The Double-Angle Identities

cosine: $\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$

$= 1 - 2\sin^2\alpha$

$= 2\cos^2\alpha - 1$

sine: $\sin(2\alpha) = 2\sin\alpha\cos\alpha$

tangent: $\tan(2\alpha) = \dfrac{2\tan\alpha}{1 - \tan^2\alpha}$

Find $\sin 2x$, $\cos 2x$, and $\tan 2x$ if $\sin x = \dfrac{8}{17}$ and x terminates in quadrant II.

$y = 8 \qquad r = 17 \qquad x = -15 \qquad 8^2 + x^2 = 17^2$

$x = \pm 15$

$$\sin(2\alpha) = 2 \sin\alpha \cos\alpha$$

$$\sin 2x = 2(\sin x)(\cos x)$$

$$= 2\left(\frac{8}{17}\right)\left(\frac{-15}{17}\right) = \boxed{\frac{-240}{289}}$$

Find $\sin 2x$, $\cos 2x$, and $\tan 2x$ if $\sin x = \frac{8}{17}$ and x terminates in quadrant II.

$y = 8 \quad r = 17 \quad x = -15$

$\cos^2 x = (\cos x)^2$

cosine: $\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$

$= 1 - 2\sin^2\alpha$

$= 2\cos^2\alpha - 1$

$$\cos 2x = 1 - 2\left(\frac{8}{17}\right)^2$$

$$= 1 - \frac{128}{289} = \boxed{\frac{161}{289}}$$

Find $\sin 2x$, $\cos 2x$, and $\tan 2x$ if $\sin x = \frac{8}{17}$ and x terminates in quadrant II.

$y = 8 \quad r = 17 \quad x = -15$

$\tan x = \frac{-8}{15}$

$$\tan(2\alpha) = \frac{2\tan\alpha}{1 - \tan^2\alpha} = \frac{2\left(\frac{-8}{15}\right)}{1 - \left(\frac{-8}{15}\right)^2}$$

$$\tan 2x = \frac{\sin 2x}{\cos 2x} = \frac{-240}{161}$$

Simplify the expression by using a double-angle formula.

$$\sin\frac{\pi}{7}\cos\frac{\pi}{7}$$

sine: $\sin(2\alpha) = 2\sin\alpha\cos\alpha$

$a = \frac{\pi}{7}$

$\frac{1}{2}\sin(2a) = \sin a \cos a$

~~$\sin\left(2\frac{\pi}{7}\right)$~~

$$\frac{1}{2}\sin\left(\frac{2\pi}{7}\right)$$

Simplify the expression by using a double-angle formula.

$$\frac{2\tan\frac{\pi}{5}}{1-\tan^2\frac{\pi}{5}}$$

$$\tan(2\alpha) = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$a = \frac{\pi}{5}$

$\tan\left(\frac{2\pi}{5}\right)$

Simplify the expression by using a double-angle formula.

$$\cos^2 \frac{3\pi}{5} - \sin^2 \frac{3\pi}{5}$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$$

$$a = \frac{3\pi}{5}$$

$$\cos\left(\frac{6\pi}{5}\right)$$

Power reducing formulas:

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

$$\cot^2 u = \frac{1 + \cos 2u}{1 - \cos 2u}$$

cosine: $$\begin{aligned} \cos(2\alpha) &= \cos^2\alpha - \sin^2\alpha \\ &= 1 - 2\sin^2\alpha \\ &= 2\cos^2\alpha - 1 \end{aligned}$$

$\cot^2 3x \sin^4 3x$

$$\sin^2 u = \frac{1-\cos 2u}{2}$$

$$\cot^2 u = \frac{1+\cos 2u}{1-\cos 2u}$$

$$\frac{1+\cos 6x}{1-\cos 6x} = \cot^2(3x)$$

$$\sin^4(3x) = \left(\sin^2(3x)\right)^2 = \left(\frac{1-\cos(6x)}{2}\right)^2$$

$$\sin^5(3x) = \sin^4(3x)\sin(3x)$$

$$\frac{1+\cos 6x}{1-\cos 6x} \qquad \left(\frac{1-\cos(6x)}{2}\right)^2$$

$$\frac{(1+\cos(6x))(1-\cos(6x))}{4}$$

$$\frac{1-\cos^2(6x)}{4}$$

$$\frac{1 \quad \cos^2(6x)}{4} = \frac{1}{4} - \frac{1}{4}\boxed{\cos^2(6x)}$$

$$\frac{1 + \cos(12x)}{2} \rightarrow \frac{1}{4} - \frac{1}{4}\left(\frac{1}{2} + \frac{1}{2}\cos(12x)\right)$$

$$= \frac{1}{4} - \frac{1}{8} - \frac{1}{8}\cos(12x)$$

$$= \boxed{\frac{1}{8} - \frac{1}{8}\cos(12x)}$$

that a power-reducing form

$$\int \left(\sin^4(x)\cos^6(x)\right) dx$$

$$\int \left(\frac{3}{2} - 2\cos(4x) + \frac{\cos(8x)}{2}\right)$$

Finally, combine the results of each integration.

$$\int \left(\sin^4(x)\cos^6(x)\right) dx = \frac{3x}{256} - \frac{\sin(4x)}{256} + \frac{\sin(8x)}{2048} + \frac{\sin^5(2x)}{320} + C$$

$$\tan^2 3x \cos^4 3x = \frac{1}{8} - \frac{1}{8}\cos 12x$$

Prove the identity.

$$\frac{2\tan x}{1+\tan^2 x} = \sin 2x$$

$\dfrac{2\tan x}{\sec^2 x}$ P

$2\tan x \cos^2 x$ R

$2\dfrac{\sin x}{\cos x}\cos^2 x$ Q

$2\sin x \cos x$ A

$\sin 2x$ D

$$\frac{\sec^2 x - 2\tan^2 x}{1+\tan^2 x} = \cos 2x$$

$1+\tan^2 x \leftarrow \sec^2 x$

$$\frac{\sec^2 x - 2\tan^2 x}{\sec^2 x} = \frac{\sec^2 x}{\sec^2 x} - \frac{2\tan^2 x}{\sec^2 x}$$

$$1 - \frac{2\tan^2 x}{\sec^2 x} \qquad \frac{\tan^2 x}{\sec^2 x} = \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}}$$

$$1 - 2\sin^2 x = \cos 2x$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\cos u = 1 - 2\sin^2\frac{u}{2}$$

$$\sin^2\frac{u}{2} = \frac{1-\cos u}{2}$$

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$$

half-angle formulas.

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$$

$$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

$$\tan\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{1+\cos u}} = \frac{\sin u}{1+\cos u} = \frac{1-\cos u}{\sin u}$$

Use a half-angle formula to find the exact value of $\sin 157.5°$. Q2 +

$$\sin 157.5° = \sin \frac{315°}{2}$$

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$$

$$\sqrt{\frac{1-\cos(315)}{2}} = \sqrt{\frac{1-\sqrt{2}/2}{2}} = \frac{\frac{2}{2}-\frac{\sqrt{2}}{2}}{2} = \frac{\frac{2-\sqrt{2}}{2}}{2}$$

$$\left(\frac{2-\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \sqrt{\frac{2-\sqrt{2}}{4}} = \boxed{\frac{\sqrt{2-\sqrt{2}}}{2}}$$

Suppose that $\tan\theta = \dfrac{12}{5}$ and $0° < \theta < 90°$. $x = 5 \quad y = 12 \quad r = 13$

Find the exact values of $\cos\dfrac{\theta}{2}$ and $\tan\dfrac{\theta}{2}$.

$$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

$$\cos\frac{\theta}{2} = \sqrt{\frac{1+(5/13)}{2}}$$

$$= \frac{\frac{13+5}{13}}{2} = \frac{\frac{18}{13}}{2} = \frac{18}{13}\cdot\frac{1}{2} = \sqrt{\frac{9}{13}}$$

$$= \frac{3}{\sqrt{13}} = \boxed{\frac{3\sqrt{13}}{13}}$$