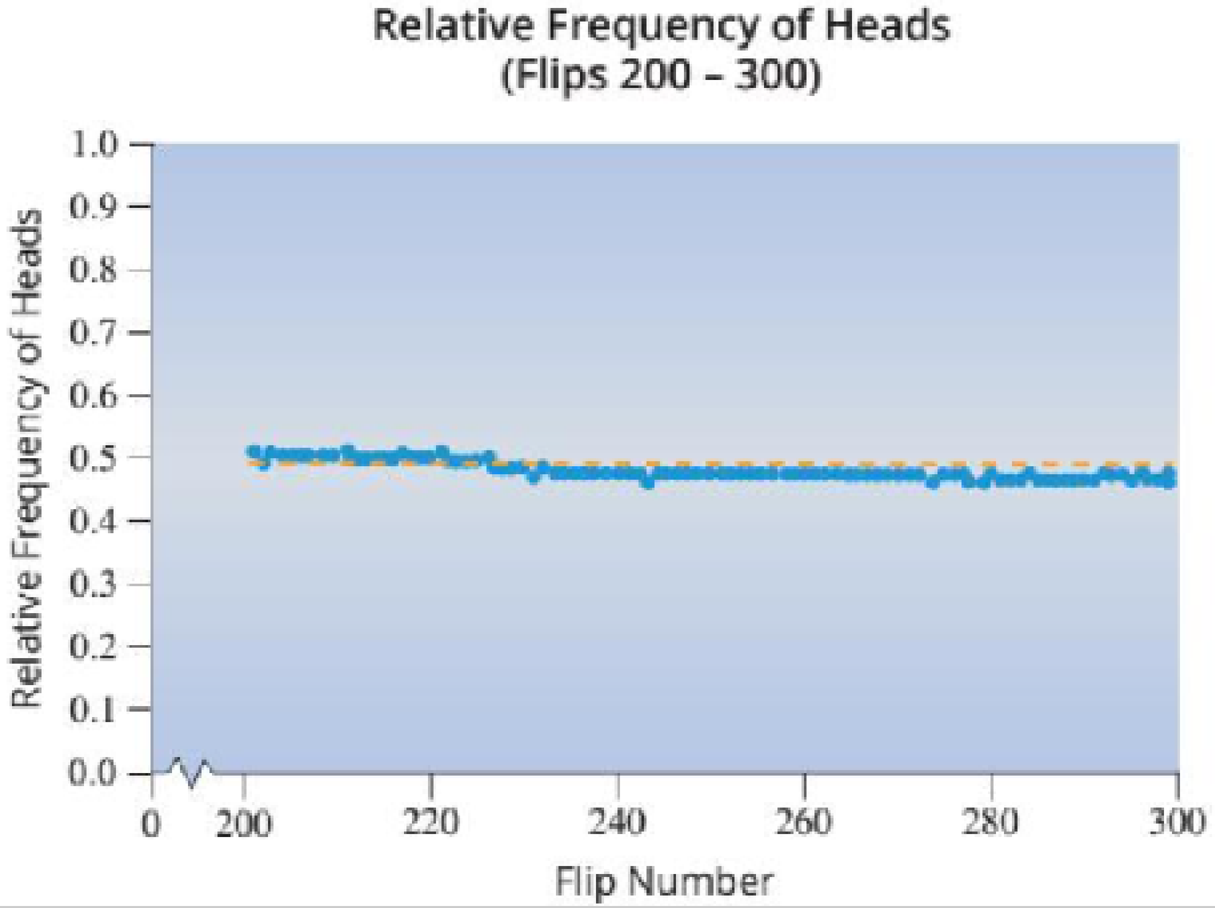
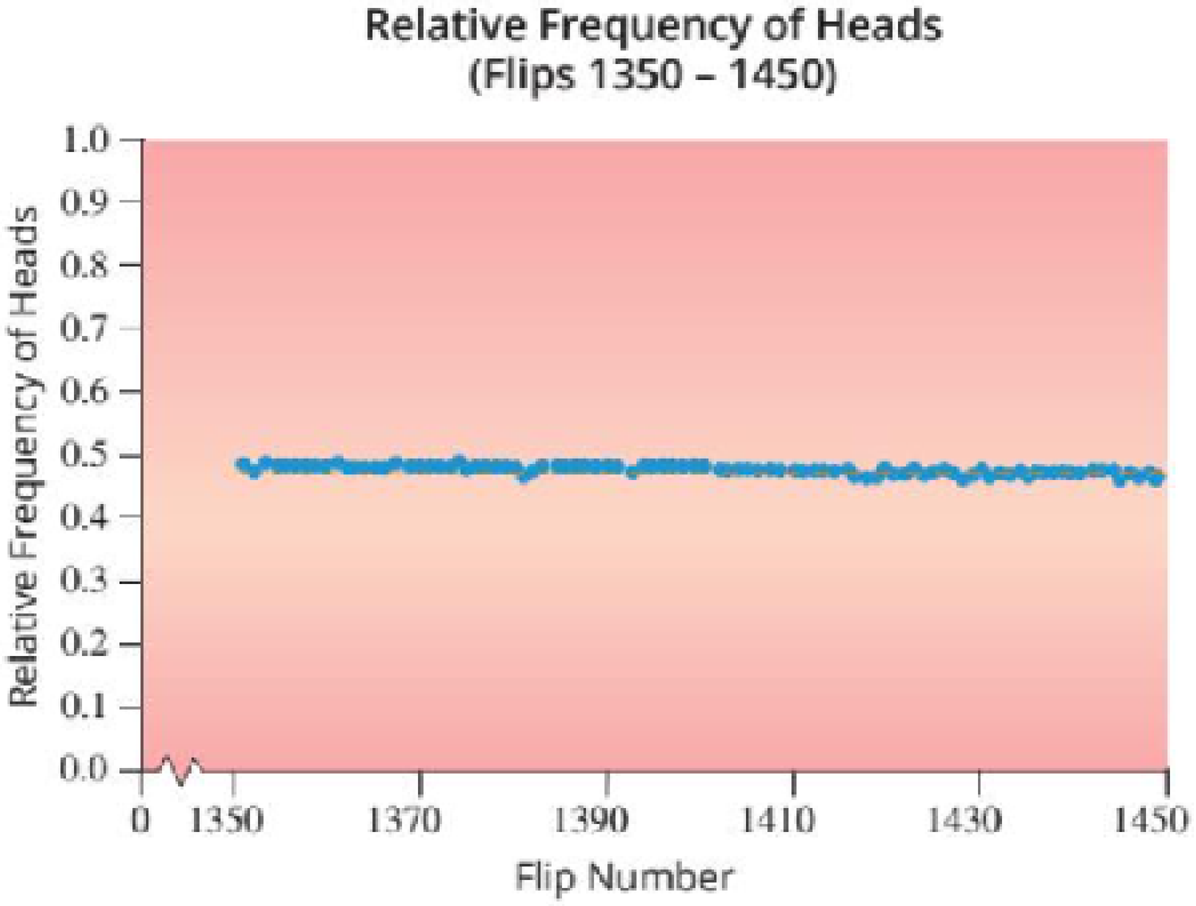

Relative Frequency of Heads
(Flips 1 – 42)
Relative Frequency of Heads
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
10
15
20
30
40
Flip Number

Figure 6.1.1


Relative Frequency of Heads
(Flips 200 – 300)
Relative Frequency of Heads
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
200
220
240
260
280
300
Flip Number


Relative Frequency of Heads
(Flips 1350 – 1450)
Relative Frequency of Heads
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1350
1370
1390
1410
1430
1450
Flip Number

Classical Probability

Using the **classical approach** to probability, the probability of an event A, denoted $P(A)$, is given by

$$P(A) = \frac{\text{number of outcomes in } A}{\text{total number of outcomes in the sample space}}.$$

DEFINITION

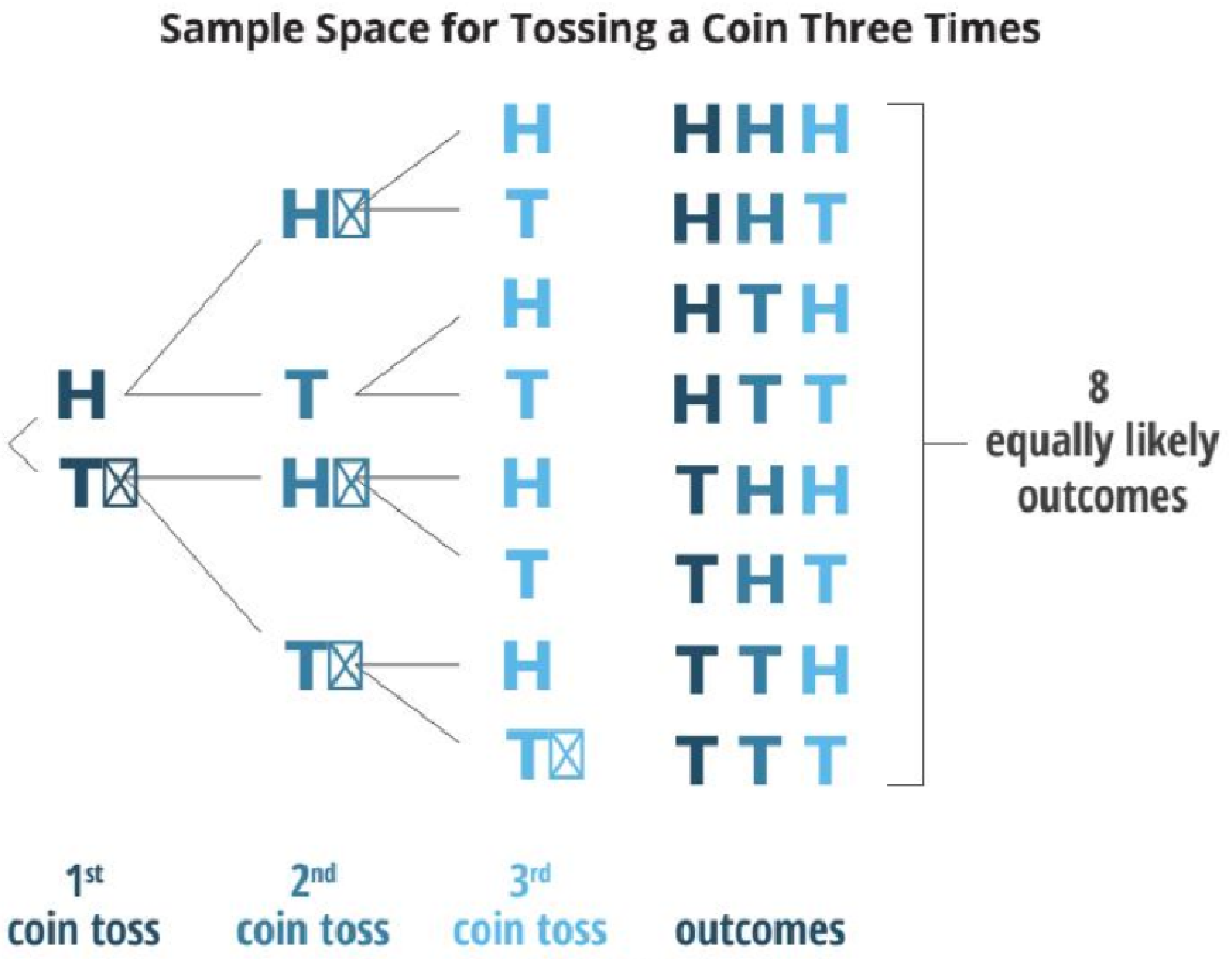

Sample Space for Tossing a Coin Three Times
H
T
H
T
H
T
H
T
H
T
H
T
H
T
HHH
HHT
HTH
HTT
THH
THT
TTH
TTT
8
equally likely
outcomes
1st
coin toss
2nd
coin toss
3rd
coin toss
outcomes

Figure 6.1.4

Subjective Approach

6.2 Addition Rules for Probability

Probability Law 1

A probability of zero means the event cannot happen. (For example, the probability of observing three heads in two tosses of a coin is zero.)

DEFINITION

Probability Law 2

A probability of one means the event must happen. (For example, if we toss a coin, the probability of getting either a head or tail is one.)

DEFINITION

Probability Law 3

All probabilities must be between zero and one inclusively. The closer the probability is to 1, the more likely the event. The closer the probability is to 0, the less likely the event. For an event A this is expressed as follows.

$$0 \leq P(A) \leq 1$$

DEFINITION

Probability Law 4

The sum of the probabilities of all outcomes must equal one. That is, if $P(A_i)$ is the probability of event A_i, and there are n such outcomes, then $P(A_1) + P(A_2) + \cdots + P(A_n) = 1$.

DEFINITION

Compound event

A **compound event** is an event that is defined by combining two or more events.

DEFINITION

$A = \{\text{annual income is greater than } \$50{,}000\}$

and

$B = \{\text{subscribes to more than one other sports magazine}\}$.

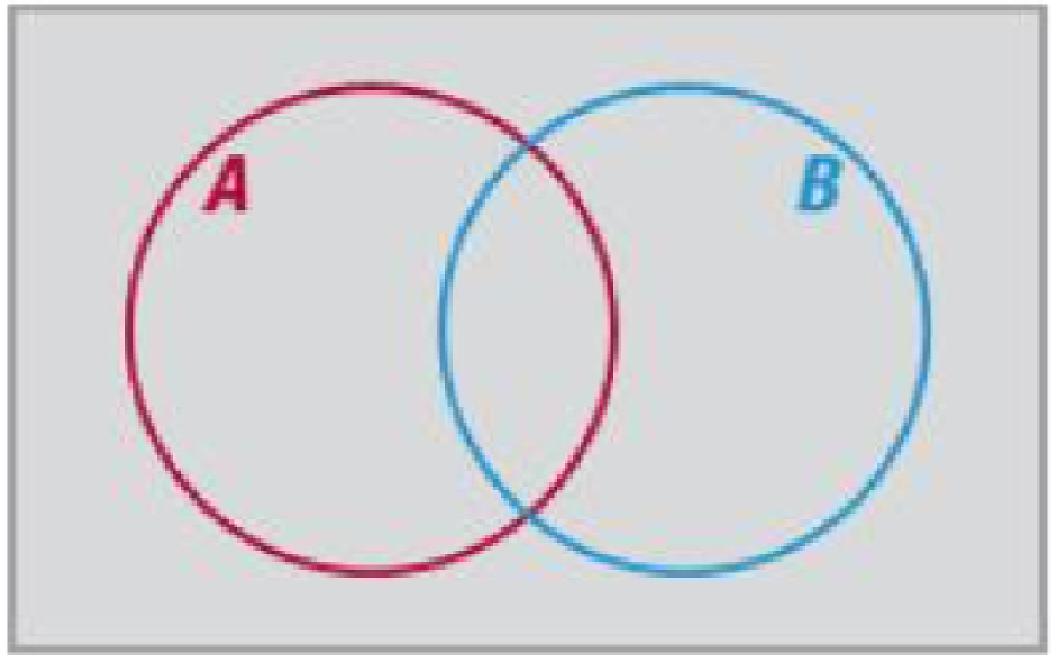


Figure 6.2.1

Union

The **union** of the events A and B is the set of outcomes that are included in event A or event B or both. Symbolically, the union of A and B is denoted $A \cup B$ and is read "A union B."

DEFINITION

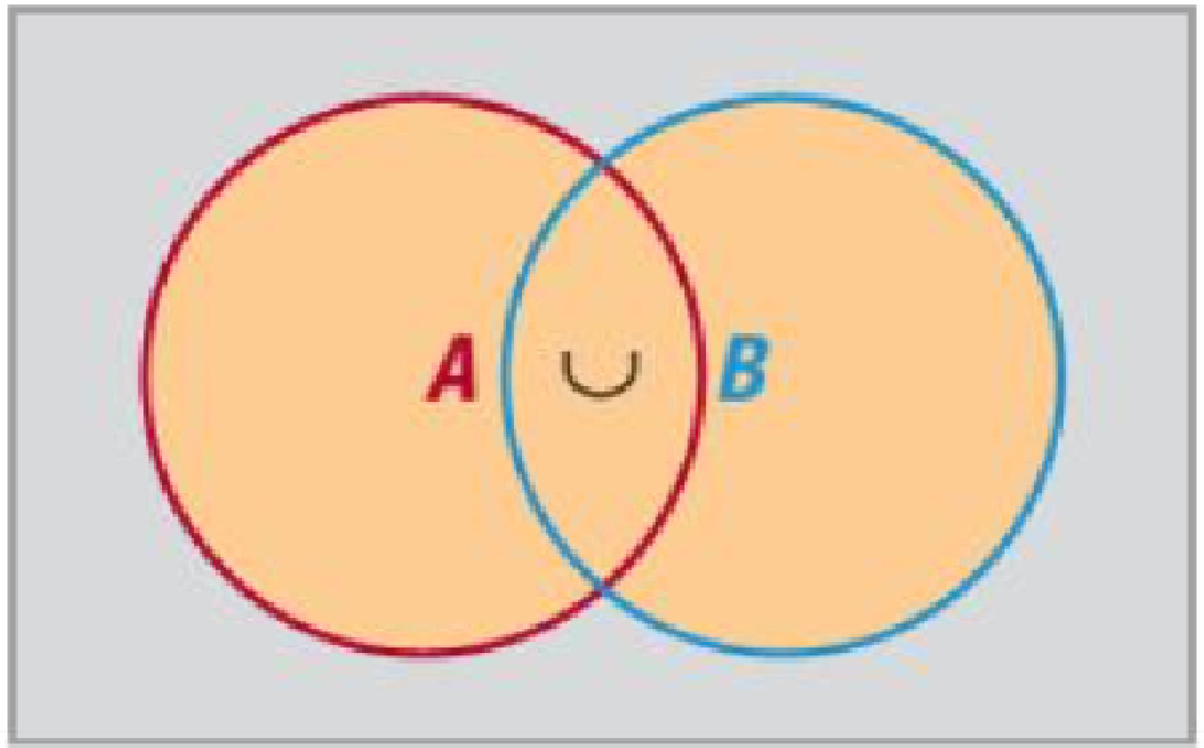


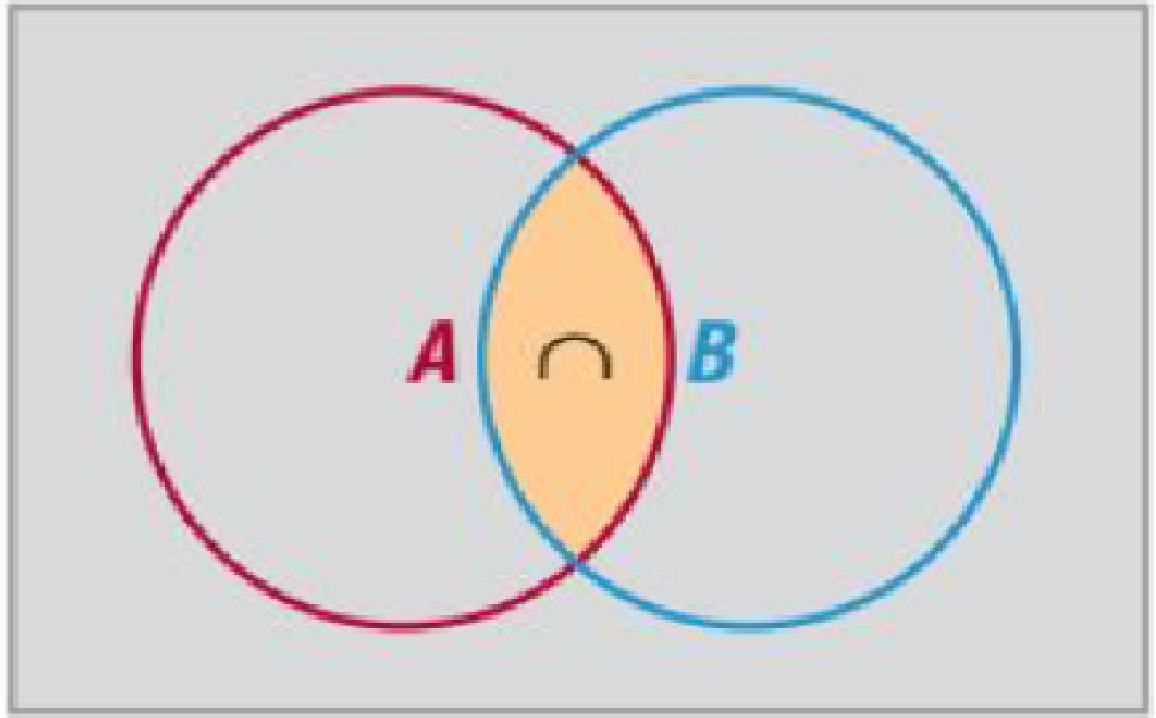
Figure 6.2.2

Intersection

The **intersection** of the events A and B is the set of all outcomes that are included in both A and B. Symbolically, the intersection of A and B is denoted $A \cap B$ and is read "A intersect B."

DEFINITION

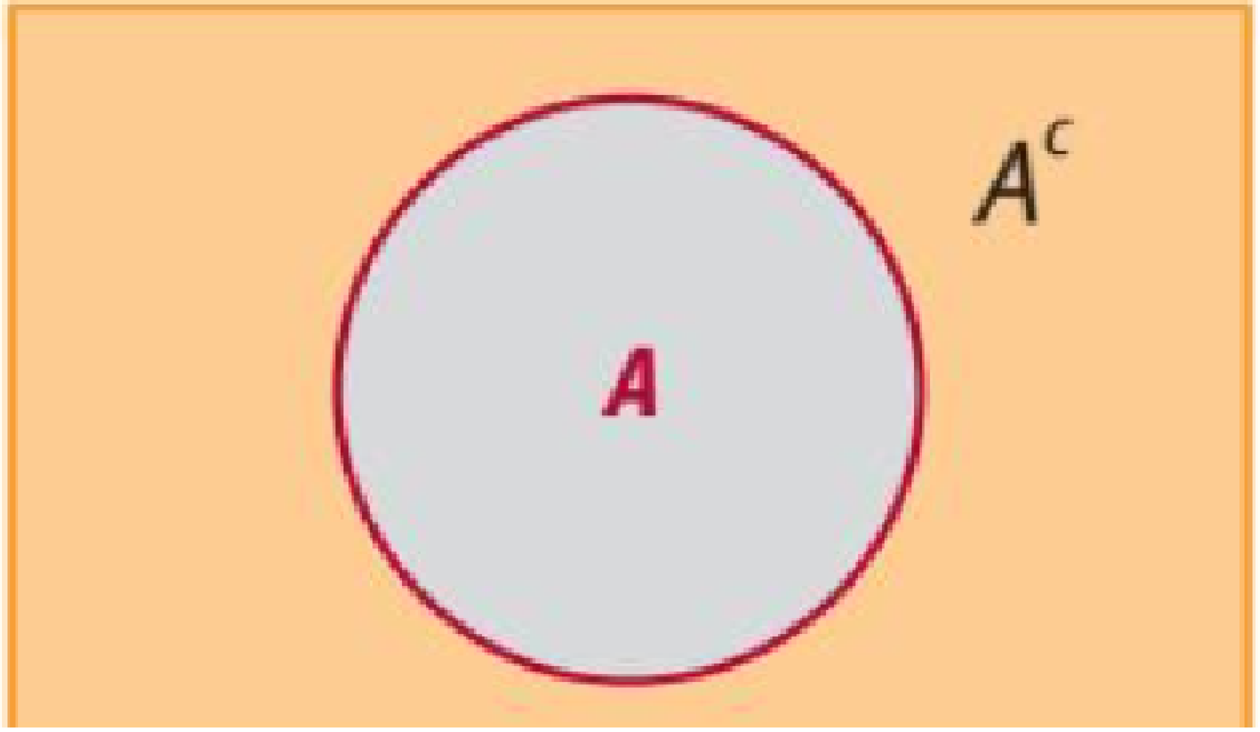
Suppose the marketing director was interested in persons who possessed an annual income greater than $50,000 and subscribed to more than one other sports magazine. That set would be called the intersection of A and B.



Complement

The **complement** of an event A is the set of all outcomes in the sample space that are not in A. Symbolically, the complement of the set A will be written as A^c.

DEFINITION



Probability Law 5: Complement of an Event

The probability of A^c is given by $P(A^c) = 1 - P(A)$.

DEFINITION

In the game of American roulette, the roulette wheel contains the numbers 1 through 36, alternating between red and black. There are two green spaces numbered 0 and 00.

a. Calculate the probability of the roulette ball landing on a red pocket.
b. Calculate the probability of the roulette ball not landing on a red pocket.
c. Calculate the odds in favor of the roulette ball landing on red.

a) $\frac{18}{38}$

b) $1 - \frac{18}{38} = \frac{20}{38}$

c)

Probability Law 8: The General Addition Rule

For any two events A and B,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

DEFINITION

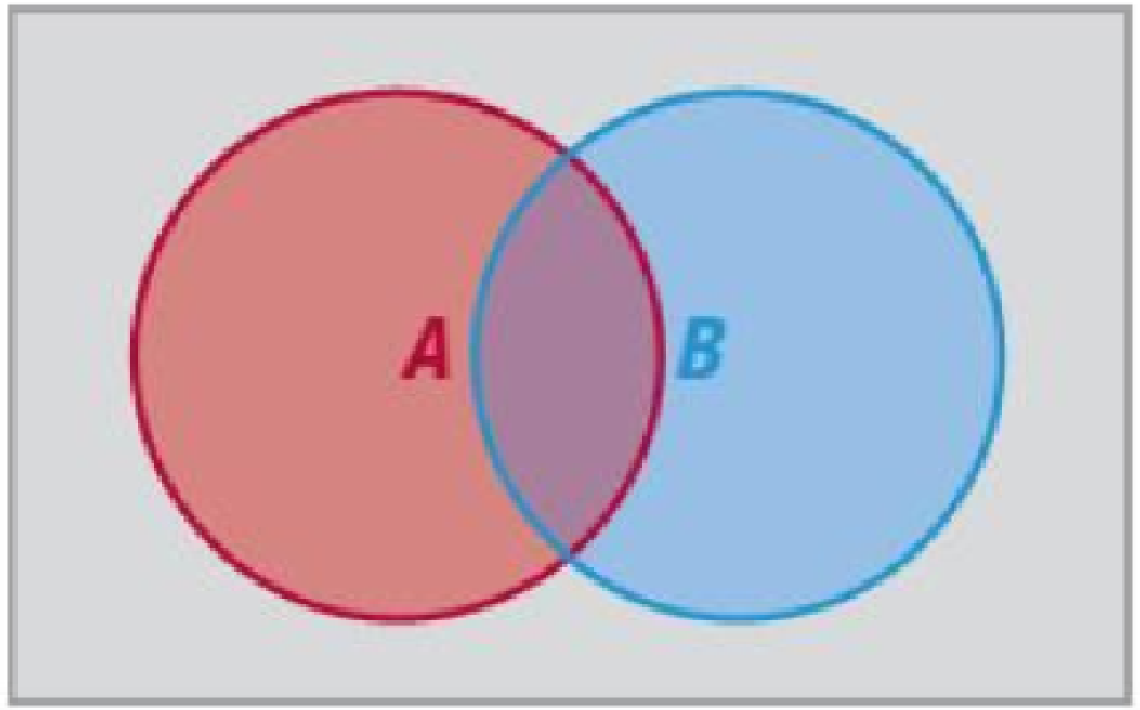


Figure 6.2.6

Odds

The **odds in favor of** an event A occurring is given by $\frac{P(A)}{P(\text{not } A)} = \frac{P(A)}{P(A^c)}$.

The **odds against** an event A occurring is given by $\frac{P(\text{not } A)}{P(A)} = \frac{P(A^c)}{P(A)}$.

DEFINITION

$r = \frac{18}{20}$

not red

odds in favor $\frac{18}{20}$

against $\frac{20}{18}$