Dont Forget To Record

Suppose that the functions g and h are defined for all real numbers x as follows.

$$g(x) = 4x^3$$
$$h(x) = x^2$$

Write the expressions for $(h+g)(x)$ and $(h-g)(x)$ and evaluate $(h \cdot g)(-1)$.

$$(h+g)(x) = (4x^3) + (x^2) = 4x^3 + x^2$$

$$(h-g)(x) = (4x^3) - (x^2) = 4x^3 - x^2$$

$$(h \cdot g)(-1) = (4x^3)(x^2) = 4x^5$$

$$4(-1)^5 = -4$$

$f(x) = 3x^2 + x - 1$

$g(x) = 2x^2 + 5$

$(f-g)(x) = (3x^2 + x - 1) - (2x^2 + 5)$

$3x^2 + x - 1 - 2x^2 - 5$

$$\boxed{x^2 + x - 6}$$

Suppose that the functions h and g are defined as follows.

$$h(x) = (x+1)(x-4)$$
$$g(x) = x-8$$

(a) Find $\left(\frac{h}{g}\right)(1)$.

$$\frac{(x+1)(x-4)}{x-8} = \frac{(2)(-3)}{-7}$$

$$= \frac{-6}{-7} = \boxed{\frac{6}{7}}$$

(b) Find all values that are NOT in the domain of $\frac{h}{g}$.

If there is more than one value, separate them with commas.

$x \neq 8$

Suppose that the functions f and g are defined as follows.

$$f(x) = \frac{2}{x+8} \qquad g(x) = \frac{9}{x}$$

$x \neq -8$ $\qquad$ $x \neq 0$

Find $\dfrac{f}{g}$. Then, give its domain using an interval or union of intervals.

Simplify your answers.

$$\frac{2/x+8}{9/x} = \frac{2}{x+8} \times \frac{x}{9} = \boxed{\frac{2x}{9x+72}}$$

$x \neq -8, 0$

$$f(x) = \frac{6}{x+7} \qquad g(x) = \frac{x}{x-4} \qquad \left(\frac{f}{g}\right)(x)$$

$x \neq -7 \qquad x \neq 4 \qquad x \neq 0$

$$\frac{\frac{6}{x+7}}{\frac{x}{x-4}} = \frac{6}{x+7} \cdot \frac{x-4}{x} = \boxed{\frac{6x-24}{x^2+7x}}$$

$x \neq -7, 0, 4$

Find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$, where $h \neq 0$, for the function below.

$$f(x) = -4x^2 - 6x + 6$$

$$f(x+h) = -4(x+h)^2 - 6(x+h) + 6$$

$$= -4(x^2 + 2xh + h^2) - 6(x+h) + 6$$

$$= -4x^2 - 8xh - 4h^2 - 6x - 6h + 6$$

Find the difference quotient $\frac{f(x+h)-f(x)}{h}$, where $h \neq 0$, for the function below.

$$f(x) = -4x^2 - 6x + 6$$

$$\cancel{+4x^2} + \cancel{6x} \cancel{-6}$$

$$(\cancel{-4x^2} - 8xh - 4h^2 \cancel{-6x} - 6h + \cancel{6}) - (-4x^2 - 6x + 6)$$

$$\frac{-8xh - 4h^2 - 6h}{h} = \boxed{-8x - 4h - 6}$$

$$f(x) = -4x^2 - 6x + 6 \qquad f'(x) = -8x - 6$$

The functions q and r are defined as follows.

$$q(x) = -2x + 1$$
$$r(x) = -2x^2 - 1$$

Find the value of $q(r(2))$.

$r(2) = -2(2)^2 - 1$
$= -8 - 1 = -9$

$q(-9) = -2(-9) + 1$
$= 18 + 1 = \boxed{19}$

$\boxed{q(r(2)) = 19}$

The functions s and t are defined as follows.

$s(-1) = 2(-1) + 1 = -1$

$$s(x) = 2x + 1$$

$$t(x) = -2x^2 - 1$$

Find the value of $t(s(-1))$.

$t(-1) = -2(-1)^2 - 1 = -3$

$$q(x) = x^2 + 4$$
$$r(x) = \sqrt{x+5}$$

Find the following.

$(q \circ r)(4) = $ ☐

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$(q \circ r)(4) = q(r(4))$

$= \sqrt{4+5} = 3$

$q(3) = (3)^2 + 4$

$= \boxed{13}$

$(r \circ q)(4) = r(q(4))$

$(4)^2 + 4 = 20$

$r(20) = \sqrt{20+5} = \boxed{5}$

$$q(x) = x^2 + 4$$
$$r(x) = \sqrt{x+5}$$

Find the following.

$(q \circ r)(4) =$ ☐

$(r \circ q)(4) =$ ☐

$$(q \circ r)(x) = q(r(x))$$
$$q(\sqrt{x+5}) =$$
$$(\sqrt{x+5})^2 + 4$$
$$x+5+4 =$$
$$\boxed{x+9}$$

$$g(x) = x^2 - 4$$

$$h(x) = \frac{x}{2}$$

d the compositions $g \circ g$ and $h \circ h$.

$$(g \circ g)(x) = g(g(x)) = g(x^2 - 4)$$

$$= (x^2 - 4)^2 - 4$$

$$= x^4 - 4x^2 - 4x^2 + 16 - 4$$

$$= x^4 - 8x^2 + 12$$

$$g(x) = x^2 - 4$$

$$h(x) = \frac{x}{2}$$

d the compositions $g \circ g$ and $h \circ h$.

$$(h \circ h)(x) = h(h(x))$$

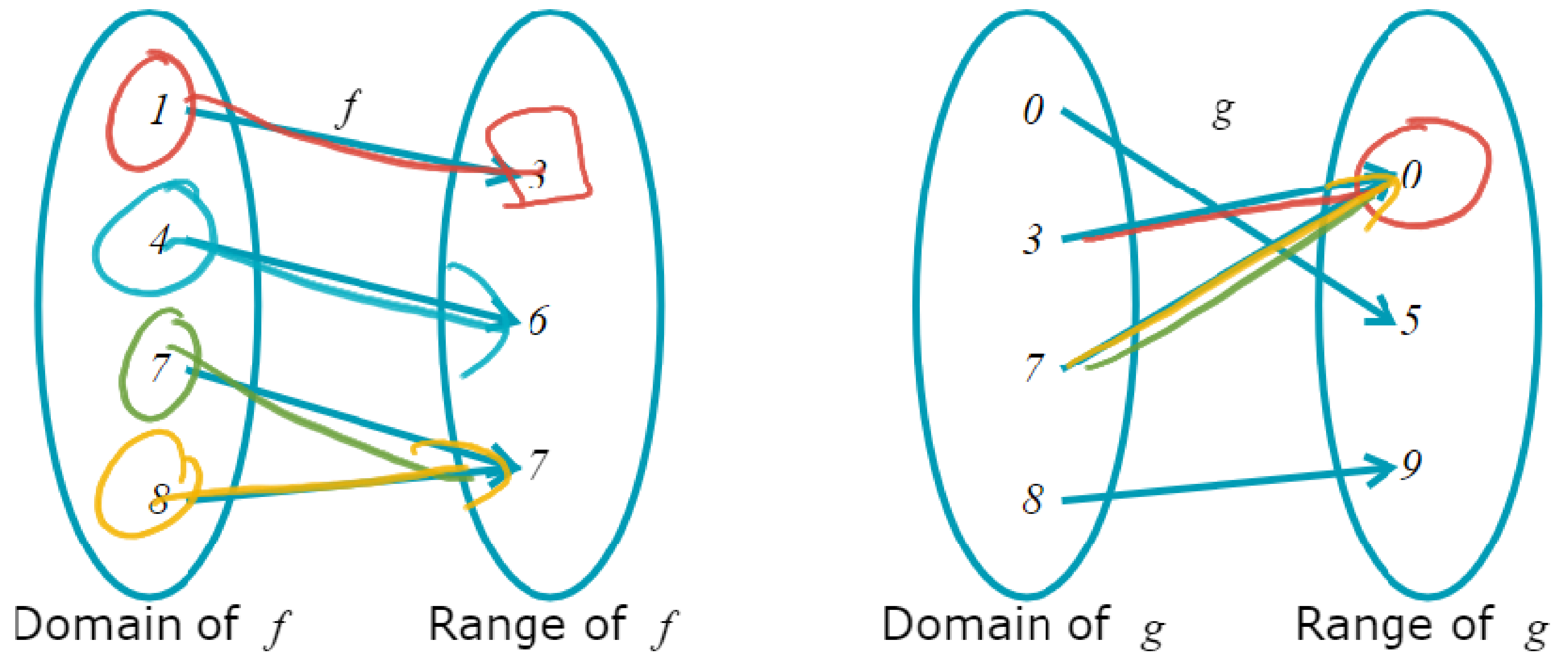
$$= h\left(\frac{x}{2}\right)$$

$$= \frac{x/2}{2} = \boxed{\frac{x}{4}}$$

Two functions f and g are defined in the figure below.

$(g \circ f)(x)$

$g(f(x))$

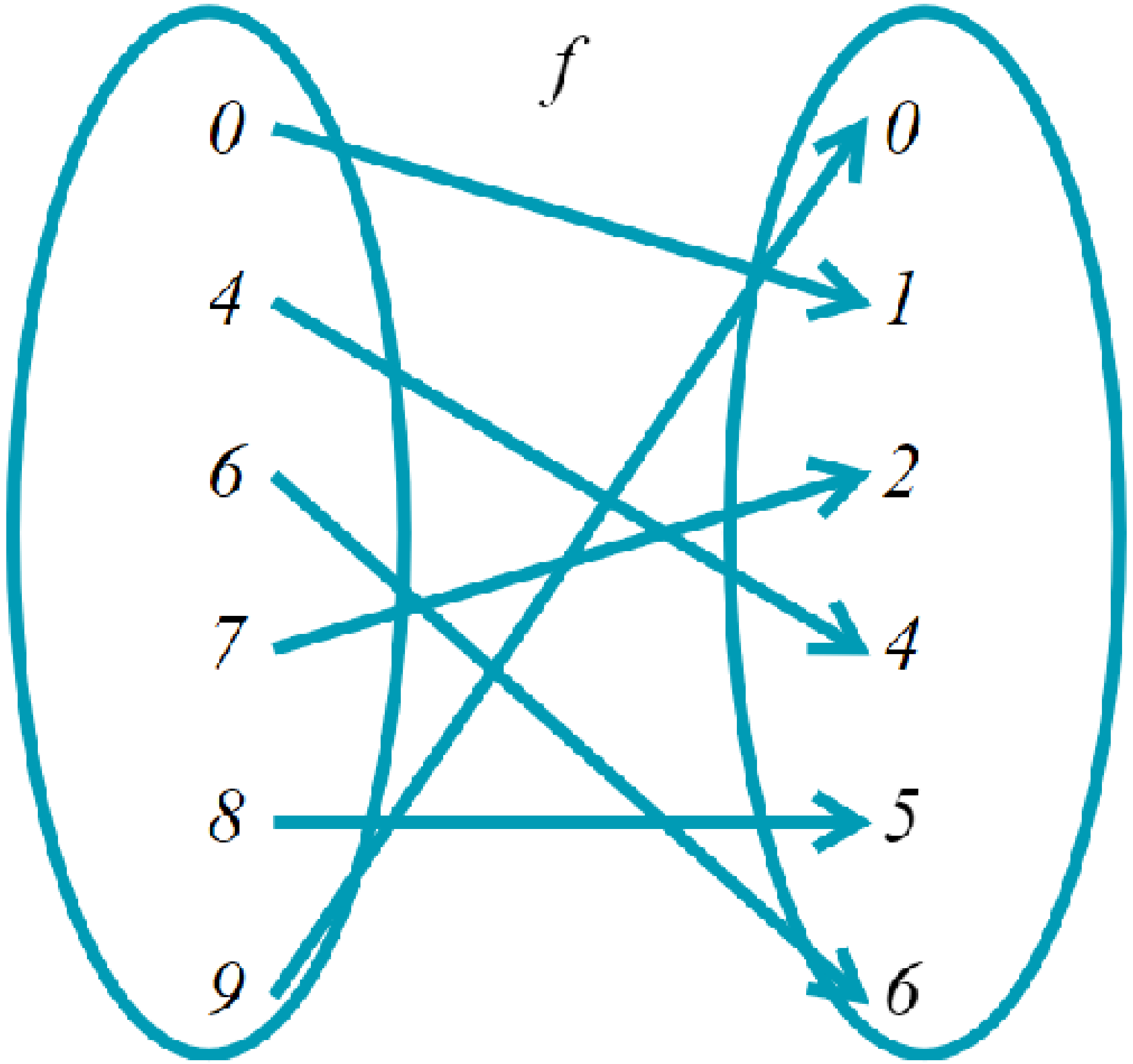


$$\boxed{\begin{array}{l}(1, 0)\\(7, 0)\\(8, 0)\end{array}}$$

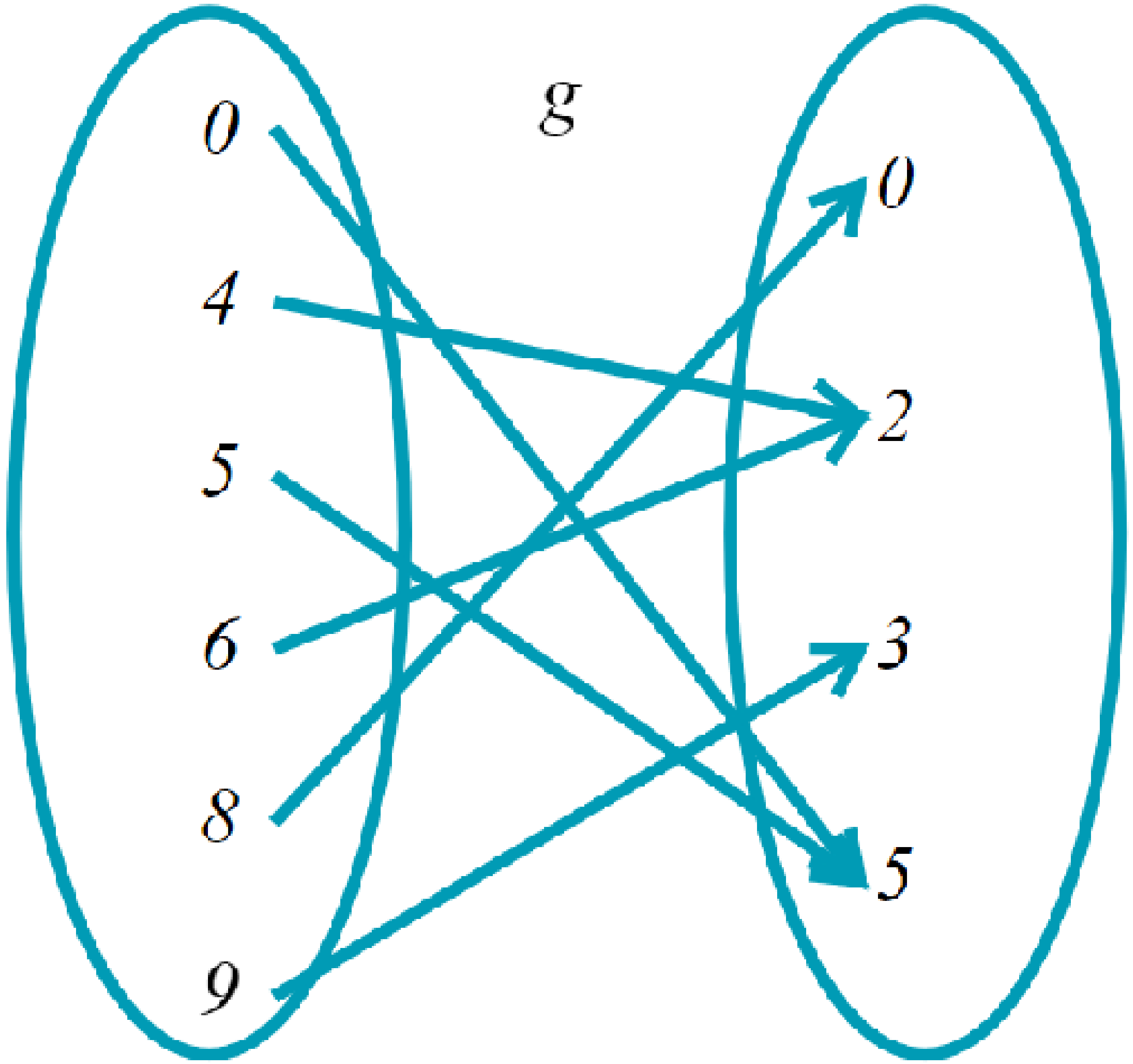
Find the domain and range of the composition $g \circ f$. Write your answers in set notation.

(a) Domain of $g \circ f$: 1, 7, 8

(b) Range of $g \circ f$: 0



Domain of f Range of f



Domain of g Range of g

(4, 2) (8, 5) $\{4, 6, 8, 9\}$ D

(6, 2) (9, 5) $\{2, 5\}$ R

For the real-valued functions $g(x) = \sqrt{5x+30}$ and $h(x) = x - 5$,

$(g \circ h)(x) =$ $g(h(x))$

$g(x-5) = \boxed{\sqrt{5(x-5)+30}}$

$= \sqrt{5x+5}$

Domain of $g \circ h$:

$5(x-5)+30 \geq 0$

$5x+5 \geq 0$

$5x \geq -5$

$x \geq -1$

$D = [-1, \infty)$

$$f(g)(x) = f(1/3x)$$

$$= \frac{1}{3(1/3x)}$$

$$= 3 \cdot \frac{1}{3x} = \frac{1}{x}$$

$$= \frac{1}{\frac{1}{x}} = x$$

(a) $f(x) = \dfrac{1}{3x}, \; x \neq 0$

$g(x) = \dfrac{1}{3x}, \; x \neq 0$

$f(g(x)) = \boxed{}$ x

$g(f(x)) = \boxed{}$ x

- ◯ f and g are inverses of each other
- ◯ f and g are *not* inverses of each other

(b) $f(x) = x + 2$

$g(x) = x + 2$

$f(g(x)) = \square$ $X+4$

$g(f(x)) = \square$ $X+4$

○ f and g are inverses of each other

~~○~~ f and g are *not* inverses of each other

$f(g(x)) = f(x+2)$

$= (x+2) + 2$

$= X + 4$

Consider the function $f(x) = \sqrt{1-x} + 2$ for the domain $(-\infty, 1]$.

find $\boxed{f^{-1}(x)}$

$$y = \sqrt{1-x} + 2 \quad \Longleftrightarrow \quad x = \sqrt{1-y} + 2$$

$$x - 2 = \sqrt{1-y}$$

$$(x-2)^2 = 1 - y$$

$$(x-2)^2 - 1 = -y$$

$$\boxed{-(x-2)^2 + 1} = y$$

The one-to-one function f is defined below.

$$f(x) = \sqrt[3]{x-10}+8$$

Find $f^{-1}(x)$, where f^{-1} is the inverse of f.

$$y = \sqrt[3]{x-10}+8$$

$$x = \sqrt[3]{y-10}+8$$

$$x-8 = \sqrt[3]{y-10}$$

$$(x-8)^3 = y-10$$

$$\boxed{(x-8)^3+10 = y}$$

The one-to-one function h is defined below.

$$h(x) = \frac{9x-8}{7x+4}$$

$$y = \frac{9x-8}{7x+4}$$

Find $h^{-1}(x)$, where h^{-1} is the inverse of h.

Also state the domain and range of h^{-1} in interval notation.

$$x = \frac{9y-8}{7y+4}$$

$h^{-1}(x) =$ $y = \frac{-4x-8}{7x-9}$

Domain of h^{-1} : $x \neq 9/7$

Range of h^{-1} : $x \neq -4/7$

$$x(7y+4) = 9y-8$$

$$7xy + 4x = 9y - 8$$

$$7xy - 9y = -4x - 8$$

$$\frac{y(7x-9)}{(7x-9)} = \frac{-4x-8}{(7x-9)}$$

$$-7x + 9 \neq 0$$

$$\frac{-7x}{-7} \neq \frac{-9}{-7}$$

$$x \neq 9/7$$