$$\int \frac{1}{x}\,dx = \ln|x|$$

$u = x+2$

$$\int \frac{1}{x+2}\,dx = \ln|x+2|$$

$du = dx$

$$\int \frac{2}{x-1} - \frac{1}{x+2}\,dx$$

$$\int \frac{2}{x-1} - \int \frac{1}{x+2}$$

$$2\ln|x-1| - \ln|x+2| + C$$

$$\frac{3}{8} + \frac{4}{7} = \frac{3(7) + 4(8)}{8(7)} = \frac{53}{56}$$

$$2x + 4 \quad - x + 1$$

$$\frac{2}{x-1} - \frac{1}{x+2} = \frac{2(x+2) - 1(x-1)}{(x-1)(x+2)}$$

$$x^2 - 1x + 2x - 2$$

$$= \boxed{\frac{x+5}{x^2 + x - 2}}$$

$$\int \frac{x+5}{x^2+x-2}\,dx = \int \frac{2}{x-1} - \frac{1}{x+2}\,dx$$

$$\frac{x+5}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$

$$x+5 = A(x+2) + B(x-1)$$

$$x+5 = A(x+2) + B(x-1)$$

$$x=1 \rightarrow \quad 6 = A(3) + \cancel{B(0)}$$

$$A = 2$$

$$x=-2 \rightarrow \quad 3 = \cancel{A(0)} + B(-3)$$

$$B = -1$$

$$\frac{2}{x-1} + \frac{-1}{x+2}$$

Case 1

$$Q(x) = (x+b_1)(x+b_2)\cdots(x+b_k)$$

$$\frac{R(x)}{Q(x)} = \frac{A_1}{(x+b_1)} + \frac{A_2}{(x+b_2)} + \cdots + \frac{A_k}{(x+b_k)}$$

$$\int \frac{x^2+2x-1}{2x^3+3x^2-2x}\,dx = \int \frac{x^2+2x-1}{x(2x-1)(x+2)}$$

$$x(2x^2+3x-2)$$

$$x(2x-1)(x+2)$$

$$2x^2+3x-2$$

$$2x^2+4x-1x-2$$

$$2x(x+2)-1(x+2)$$

$$(2x-1)(x+2)$$

$$\int \frac{x^2+2x-1}{x(2x-1)(x+2)} = \int \frac{A}{x} + \frac{B}{2x-1} + \frac{C}{x+2}$$

$$x^2+2x-1 = A(2x-1)(x+2) + B(x)(x+2) + C(x)(2x-1)$$

$$x=0 \rightarrow -1 = A(-1)(2) \rightarrow -1 = -2A \rightarrow A = 1/2$$

$$x=-2 \rightarrow -1 = C(-2)(-5) \rightarrow C = \frac{-1}{10}$$

$$x=1/2 \rightarrow \frac{1}{4} = B\left(\frac{1}{2}\right)\left(\frac{5}{2}\right) \rightarrow B = \frac{1}{5}$$

$$\int \frac{x^2+2x-1}{x(2x-1)(x+2)} = \int \frac{A}{x} + \frac{B}{2x-1} + \frac{C}{x+2}$$

$$\int \frac{1/2}{x} + \frac{1/5}{2x-1} + \frac{-1/10}{x+2}\,dx$$

$u = 2x-1$

$du = 2\,dx$

$$\frac{1}{2}\ln|x| + \frac{1}{5}\left(\frac{1}{2}\right)\ln|2x-1| - \frac{1}{10}\ln|x+2|$$

$$1/2\ln|x| + 1/10\ln|2x-1| - 1/10\ln|x+2| + C$$

$$\int \frac{x^3+x}{x-1}\,dx$$

$$\begin{array}{r}
x^2 + x + 2 \\
x-1 \enclose{longdiv}{x^3 + 0x^2 + x + 0} \\
-x^3 + x^2 \\
\hline
x^2 + x + 0 \\
-x^2 + x \\
\hline
2x + 0 \\
-2x + 2 \\
\hline
2
\end{array}$$

$$\int x^2 + x + 2 + \frac{2}{x-1}\,dx$$

$$\boxed{\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 2\ln|x-1| + C}$$

Case 2

$$\frac{A_1}{x+b_1} + \frac{A_2}{(x+b_1)^2} + \dots + \frac{A_r}{(x+b_1)^r}$$

ex

$$\frac{x^3-x+1}{x^2(x-1)^3} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2} + \frac{E}{(x-1)^3}$$

$$\int \frac{x^4-2x^2+4x+1}{x^3-x^2-x+1}\,dx = \int x+1+\frac{4x}{(x+1)(x-1)^2}$$

$$\int x+1+\frac{4x}{x^3-x^2-x+1}\,dx$$

$$x^2(x-1)-1(x-1)$$

$$(x^2-1)(x-1)$$

$$(x-1)(x+1)(x-1)$$

$$\frac{4x}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$4x = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$$

$$x=1 \rightarrow 4 = C(2) \rightarrow C=2$$

$$x=-1 \rightarrow -4 = A(4) \rightarrow A=-1$$

$$x=0 \rightarrow 0 = (-1)(1) + B(-1) + (2)(1)$$

$$0 = -1 - B + 2 \rightarrow B = 1$$

$$\int x+1+\frac{4x}{(x+1)(x-1)^2}$$

$u = x-1 \quad du = dx$

$$\int 2u^{-2} = \frac{-2}{u}$$

$$\int x+1+\frac{-1}{x+1}+\frac{1}{x-1}+\frac{2}{(x-1)^2}\,dx$$

$$\frac{1}{2}x^2+x-\ln|x+1|+\ln|x-1|-\frac{2}{x-1}+C$$

$$
\begin{array}{r}
\boxed{x + 1} \\
x^3 - x^2 - x + 1 \,\big)\, x^4 + 0x^3 - 2x^2 + 4x + 1 \\
- x^4 + x^3 + x^2 \mp x \\
\hline
x^3 - x^2 + 3x + 1 \\
- x^3 + x^2 + x \mp 1 \\
\hline
\boxed{4x}
\end{array}
$$

Case 3

$$\frac{x}{(x-2)(x^2+1)(x^2+4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{x^2+4}$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{2x^2 - x + 4}{x^3 + 4x}\,dx \quad \int \frac{A}{x} + \frac{Bx + C}{x^2 + 4}$$

$x(x^2+4)$

$$2x^2 - x + 4 = A(x^2+4) + (Bx + C)(x)$$

$$= Ax^2 + 4A + Bx^2 + Cx$$

$$2x^2 - x + 4 = (A+B)x^2 + Cx + 4A$$

$B = 1$ $\quad$ $C = -1$ $\quad$ $A = 1$

$$\int \frac{1}{x} + \boxed{\frac{x-1}{x^2+4}}\,dx$$

$$u = x^2+4$$

$$du = 2x\,dx$$

$$\frac{1}{2}du = x\,dx$$

$$\int \frac{1}{x} + \frac{x}{x^2+4} - \frac{1}{x^2+4}\,dx$$

$$\frac{1/2}{u}$$

$$\boxed{\ln|x| + \frac{1}{2}\ln|x^2+4| - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C}$$

$$\frac{1}{2}\ln|x^2+4|$$